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5-1 Activity: Tables and Trends

Polynomial Functions

Work in small groups and use tables to see how polynomial functions behave. These are cubic polynomial functions with leading coefficient 1. Each of these graphs crosses the x-axis three different times between -5 and 5.

Graph A: Complete the table for $P(x) = x^3 - 2x^2 - 11x + 12$.

x	−5	−4	−3	−2	−1	0	1	2	3	4	5
$P(x)$	−108	−40	0	18	20	12	0	−10	−12	0	32

Describe the trend of the graph including end behavior. up, down, up

Graph B: Complete the table for $P(x) = -x^3 + 4x^2 - x - 4$.

x	−5	−4	−3	−2	−1	0	1	2	3	4	5
$P(x)$	226	128	62	22	2	−4	−2	2	2	−8	−34

Describe the trend of the graph including end behavior. down, up, down

Graph C: Complete the table for $P(x) = 3x^3 - 6x^2 - 16x + 32$.

x	−5	−4	−3	−2	−1	0	1	2	3	4	5
$P(x)$	−413	−192	−55	16	39	32	13	0	11	64	177

Describe the trend of the graph including end behavior. up, down, up

Graph D: Complete the table for $P(x) = x^3 - 2x^2 - 5x + 6$.

x	−5	−4	−3	−2	−1	0	1	2	3	4	5
$P(x)$	−144	−70	−24	0	20	6	0	−4	0	18	56

Describe the trend of the graph including end behavior. up, down, up

As a group, explain how you know the graph does not turn again outside the range -5 to 5.
Answers may vary. Sample: As absolute values of x increase, x^3 dominates the behavior of the function.

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5-2 Puzzle: Made in the Shade

Polynomials, Linear Factors, and Zeros

Find the zeros of each polynomial below. For each corresponding row, shade in each number that is a zero. The illustration made from shading the squares suggests the answer to the riddle below.

A. $P(x) = x(x^2 - 1)$
−1, 0, 1

B. $P(x) = x(x + 2)(x + 1)(x^2 + 2x - 3)$
−3, −2, −1, 0, 1

C. $P(x) = x(x + 4)(x + 3)(x + 1)(x - 1)$
−4, −3, −1, 0, 1

D. $P(x) = x(x^2 - 25)(x^2 + 4x + 3)$
−5, −3, −1, 0, 5

E. $P(x) = (x^2 + x - 20)(x + 2)(x^2 + 4x + 3)$
−5, −3, −2, −1, 4

F. $P(x) = (x^2 - 9)(x^2 - 25)$
−5, −3, 3, 5

G. $P(x) = (x^2 + 9x + 20)(x^2 - 5x + 6)(x - 5)$
−5, −4, 2, 3, 5

H. $P(x) = (x^2 - 5x + 6)(x^2 - 9x + 20)$
2, 3, 4, 5

I. $P(x) = x^2 - 6x + 9$
3

J. $P(x) = (x^2 - 4x + 4)(x^2 - 4x + 4)$
2

K. $P(x) = x(x^2 - 2x + 1)(x - 2)$
0, 1, 2

A	−5	−4	−3	−2	**−1**	**0**	**1**	2	3	4	5
B	5	−5	−4	**−3**	**−2**	**−1**	**0**	**1**	2	3	4
C	4	5	−5	**−4**	**−3**	−2	**−1**	**0**	**1**	2	3
D	3	4	**5**	**−5**	−4	**−3**	−2	**−1**	**0**	1	2
E	2	3	**4**	5	**−5**	−4	**−3**	**−2**	**−1**	0	1
F	1	2	**3**	4	**5**	**−5**	−4	**−3**	−2	−1	0
G	0	1	**2**	**3**	4	**5**	**−5**	**−4**	−3	−2	−1
H	−1	0	1	**2**	**3**	**4**	**5**	−5	−4	−3	−2
I	−2	−1	0	1	2	**3**	4	5	−5	−4	−3
J	−3	−2	−1	0	1	**2**	3	4	5	−5	−4
K	−4	−3	−2	−1	**0**	**1**	**2**	3	4	5	−5

Riddle: This grows above the ground, but the solutions to the polynomials above lie beneath. And as it grows, it provides shade to those underneath. What is it? a tree

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TEACHER INSTRUCTIONS

5-3 Game: Discovering Your Roots

Solving Polynomial Equations

Provide the host with the following equations and their solutions.

