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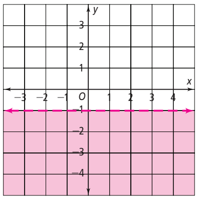
2-8 Puzzle: Identify the Inequality

Two-Variable Inequalities

Four different inequalities are described below. Write each inequality in the space provided and graph it on the grid.

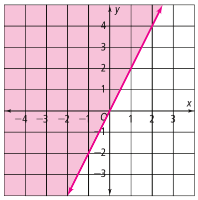
1. a. I am not in the second quadrant.
   b. One point on my border is $(-2, -1)$. It is not one of my solutions.
   c. My border is a horizontal line.

   $y < -1$



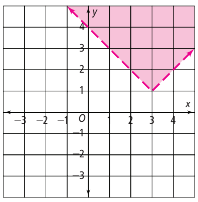
2. a. I am not in the fourth quadrant.
   b. $(0, 0)$ is one of my solutions and on my border.
   c. My border is the graph of a line.
   d. One point on my border is $(2, 4)$, which is also one of my solutions.

   $y \geq 2x$



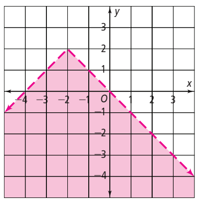
3. a. I am not in the third quadrant.
   b. My lowest point is $(3, 1)$.
   c. My border is the graph of an absolute value function.
   d. One point on my border is $(0, 4)$. It is not one of my solutions.

   $y > |x - 3| + 1$



4. a. I am not in the first quadrant.
   b. My highest point is $(-2, 2)$.
   c. My border is the graph of an absolute value function.
   d. $(0, 0)$ is on my border but is not one of my solutions.

   $y < -|x + 2| + 2$



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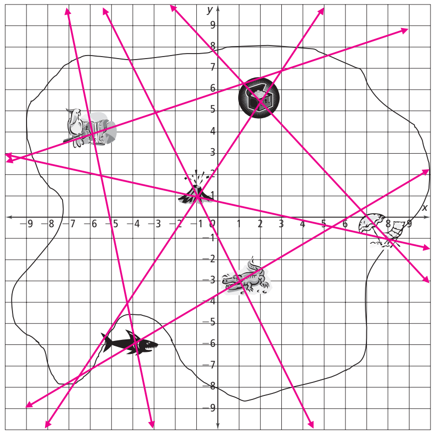
3-1 Puzzle: X Marks the Spot

Solving Systems Using Tables and Graphs

René "Axes" Descartes was a mathematical pirate. He subtracted poor crewmates, added new ones, multiplied his treasure, and divided the goods. Upon arriving at Treasure Island, he would give each of his crewmates a system of equations whose solution, he claimed, was the location of the treasure. Below is a map of the island and the systems he gave to each of his crewmates. Graph each of his crewmates' systems, and list the solution along with the location the crewmates were actually sent.



Galois, the dueler: $\begin{cases} y = -5x - 26 \\ 5y = 3x - 18 \end{cases}$
(−4, −6); shark

Poisson, the distributer: $\begin{cases} y = -\frac{13}{12}x + \frac{23}{3} \\ y = \frac{3}{2}x + \frac{5}{2} \end{cases}$
(2, 5.5); treasure chest

Fermat, the marginal: $\begin{cases} 2x + y = -1 \\ 3x - 2y = -5 \end{cases}$
(−1, 1); volcano

Lagrange, the multiplier: $\begin{cases} 2x = -y - 1 \\ 5y - 3x = -18 \end{cases}$
(1, −3); alligator

Pascal, the lamb: $\begin{cases} 2x + 9y = 7 \\ 13x + 12y = 92 \end{cases}$
(8, −1); beach

Poincaré, the conjecturer: $\begin{cases} 2y = -10x - 52 \\ 3y - x = 18 \end{cases}$
(−6, 4); waterfall

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TEACHER INSTRUCTIONS

3-2 Game: Risk and Reward

Solving Systems Algebraically

Provide the host with the following questions and answers for the given categories and point values.

Vocabulary

Points	Question: What is . . .	Answer
10	a system of equations?	when two or more equations are considered simultaneously
20	a solution of a system?	an ordered pair that is a solution of all equations in a system
30	an equivalent system?	systems that have the same solutions
40	a substitution method?	solving one equation for a variable, substituting it in the other equation, and solving for the remaining variable
50	an elimination method?	multiplying equations so like terms have opposite coefficients, adding the equations, and solving the result

Substitution

Points	System	Answer
10	$\begin{cases} y = 3x - 1 \\ 2x + y = 9 \end{cases}$	(2, 5)
20	$\begin{cases} 3y - 2x = 4 \\ x = 3 - y \end{cases}$	(1, 2)
30	$\begin{cases} 2x - y = -8 \\ 3x + 2y = -5 \end{cases}$	(−3, 2)
40	$\begin{cases} 3x - 6y = 30 \\ 2y = x + 6 \end{cases}$	No solution
50	$\begin{cases} 2a + 4b = -5 \\ 3a - 5b = 9 \end{cases}$	(0.5, −1.5)

