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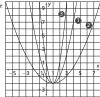
Activity: Sharing Vertices

Quadratic Functions and Transformations

This is an activity that can be done alone or in groups of two or three students. Your teacher may discuss each group's results once everyone has finished.

- · Twelve different quadratic functions are given below.
- Your job is to find 4 sets of 3 functions that have the same vertex
- Fill in your results at the bottom of the page and then check each set using a graphing calculator.

By graphing the following three functions, you can see



3. $y = -3x^2 + 12x - 9$

6. $y = \frac{1}{4}x^2 + \frac{3}{2}x + \frac{17}{4}$

9. $y = -\frac{1}{4}x^2 + x + 2$

12. $y = 3x^2 - 12x + 9$

Example Set

Vertex: (1, -2) $y = x^2 - 2x - 1$ $y = \frac{1}{4}x^2 - \frac{1}{2}x - \frac{7}{4}$ $y = 3x^2 - 6x + 1$

4. $y = -\frac{1}{4}x^2 - \frac{3}{2}x - \frac{17}{4}$

7. $y = \frac{1}{4}x^2 - x - 2$



$$5. \ \ y = -3x^2 - 18x - 29$$

8.
$$y = -x^2 + 4x - 1$$

$$-11 11. y = 3x^2 + 18x + 29$$

10.
$$y = -x^2 - 6x - 11$$
 11. $y = 3x^2 + 18x + 1$

Set A
 Set B
 Set C
 Set D

 Vertex:
$$(-3, 2)$$
 Vertex: $(-3, -2)$
 Vertex: $(2, 3)$
 Vertex: $(2, -3)$

 1. $y = x^2 + 6x + 11$
 4. $y = -\frac{1}{4}x^2 - \frac{3}{2}x - \frac{17}{4}$
 3. $y = -3x^2 + 12x - 9$
 2. $y = x^2 - 4x + 1$

6.
$$y = \frac{1}{4}x^2 + \frac{3}{2}x + \frac{17}{4}$$
 5. $y = -3x^2 - 18x - 29$ 8. $y = -x^2 + 4x - 1$ 7. $y = \frac{1}{4}x^2 - x - 2$

11.
$$y = 3x^2 + 18x + 29$$
 10. $y = -x^2 - 6x - 11$ 9. $y = -\frac{1}{4}x^2 + x + 2$ 12. $y = 3x^2 - 12x + 9$

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4-2

Activity: Decoding Device

Standard Form of a Quadratic Function

This is an activity that can be done alone or in groups of two or three students. Your teacher may wish to discuss each group's results once everyone has finished.

- Using standard form $y = ax^2 + bx + c$ and vertex form $y = a(x h)^2 + k$, identify the ordered pairs in Exercises 1-14 below.
- · Simplify fractions for the correct numerator (num.) and denominator (den.).
- Find the letter corresponding to the ordered f 3 f H f X f B f U f M f H f Y f B f X f Qpairs using the table at the right. For example, 4 if your result is $y = 2x^2 + 7x + 9$, then $(a, c) = (2, 9) \rightarrow G$.
- $\bullet \;\;$ Fill in the letters to find the name of a cryptographic device invented by an ancient 7 N F W L Y U Z Q U L Greek scholar.



P	0	L	Υ	В	1	U	S	
1	2	3	4	5	6	7	8	

S Q U A R E 14

Convert to Standard Form

1.
$$y = 5(x+3)^2 - 36$$
; (a, c)
 $y = 5x^2 + 30x + 9$; (5, 9); P

2.
$$y = (x + \frac{3}{2})^2 - \frac{1}{4}$$
; (a,b)
 $y = x^2 + 3x + 2$; $(1,3)$; 0

3.
$$y = x(x+3) + 6x$$
; (b, c)
 $y = x^2 + 9x + 0$; $(9, 0)$; L

4.
$$y = 3[x(x-5) + 2]; (a, c)$$

 $y = 3x^2 - 15x + 6; (3, 6); Y$

5.
$$y = 2x\left(\frac{1}{2}x + \frac{3}{4}\right) - 2 \cdot 3$$
; (mum. b ; den. b) 6. $y = 2x\left(x + 3 - \frac{4}{5}x\right) - 5x$; (den. a ; num. a) $y = x^2 - \frac{3}{2}x + 6$; (3, 2); B $y = \frac{2}{5}x^2 + x$; (5, 2); I