	Equation	Solution
1.	$(x^2 - 9)(x^2 + 6x + 9) = 0$	$-3, 3$
2.	$(x^2 - 1)(x^2 + 16) = 0$	$\pm 1, \pm 4i$
3.	$(x^2 + 9)(2x + 9) = 0$	$\pm 3i, -\frac{9}{2}$
4.	$(x^2 + 9)(x^2 + 4) = 0$	$\pm 3i, \pm 2i$
5.	$(x^2 + 25)(x^2 - 4)(x + 4) = 0$	$-4, \pm 5i, \pm 2$
6.	$(x^2 + 100)(x^2 - 100) = 0$	$\pm 10i, \pm 10$
7.	$(x^2 + 49)(3x - 5) = 0$	$\pm 7i, \frac{5}{3}$
8.	$(x^2 - 81)(3x^2 - 27) = 0$	$\pm 9, \pm 3$
9.	$(x^2 - 5x + 6)(3x^2 + 27) = 0$	$3, 2, \pm 3i$
10.	$(x^2 - 6x + 9)(9x^2 - 81) = 0$	± 3
11.	$(x^2 + 10x + 25)(3x^2 + 27) = 0$	$-5, \pm 3i$
12.	$(x^2 + 1)^2(2x + 3)^2 = 0$	$\pm i, -\frac{3}{2}$
13.	$(x^2 - 2)(2x - 3)^2 = 0$	$\pm\sqrt{2}, \frac{3}{2}$
14.	$(x^2 + 2)(2x - 4)^2 = 0$	$\pm\sqrt{2}i, 2$
15.	$(x^2 + 2x)(2x^2 - 16) = 0$	$-2, 0, \pm 2\sqrt{2}$
16.	$(x^2 + 3x)(3x^2 - 24) = 0$	$-3, 0, \pm 2\sqrt{2}$
17.	$(x^2 - 6x + 9)(x^2 - 10x + 25) = 0$	$3, 5$
18.	$(x^2 + 2x + 1)(x^2 + 10x + 25) = 0$	$-1, -5$

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5-3 Game: Discovering Your Roots

Solving Polynomial Equations

This is a game for three students—a host and two players. Players alternate turns. The host will ask a player to solve an equation below in a reasonable amount of time. Players are to write all solutions to the given equation. Players earn 5 points for a correct answer and lose 3 points for an incorrect or incomplete answer. See Teacher Instructions page.

	Equation	Player 1	Player 2
1.	$(x^2 - 9)(x^2 + 6x + 9) = 0$		
2.	$(x^2 - 1)(x^2 + 16) = 0$		
3.	$(x^2 + 9)(2x + 9) = 0$		
4.	$(x^2 + 9)(x^2 + 4) = 0$		
5.	$(x^2 + 25)(x^2 - 4)(x + 4) = 0$		
6.	$(x^2 + 100)(x^2 - 100) = 0$		
7.	$(x^2 + 49)(3x - 5) = 0$		
8.	$(x^2 - 81)(3x^2 - 27) = 0$		
9.	$(x^2 - 5x + 6)(3x^2 + 27) = 0$		
10.	$(x^2 - 6x + 9)(9x^2 - 81) = 0$		
11.	$(x^2 + 10x + 25)(3x^2 + 27) = 0$		
12.	$(x^2 + 1)^2(2x + 3)^2 = 0$		
13.	$(x^2 - 2)(2x - 3)^2 = 0$		
14.	$(x^2 + 2)(2x - 4)^2 = 0$		
15.	$(x^2 + 2x)(2x^2 - 16) = 0$		
16.	$(x^2 + 3x)(3x^2 - 24) = 0$		
17.	$(x^2 - 6x + 9)(x^2 - 10x + 25) = 0$		
18.	$(x^2 + 2x + 1)(x^2 + 10x + 25) = 0$		
	Total		

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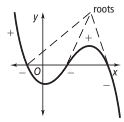
5-4 Activity: Researching the Factors
Dividing Polynomials

Work in small groups for this activity.