Elimination

Points	System	Answer
10	$\begin{cases} x + 3y = -9 \\ -x + y = -7 \end{cases}$	(3, −4)
20	$\begin{cases} 2x + 6y = -12 \\ 3x - 2y = 4 \end{cases}$	(0, −2)
30	$\begin{cases} 2x + y = 2 \\ 3y = -6x + 6 \end{cases}$	Infinite solutions
40	$\begin{cases} 5x + 2y = 2 \\ 3x + 5y = 24 \end{cases}$	(−2, 6)
50	$\begin{cases} 0.7x + 2y = 1.61 \\ 2x - 0.3y = 0.39 \end{cases}$	(0.3, 0.7)

Application

Points	Question	Answer
10	The sum of two numbers is 12, and their difference is 6. What are the two numbers?	3, 9
20	Twice a boy's age is 9 more than his brother's. The average of their ages is 12. How old are they?	11, 13
30	A girl has $6 in 36 coins (all quarters and nickels). How many of each does she have?	21 quarters, 15 nickels
40	A girl's cell phone costs $10 per month plus 10¢ per minute. Her friend's costs $3 per month plus 15¢ per minute. For what amount of talking time will the cost of these plans be the same?	140 min
50	How many liters of a 60% salt mixture must be added to 2 liters of a 30% salt mixture to yield a 48% mixture?	3 l

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3-2 Game: Risk and Reward

Solving Systems Algebraically

This is a game for three students. One student is the host and the other two are players.

Your teacher will provide the host with a sheet of questions and answers. Use the scorecard below to record the score and keep track of the questions in each category that have been asked.

Rules

- Decide which player goes first. Players alternate turns.
- During a turn, a player selects a category from the list below. The host will ask the available question with the lowest point value. There are five questions in each of the four categories worth 10, 20, 30, 40, or 50 points, for a total of 20 questions.
- If the player answers correctly within a reasonable amount of time, the player earns the point value of that question.
- If the player answers incorrectly, the player loses the point value and the other player has the option of answering the question to earn or lose the point value.
- Play continues until all the questions have been used. The player with the highest point total wins.

Categories

- **Vocabulary** Provide a definition for a given vocabulary.
- **Substitution** Solve a system of equations using the substitution method.
- **Elimination** Solve a system of equations using the elimination method.
- **Application** Solve an application problem using a system of equations.

See Teacher Instructions page.

Points	Category	Player 1	Player 2
	Total		

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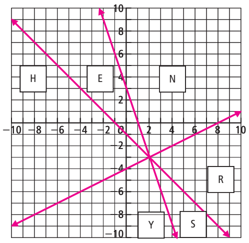
3-3 Puzzle: Shady Places

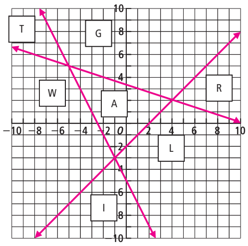
Systems of Inequalities

Graph the systems using the respective coordinate grids below.

$y = -x - 1$
$y = -3x + 3$
$2y = x - 8$

$y = x - 2$
$y = -2x - 5$
$3y = -x + 10$





Now find the letter(s) in the region corresponding to the solution of each system of inequalities. Then fill in these letters to find the beginning of a famous quote.

1. $\begin{cases} y > -x - 1 \\ 2y < x - 8 \end{cases}$ **R**
2. $\begin{cases} 3y > -x + 10 \\ y < -2x - 5 \end{cases}$ **T**
3. $\begin{cases} y > -x - 1 \\ y < -3x + 3 \end{cases}$ **E**
4. $\begin{cases} y < x - 2 \\ y < -2x - 5 \end{cases}$ **I**
5. $\begin{cases} y < -x - 1 \\ y > -3x + 3 \end{cases}$ **S**
6. $\begin{cases} 3y < -x + 10 \\ y > -2x - 5 \end{cases}$ **A, L**
7. $\begin{cases} y < -3x + 3 \\ 2y > x - 8 \end{cases}$ **H, E**
8. $\begin{cases} 3y > -x + 10 \\ y > -2x - 5 \end{cases}$ **G, R**
9. $\begin{cases} y < -x - 1 \\ 2y < x - 8 \end{cases}$ **Y, S**
10. $\begin{cases} 3y < -x + 10 \\ y > x - 2 \end{cases}$ **W, A**
11. $\begin{cases} y > -x - 1 \\ 2y > x - 8 \end{cases}$ **E, N**
12. $\begin{cases} 3y < -x + 10 \\ y > x - 2 \\ y > -2x - 5 \end{cases}$ **A**

T	H	E	G	R	A	S	S	I	S	A	L	W	A	Y	S	G	R	E	E	N	E	R
2	7	7	8	8	12	5	5	4	5	6	6	10	10	9	9	8	8	3	11	11	3	1

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3-4 Activity: Parking Program

Linear Programming

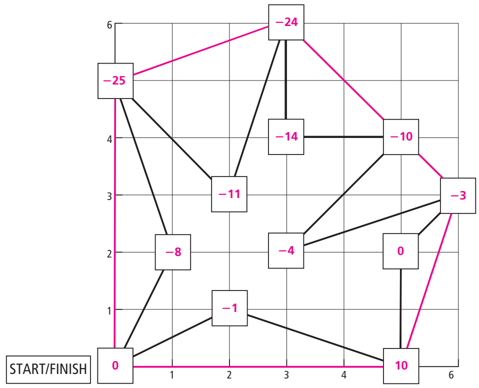
This activity can be done alone or in small groups.