7. $y = 3\left[x(x - 5) + \frac{2}{7}\right]$; (num. c ; den. c)

8. $y = \frac{1}{8}(3x + 2)(3x + 4)$; (num. a ; den. a) $y = 3x^2 - 15x + \frac{6}{5}$; (6, 7); U $y = \frac{9}{6}x^2 + \frac{9}{6}x + 1$; (9, 8); S

$$y = x^2 - \frac{3}{2}x + 6; (3, 2); B$$

$$y = \frac{2}{5}x^2 + x; (5, 2); 1$$
7. $y = 3\left[x(x - 5) + \frac{2}{7}\right]; (\text{num. } c; \text{den. } c)$

$$y = 3x^2 - 15x + \frac{6}{7}; (6, 7); U$$

$$y = \frac{9}{5}x^2 + \frac{9}{6}x + 1; (9, 8); 5$$

Convert to Vertex Form

9.
$$y = x^2 - 16x + 66$$
; (h, k)
 $y = (x - 8)^2 + 2$; $(8, 2)$; S
11. $y = 2x^2 - 28x + 106$; (h, k)

10.
$$y = x^2 - 2x + 10$$
; (h, k)
 $y = (x - 1)^2 + 9$; $(1, 9)$; Q
12. $y = -3x^2 + 1$; (h, k)

11.
$$y = 2x^2 - 28x + 106; (h, k)$$

 $y = 2(x - 7)^2 + 8; (7, 8); U$

12.
$$y = -3x^2 + 1$$
; (n, k)
 $y = -3(x - 0)^2 + 1$; $(0, 1)$; A
14. $y = -\frac{1}{5}x^2 + \frac{8}{5}x + \frac{24}{5}$; (h, k)

13.
$$y = \frac{1}{3}x^2 - \frac{10}{3}x + \frac{40}{3}$$
; (h, k)
 $y = \frac{1}{3}(x - 5)^2 + 5$; $(5, 5)$; R

14.
$$y = -\frac{1}{5}x^2 + \frac{8}{5}x + \frac{24}{5}$$
; (h, k)
 $y = -\frac{1}{5}(x - 4)^2 + 8$; (4, 8); E

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4-3

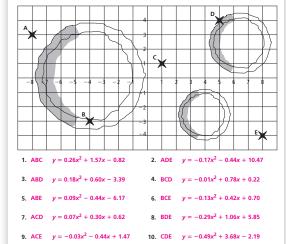
Activity: Flight Path

Modeling With Quadratic Functions

This is an activity for groups of two to four students. You will need a graphing calculator.

A spacecraft is going to a distant planet to collect and transmit data to the scientists at mission control. The figure below represents an overhead view of a region of interest on the surface of the planet. The five stars in the figure mark specific

The spacecraft will take a flight path that must be parabolic and passes over exactly three of the targeted areas. There are ten different flight paths that pass over three of the points. Find the ten equations that model these flight paths. Round your answers to the nearest hundredth.



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4-4

Game: Factor This!

Factoring Quadratic Expressions

Game Play

This is a game for four students senarated into two teams. Each team begins by secretly developing eight factorable polynomial expressions and writing them in standard form at the bottom of the page. Teams exchange pages and attempt to factor the opponent's functions. Then they write the factored form next to each

Rules

- You must have two of each of the following types of polynomials.

 A. the binomial factors have all integer coefficients and terms, and the leading coefficient equals 1 **Example:** $(x+2)(x-7)=x^2-5x-14$ **B.** the binomial factors have all integer coefficients and terms, and one of the factors has
- leading coefficient *not* equal to 1 **Example:** $(2x + 1)(x 3) = 2x^2 5x 3$
- C. the binomial factors have all integer coefficients and terms, and both of the factors have leading coefficients *not* equal to 1 **Example:** $(3x-2)(2x-9) = 6x^2 31x + 18$ D. the binomial factors have fractional coefficients and terms, and one of the factors has leading coefficient equal to 1 and all numerators equal 1

Example:
$$\left(x - \frac{1}{7}\right)\left(\frac{1}{3}x + \frac{1}{5}\right) = \frac{1}{3}x^2 + \frac{16}{105}x - \frac{1}{35}$$

For each factor, all integer coefficients and terms must be between -10 and 10, and fractions must have numerators equal to 1 and denominators between -7 and 7.

The team that earns the most points wins.