The polynomial $P(x) = x^4 + x^3 - 28x^2 + 20x + 48$ can be factored into exactly four distinct linear factors involving real numbers only. Write the polynomial in factored form $P(x) = (x - a)(x - b)(x - c)(x - d)$.

Notice that when the value of a polynomial changes from negative to positive (or from positive to negative) there is a root in between, as shown in the example at the right.



- Complete the following table to help find possible values for the roots of the polynomial.

x	−7	−5	−3	0	3	5	7
$P(x) = x^4 + x^3 - 28x^2 + 20x + 48$	594	−252	−210	48	−36	198	1560

- $P(x) = (x - a)(x - b)(x - c)(x - d)$. Devise a plan to find a, b, c, and d. Describe your plan in writing. Some possible strategies are shown at the right. Consider the advantages and disadvantages of each approach. Explore the use of repeated synthetic division on successive quotients.

  - Guess and Check
  - Synthetic Division
  - Graph and Check
  - Factoring

  Answers may vary. Sample: From the table, there is a root between –7 and –5. Try –6. There is a root between –3 and 0. Try –2 and –1. There is a root between 0 and 3. Try 1 and 2. There is a root between 3 and 5. Try 4. Use substitution to test different possibilities. $P(-6) = 0$ and $P(4) = 0$. Two of the four roots are –6 and 4.

- Write the polynomial in factored form. Show your group's work with your plan. You may use a combination of methods.

  Answers may vary. Sample:
  $P(x) = (x + 6)(x - 4)(x - 2)(x + 1)$

Wrap Up

Summarize your results in a complete logical and informative solution.

Answers may vary. Sample: A good solution should show how each of the four roots are found and should give a reason for each step along the way.

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5-5 Game: Theory at Play
Theorems About Roots of Polynomial Equations

This game is for two to four students. To play the game, make 20 game cards by cutting along the solid lines on the following sheet. Then fold each card along the dashed lines so the printed sides are shown. You can tape the cards closed. Lay the cards on a table so the category side is face up.

Decide who goes first. Players take turns selecting a card from the table. Do not return a card to the table once it has been selected. Each card is labeled by a category. The category determines what you must find.

- **Category A:** List the possible rational roots.
- **Category B:** A polynomial with rational coefficients has the given roots. Find two additional roots.
- **Category C:** Determine the maximum number of positive real roots.
- **Category D:** Determine the maximum number of negative real roots.

The answers are on the other side of the card. Players check each other's answers. Score each category as follows.

- **Category A:** Each correct possible rational root is worth 1 point. An incorrect root is worth −1 point. There is no penalty for not giving a possible rational root.
- **Category B:** Each correct root is worth 2 points. An incorrect root is worth 0 points.
- **Category C:** A correct answer is worth 4 points. An incorrect answer is worth 0 points.
- **Category D:** A correct answer is worth 4 points. An incorrect answer is worth 0 points.

The game continues until there are no more cards on the table. The player who earns the most points wins the game.

Check students' work.