Five streets border a park. The border can be modeled using the system:

$$\begin{cases} y \le -x + 9 \\ y \ge 3x - 15 \\ 3y \le x + 15 \\ x \ge 0 \\ y \ge 0 \end{cases}$$

A park ranger is building a bike path through the park. She is curious about the path's elevation at 12 distinct points. The following function models the elevation (meters).

$$f(x, y) = 2x - 5y$$



1. Use the system of linear inequalities above to graph the park boundaries. You do *not* need to actually shade when graphing.
   See graph.
2. Begin at START and follow the bike path to the end. Note the elevation at the 12 test points.
   See graph.
3. What and where are the maximum and minimum values of $f(x, y)$ in this region?
   maximum of 10 m at (5, 0); minimum of −25 m at (0, 5)

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3-5 Activity: Writing Systems

Systems With Three Variables

This activity is for groups of three students. Your teacher will determine whether you can use your calculator.

Activity

- Each group makes four systems of equations with three variables.

Rules

- All equations must have integer coefficients between −6 and 6.
- Not all coefficients can be 1 or −1.
- At most, one coefficient can be 0.
- No equation is a constant multiple of the other.

Phase 1

Each group member writes one equation according to the rules above. The group works together to solve the system that the three equations form.

Check students' work.

Phase 2

Each group member writes one equation according to the rules above. Each member works separately to solve the system that the three equations form.

Check students' work.

Phase 3

The group members write one equation according to the rules above. Then the group finds two more equations to make a system of three equations that has an infinite number of solutions. The two additional equations may violate one of the rules above.

Check students' work.

Phase 4

Two group members each write one equation according to the rules above. Then the group finds a third equation to make a system of three equations that has no solution.

Check students' work.

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3-6 Game: Matrix Mania

Solving Systems Using Matrices

This is a game for two teams of two students.

Game Play

There are two rounds in this game. Play each round as follows:

- **Select:** At the beginning of the round each team selects systems, as determined below, that the other team needs to solve.
- **Exchange:** Teams exchange their systems.
- **Solve:** Teams find solutions for each system.
- **Verify:** At end of the game, teams check their opponent's solutions. Your teacher will provide the correct solutions.

Rules

- Find systems in the matrices below.
- Determine systems by a matrix element. The matrix element corresponds to the first term in the system. For example, a_{23} from matrix A below corresponds to the following system.

$$\begin{cases} 3x + 0y = -6 \\ -1x - 11y = 2 \end{cases}$$

- The team that scores the most points wins.

Scoring

- Each correct solution is worth 2 points.
- The team that finishes a round first earns 1 point.

Round 1

Each team selects four systems of two equations with two unknowns. There are 16 possible systems starting in the first four rows and first four columns.

$$A = \begin{bmatrix} 1 & -2 & 4 & 10 & 2 & 0 \\ 2 & 1 & 3 & 0 & -6 & 30 \\ 3 & -4 & -1 & -11 & 2 & 78 \\ -1 & 1 & -1 & -1 & 12 & 13 \\ -2 & 1 & 2 & 5 & -13 & -23 \end{bmatrix}$$

a_{11}: (2, −1); a_{12}: (−3, 1); a_{13}: (−2, 1); a_{14}: (1, −5); a_{21}: (1,1); a_{22}: (3, −1); a_{23}: (−2, 0); a_{24}: (−8, −5); a_{31}: (5, 4); a_{32}: (2, 3); a_{33}: (−13, 1); a_{34}: (−7, 0.5); a_{41}: (−3, −4); a_{42}: (1, 2); a_{43}: (−14, 12); a_{44}: (−1, 1)

Round 2

Each team selects two systems of three equations with three unknowns. There are six possible systems starting in the first three rows and first two columns.

$$B = \begin{bmatrix} 1 & 2 & -1 & 2 & -17 \\ 2 & 0 & 1 & 3 & -7 \\ -1 & -1 & 2 & 0 & 5 \\ -2 & -2 & 3 & 3 & -1 \\ -2 & 1 & 1 & 3 & 4 \end{bmatrix}$$

b_{11}: (1, 1, 1); b_{12}: (−7, −1, −2); b_{21}: (3, −9, −3); b_{22}: (−1, 2, −3); b_{31}: (−4, −2, −3); b_{32}: (5, 5, −2)