Scoring

Correctly factoring:

- Type A -1 point each
- Type B 2 points each
- Type C 3 points each
- Type D 4 points each

function

• Correctly identifying the opponent's

function as unfactorable - 5 points

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Puzzle: Dominoes

Quadratic Equations

Complete this puzzle with a partner

Materials

- Twenty-four small index cards
- · Transparent tape

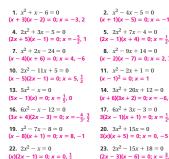
Goal

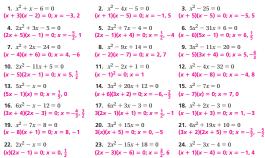
- · Find the solutions to the 24 quadratic equations given below
- Make a card for each equation, writing the two solutions and the equation number, as shown in the example below.
- Tape the index cards together end to end to form the longest string possible. There is notes cause sogener ratio or elia to form the nongest string possible. You can only tape together two cards when they share at least one solution (when the ends match). $\frac{\text{Longest possible string uses all twenty-four cards.}}{\text{Answers may vary. Sample:}} \\ + x - 6 = 0 \rightarrow (x + 3)(x - 2) = 0 \rightarrow x = -3 \text{ or } x = 2, \\ 7, 14, 21, 4, 11. \\ 7, 14, 21, 4, 11.$

 $x^2 + x - 6 = 0 \rightarrow (x + 3)(x - 2) = 0 \rightarrow x = -3 \text{ or } x = 2$

2







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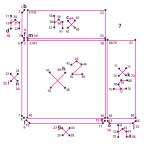
4-6

Puzzle: Dot Mania

Completing the Square

This puzzle can be done individually or with a partner. You will need three different-colored pencils or pens.

There are fifteen quadratic expressions labeled a, through o, below. Find the value that needs to be added to make an expression that is a perfect square. Then replace each letter with its perfect-square value in the dot-to-dot puzzl



Next connect the following sets of numbers with a segment. Stop your segment after completing each set and then move to the next set and start a new segment. Use different-colored pencils or pens as indicated.

Color 1	Color 2	Color 3
1-4	37-41	51-55
5-8	42-46	56-61
9-12	47-48	62-63
13-16	49-50	64-65
17-22		66-70
23-24		71-72
25-26		73-74
27-28		75-80
29-30		
31-36		

o. $\frac{2}{7}x^2 - 4x$ **14**

a. $x^2 + 12x$ **b.** $x^2 - 2x$ c. $x^2 + 16x$ **d.** $2x^2 - 12x$ **e.** $3x^2 + 12x$ f. $\frac{1}{4}x^2 - 2x$ g. $\frac{1}{3}x^2 + 6x$ i. $x^2 - 10x$ h. $x^2 - 14x$ j. $\frac{3}{5}x^2 + 6x$ **k.** $2x^2 - 24x$ 1. $x^2 + 6x$

n. $\frac{3}{2}x^2 + 12x$ 24 When the segments are connected, you will have a diagram and can answer the following questions.

- 1. What is the area of the region that "completes the square"? h²
- 2. What is the connection between the figure you made and the algebraic process of completing the square Check students' work.

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The Quadratic Formula

Provide the host with the following questions and answers for the given categories and point values

Vocabulary

Points	Question	Answer				
10	quadratic equation	an equation that can be written in the form $ax^2 + bx + c = 0$				
20	vertex	the lowest (or highest) point on the graph of a quadratic function, or the intersection of a parabola with its axis of symmetry				
30	parabola	the graph of a quadratic function				
40	discriminant	for a quadratic function $f(x) = ax^2 + bx + c$, the value $b^2 - 4ac$				
50	quadratic formula	$x=-b\pm\sqrt{b^2-4ac}$, the solutions of the quadratic function $f(x)=ax^2+bx+c$				

With

Points	Question	Answer
10	$x^2 + 2x - 8 = 0$	-4, 2
20	$9x^2 + 16 = -24x$	$-\frac{4}{3}$
30	$2x^2 + 4x - 6 = 0$	-3, 1
40	$x^2 - 10x + 18 = 0$	$5\pm\sqrt{7}$
50	$4x^2 + 4x - 1 = 0$	$\frac{-1\pm\sqrt{2}}{2}$

Number/Type

Points	Question	Answer
10	$4x^2 + 4x - 3 = 0$	2 real
20	$25x^2 = 30x - 9$	1 real
30	$4x^2 - 7x + 5 = 0$	2 complex
40	$3x^2 = 5x - 2$	2 real
50	$\frac{1}{2}x^2 + \frac{4}{3}x + 1 = 0$	2 complex