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5-5 Game: Theory at Play
Theorems About Roots of Polynomial Equations

Answer $\pm1, \pm\frac{1}{3}$	**Answer** $\pm1, \pm\frac{1}{2}$	**Answer** $\pm1, \pm5$	**Answer** $\pm1, \pm\frac{1}{2}, \pm\frac{5}{2}, \pm5$
Category A $3x^3 + 7x^2 - 103x - 1$	**Category A** $2x^3 + x^2 - x - 1$	**Category A** $x^3 + 2x^2 + 5$	**Category A** $2x^4 - 12x + 5$
Answer $\pm1, \pm\frac{1}{4}, \pm\frac{1}{2}$	**Answer** $\pm1, \pm7$	**Answer** $\pm1, \pm2, \pm\frac{1}{3}, \pm\frac{2}{3}$	**Answer** $\pm1, \pm\frac{1}{4}, \pm\frac{1}{2}$
Category A $4x^3 + 7x^2 - 103x - 1$	**Category A** $x^5 + x^2 - x - 7$	**Category A** $3x^6 - 5x^2 - 6$	**Category A** $4x^3 - 3x^2 + 1$
Answer $3 - 2\sqrt{2}, 3 - i$	**Answer** $3 - \sqrt{5}, -i$	**Answer** $\sqrt{11}, 1 - 2i$	**Answer** $3 - 2\sqrt{3}, 1 - \sqrt{2}$
Category B $3 + 2\sqrt{2}, 3 + i$	**Category B** $3 - \sqrt{5}, -i$	**Category B** $-\sqrt{11}, 1 - 2i$	**Category B** $3 + 2\sqrt{3}, 1 - \sqrt{2}$
Answer one	**Answer** three or one	**Answer** three or one	**Answer** zero
Category C $3x^3 + 7x^2 - x - 35$	**Category C** $x^3 - 2x^2 + 3x - 5$	**Category C** $-x^6 + 7x^4 - 7x + 12$	**Category C** $2x^3 + 7x^2 + 5x + 1$
Answer two or none	**Answer** zero	**Answer** one	**Answer** three or one
Category D $3x^3 + 7x^2 - x - 35$	**Category D** $x^3 - 2x^2 + 3x - 5$	**Category D** $-x^6 + 7x^4 - 7x + 12$	**Category D** $2x^3 + 7x^2 + 5x + 1$

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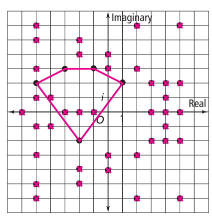
5-6 Puzzle: An Unlikely Find
The Fundamental Theorem of Algebra

A complex number plane with several points marked with dots is shown below. Most of these points represent solutions to the questions below.

- Write the missing zero(s) for each polynomial function.
- On the complex number plane, cross out the points representing the missing zeros you find, as well as the given zeros.
- When you finish crossing out the points representing the zeros, connect the remaining points to find the solution to the puzzle.
- Write the answer to the puzzle.

Find the missing zeros.

1. degree 4 polynomial function with zeros $2 + 3i, 4 - 2i$ **$2 - 3i, 4 + 2i$**
2. degree 4 polynomial function with zeros $-5 + 3i, 3 - 2i$ **$-5 - 3i, 3 + 2i$**
3. degree 6 polynomial function with zeros $4 + i, 5 - 2i, -5 + i$ **$4 - i, 5 + 2i, -5 - i$**
4. degree 6 polynomial function with zeros $-2 - 4i, -5 - 5i, -4 - i$ **$-2 + 4i, -5 + 5i, -4 + i$**
5. degree 5 polynomial function with zeros $4, -3i, 4i$ **$3i, -4i$**
6. degree 5 polynomial function with zeros $5, 3, -2, -5 + 6i$ **$-5 - 6i$**
7. degree 2 polynomial function with zero $-2 - 5i$ **$-2 + 5i$**
8. degree 2 polynomial function with zero $5 - 6i$ **$5 + 6i$**
9. degree 3 polynomial function with zeros $-1, 2 - 6i$ **$2 + 6i$**
10. degree 2 polynomial function with zeros $-6, -3$ **no more solutions**



Puzzle: You might find one of these in the rough: **a diamond**

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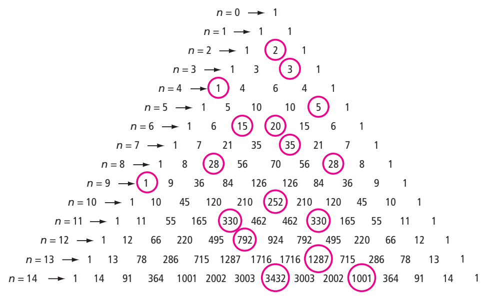
5-7 Puzzle: Pyramid Power

The Binomial Theorem

Find each of the specified coefficients of $(a + b)^n$ and circle them in the diagram.

1. the coefficient in $(a + b)^2$ not equal to the others **2**
2. the coefficient of ab^2 in $(a + b)^3$ **3**
3. the coefficient of a^4 in $(a + b)^4$ **1**
4. the coefficient of ab^4 in $(a + b)^5$ **5**
5. the coefficients of a^4b^2 and a^3b^3 in $(a + b)^6$ **15, 20**
6. the coefficient of a^3b^4 in $(a + b)^7$ **35**
7. the coefficients of a^6b^2 and of a^2b^6 in $(a + b)^8$ **28, 28**
8. the coefficient of a^9 in $(a + b)^9$ **1**
9. the coefficient of a^5b^5 in $(a + b)^{10}$ **252**
10. the coefficients of a^7b^4 and of a^4b^7 in $(a + b)^{11}$ **330, 330**
11. the coefficient of a^7b^5 in $(a + b)^{12}$ **792**
12. the coefficient of a^5b^8 in $(a + b)^{13}$ **1287**
13. the coefficients of a^7b^7 and a^4b^{10} in $(a + b)^{14}$ **3432, 1001**

What is the sum of the circled numbers? **7562**



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5-8 Activity: Savings Plans

Polynomial Models in the Real World

Polynomials can play a role in determining the amount of money you accumulate in a savings account. Suppose you make an annual deposit of \$1 into an account that pays simple interest compounded annually. The interest rate is represented by a decimal; for example, 4% = 0.04. At the end of the first year, you will have \$(1 + 0.04), at the end of the second year you will have $\$(1 + 0.04)^2$, and so on. If the interest rate is represented by x, at the end of the first year you will have \$(1 + x), at the end of the second year you will have $\$(1 + x)^2$, and so on.

Complete the table below by using x to represent the interest rate. The amount you finish with for one year is the amount you start with the next year. Each year you deposit \$1, so the sum of the three polynomials in the far right column of each row is the total amount in the account after three years.

Year 1 (dollars)		Year 2 (dollars)		Year 3 (dollars)	
Deposit	End	Start	End	Start	End
1	$(1 + x)$	$(1 + x)$	$(1 + x)^2$	$(1 + x)^2$	$(1 + x)^3$

Year 2 (dollars)		Year 3 (dollars)	
Deposit	End	Start	End
1	$(1 + x)$	$(1 + x)$	$(1 + x)^2$

Year 3 (dollars)	
Deposit	End
1	$(1 + x)$

1. Expand the sum of the three polynomials in the far right column of each row. Show your work. Label the sum $S_3(x)$. Then compare your results to your classmates' to make sure all the calculations are correct.

   $S_3(x) = x^3 + 4x^2 + 6x + 3$

2. Explain how to change $S_3(x)$ so it gives the total amount in the account over three years for an annual payment of \$1000.

   Multiply each power of 1 + x by 1000. Then add to get $1000(x^3 + 4x^2 + 6x + 3)$.

3. Explain how to find polynomials that give the total amounts in the account over four and five years for an annual payment of \$1.

   Simplify $(1 + x)^4 + (1 + x)^3 + (1 + x)^2 + (1 + x)$ to get $S_4(x) = x^4 + 5x^3 + 10x^2 + 10x +$
   Simplify $(1 + x)^5 + (1 + x)^4 + (1 + x)^3 + (1 + x)^2 + (1 + x)$ to get $S_5(x) = x^5 + 6x^4 + 15x^3 + 20x^2 + 15x + 5$.

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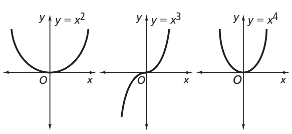
5-9 Game: Parent Functions

Transforming Polynomial Functions

In this game you will work in teams, using the parent functions graphed at the right.