Game: Risk and Reward

Points	Question	Answer
10	quadratic equation	an equation that can be written in the form $ax^2 + bx + c = 0$
20 vertex		the lowest (or highest) point on the graph of a quadratic function, or the intersection of a parabola with its axis of symmetry
30	parabola	the graph of a quadratic function
40	discriminant	for a quadratic function $f(x) = ax^2 + bx + c$, the value $b^2 - 4ac$
50	quadratic formula	$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$, the solutions of the quadratic function $f(x)=ax^2+bx+c$
	20 30 40	10 quadratic equation 20 vertex 30 parabola 40 discriminant

Without

	Points	Question	Answer
1	10	$x^2 - 4x - 21 = 0$	-3, 7
	20	$4x^2 - 36x + 81 = 0$	9 2
1	30	$6x^2 + 11x - 10 = 0$	$-\frac{5}{2}$, $\frac{2}{3}$
	40	$x^2 + 6x - 11 = 0$	$-3\pm2\sqrt{5}$
1	50	$6x^2 + 2x = 1$	-1 ± √7

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4-7

m. $\frac{2}{3}x^2 - 12x$ 54

Game: Risk and Reward

The Quadratic Formula

This is a game for three students. One student is the host and the other two are players.

Your teacher will provide the host with a sheet of questions and answers. Use the scorecard below to record the score and keep track of the questions in each category that have been asked.

Rules

- Decide which player goes first.
 Players alternate turns.
- During a turn, a player selects: category from the list below. The host will ask the first available question with the lowest point value. There are five questions in each of the four categories worth 10, 20, 30, 40, or 50 points for a total of 20 questions
- . If the player answers correctly within a reasonable amount of time, the player earns the point value of that question.
- If the player answers incorrectly, the player loses the point value, and the other player has the option to answer the question to earn or lose the point
- Play continues until all the questions have been used. The player with the highest point total wins.

Categories

- · Vocabulary: Provide a definition for a given vocabulary.
- With: Solve the quadratic equation using the quadratic formula.
- Without: Solve the quadratic equation without using the quadratic formula.
- Number/Type: State the number of solutions and the type (real or complex).

Points	Category	Player 1	Player 2
10	Vocabulary		
20	Vocabulary		
30	Vocabulary		
40	Vocabulary		
50	Vocabulary		
10	With		
20	With		
30	With		
40	With		
50	With		
10	Without		
20	Without		
30	Without		
40	Without		
50	Without		
10	Number/Type		
20	Number/Type		
30	Number/Type		
40	Number/Type		
50	Number/Type		
	Total		

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4-8

Puzzle: Complex Cross numbers

Complex Numbers

Simplify the clues to complete the cross number puzzle. Write answers in the form a+bi. If either a or b equals zero, do not enter a "0" in the puzzle. Simply omit it. Write each term, sign (if negative), and operation (addition or subtraction) in a

Example: 2 + 3i is filled in as 2 + 3i, and -2 - 3i is filled in as -2 - 3i

8	+	i		² 6	+	42 <i>i</i>		⁷ –		13	9	-	3 <i>i</i>
+			³ 5	+	35 <i>i</i>			10		15 14	_	32 <i>i</i>	
6i		⁴ 2	+	4i		8 —		+		+	18 <i>i</i>		16_
	5 3	-	6i			9 4	-	i		8i		17 _	4i
7	+	12 <i>i</i>			10 24	_	2 <i>i</i>					3	
-	5i		11 _	1	+	3i				19	¹⁸ 26	+	8 <i>i</i>
11 <i>i</i>			i		10 <i>i</i>				20	28	_	i	
	12 _	21 <i>i</i>				21 _	5	+	5i		2 <i>i</i>		

ACROSS

- 1. (-6 + 7i) + (14 6i)2. (6 + 2i)(3 + 6i) 6 + 42i
- 3. (4+3i)(5+5i) 5 + 35i
- 4. (8+2i)-(6-2i) 2 + 4i
- 5. $\frac{9-3i}{1+i}$ 3 6i
- 6. (14+5i)+(-7+7i)
- 7 + 12i9. (16 + 11i) (12 + 12i)
- 10. 5 (-19 + 2i) 24 2i 11. -5 + (4 + 3i) -1 + 3i
- 12. (-3i)(7i)(-i) -21i