Answer each question below. When your team is done, your teacher will check your answers and tally the scores to see who wins.

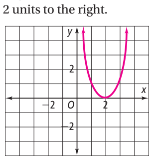


Write the function. (1 point each)

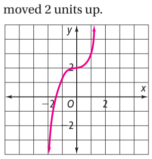
1. The graph of the quadratic parent function is moved 3 units right and 2 units down. $y = (x - 3)^2 - 2$
2. The graph of the cubic parent function is vertically stretched by a factor of 3. $y = 3x^3$
3. The graph of the fourth degree parent function is moved 3 units to the left. $y = (x + 3)^4$
4. The graph of the quadratic parent function is moved 3 units to the left and 2.5 units up. $y = (x + 3)^2 + 2.5$

Sketch the graph. (3 points each)

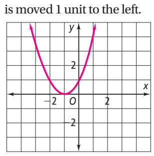
5. The graph of the fourth degree parent moved 2 units to the right.



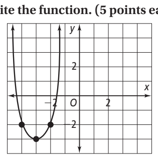
6. The graph of the cubic parent function is moved 2 units up.



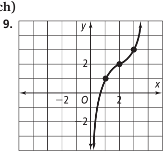
7. The graph of the quadratic parent function is moved 1 unit to the left.



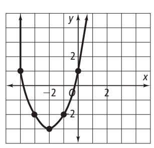
Write the function. (5 points each)

8. 

   $y = (x + 3)^4 - 3$

9. 

   $y = (x - 2)^3 + 2$

10. 

   $y = (x + 2)^2 - 3$

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6-1 Game: The Spiral Boardwalk

Roots and Radical Expressions

This is a game for two or more players. Each player will use a pencil to trace his or her path on the spiral of the game board provided.

- Make cards by cutting out each root expression below. Mix up the cards and place them face down on the table.
- Each player starts at the dot closest to the center of the spiral and moves along the Xs.
- In turn, a player selects an expression from the table.
- The player simplifies the expression.
- If the simplified expression is an integer, the player advances (clockwise) or retreats (counterclockwise) by the number of steps indicated by the integer. A positive integer is an advance. A negative integer is a retreat. Players do not retreat farther than the starting dot. If the simplified expression is not an integer, the player does not move.
- When you have finished with a card, place it back in the pile (face down) and shuffle the deck before your opponent goes.
- Players check each other's answers. If an answer is incorrect, the player loses his or her turn. If players cannot agree on an answer, they should ask the teacher.
- The player to advance the farthest or cross the finish line first wins.

$\sqrt[6]{64}$ **2**	$\sqrt[3]{125}$ **5**	$\sqrt[4]{81}$ **3**	$\sqrt[3]{-27}$ **−3**	$-\sqrt[3]{-64}$ **4**
$\sqrt[3]{8}$ **2**	$\sqrt[5]{-32}$ **−2**	$\sqrt[4]{625}$ **5**	$-\sqrt{16}$ **−4**	$-\sqrt[3]{-27}$ **3**
$\sqrt[3]{64}$ **4**	$\sqrt[10]{2^{10}}$ **2**	$\sqrt[7]{3^7}$ **3**	$-\sqrt[3]{-125}$ **5**	$-\sqrt[4]{16}$ **−2**
$-\sqrt[3]{8}$ **−2**	$\sqrt[8]{3^8}$ **3**	$\sqrt[4]{10}$ **$\sqrt[4]{10}$**	$\frac{\sqrt[4]{16}}{2}$ **1**	$\frac{-\sqrt[3]{-125}}{5}$ **1**
$\sqrt[3]{\frac{16}{2}}$ **2**	$2\sqrt[3]{27}$ **6**	$-2\sqrt[4]{81}$ **−6**	$\sqrt[3]{27} \times \sqrt[4]{-16}$ **−6**	$0.5\sqrt[3]{64}$ **2**