- **13.** negative signed solution **6.** $\frac{29+3i}{1+2i}$ **7 11** of $x^2 + 18x + 90 = 0$ -9 - 3*i*
- **15.** (4 2*i*)(6 5*i*) **14 32***i* 17. $-\sqrt{-16}$ -4*i*
- 10. $(5+i)^2$ 24 + 10*i* **19.** (6+i)(-4+2i) **-26 + 8***i* **11.** $-\sqrt{-1}$ **-***i*
- 20. (16+4i)-(44+5i) -28-i21. (1+2i)(1+3i)-5+5i13. (-4-2i)(2-3i) -14+8i14. -3(-3+6i) 9 18i
- **16.** $(i^2)(4i)$ **-4***i*
- 1. positive signed solution of $x^2 16x + 100 = 0$ 8 + 6*i* 17. $-2\left(\frac{3}{2} - \frac{1}{2}i\right)$ -3 + i
- 3. (8+12i)-(3+6i)
- 5. 10 (7 5i) 3 + 5i
- **18.** -2(-13+i) **26 2**i19. (7i)(4i) -28

7. (2+5i) - (12+4i) -10+i8. (-7+3i) + (3-6i) -4-3i

20. (2-2i)-(2+3i) -5*i*

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1-9

Game: Global Exploration

Quadratic Systems

The team assigned to check answers will need the following solutions, which

North America

Europe

Central (6 pts)	$y > (x + 2)^2 - 3$	West (5 pts)	$y < x^2 - 3$
	$y < -(x+1)^2 + 2$		$y > -(x-1)^2$
North	$y > (x + 2)^2 - 3$	North	$y > x^2 - 3$
	$y > -(x+1)^2 + 2$		$y > -(x-1)^2$
South (1 pt)	$y < (x + 2)^2 - 3$	South (1 pt)	$y < x^2 - 3$
	$y < -(x+1)^2 + 2$		$y < -(x-1)^2$

$y < (x + 2)^2 - 3$ East (1 pt) $y > -(x+1)^2 + 2$

Central (1 pt) $y > x^2 - 3$ $y<-(x-1)^2$

South America Asia

Africa

North (5 pts)	$y > (x + 1)^2$	
	$y > -x^2 + 4$	Australia
Central	$y > (x + 1)^2$	North (4 pt
	$y < -x^2 + 4$	
South (1 pt)	$y < (x + 1)^2$	Fact

$y<-x^2+4$ $y<(x+1)^2$ $y > -x^2 + 4$

$y < (x-3)^2 - 2$ West (1 pt) $y > -(x-2)^2 + 1$

$y > -(x - 3)^2 + 4$ West (1 pt) y < -2x + 4

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4-9

Game: Global Exploration

Quadratic Systems

This game is for three (or more) students separated into three teams. One team checks the answers, while the other two teams compete.

Game Play

North (5 pts)

- The game board sheet shows six separate continents placed on grids. Curves divide each continent into three or four regions (for example, Central, North, South, and East) with different point values as shown below.
- · Determine which team goes first.
- · Begin your turn by identifying the region you wish to explore.
- · Explore a territory by correctly writing the system of inequalities that includes the territory as its solution. One solution per continent has been done for you.
- The checking team and opposing team should also try to identify the system
- The checking team compares your answer to the provided solutions. If correct, the territory is yours. If wrong, the correct system is not revealed and the territory remains available. In either case, your turn is over.
- Teams take turns until all territories have been explored—most points win!

North America Europe Central (6 pts) West (5 pts) $y > (x+2)^2 - 3$ $y > x^2 - 3$ North North $y > -(x-1)^2$ $y > -(x+1)^2 + 2$ South (1 pt) South (1 pt) East (1 pt) Central (1 pt) South America Asia

South (6 pts) Central (4 pts) $y > (x-3)^2 - 2$ $y > -(x-2)^2 + 1$ $y > (x+1)^2 - 3$ North y > 0.5x + 1South (1 pt) Central (1 pt) West (1 pt) Africa

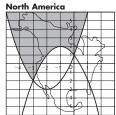
$y > (x+1)^2$ $y < -x^2 + 4$ North (4 pts) South (1 pt) $y < -(x-3)^2 + 4$ y > -2x + 4West (1 pt) West (1 pt)

Australia

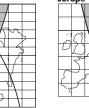
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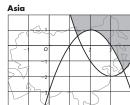
Game: Global Exploration

Quadratic Systems

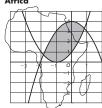


South America





Africa



Australia

