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5-7 Puzzle: Pyramid Power

The Binomial Theorem

Find each of the specified coefficients of $(a + b)^n$ and circle them in the diagram.

1. the coefficient in $(a + b)^2$ not equal to the others **2**
2. the coefficient of ab^2 in $(a + b)^3$ **3**
3. the coefficient of a^4 in $(a + b)^4$ **1**
4. the coefficient of ab^4 in $(a + b)^5$ **5**
5. the coefficients of a^4b^2 and a^3b^3 in $(a + b)^6$ **15, 20**
6. the coefficient of a^3b^4 in $(a + b)^7$ **35**
7. the coefficients of a^6b^2 and of a^2b^6 in $(a + b)^8$ **28, 28**
8. the coefficient of a^9 in $(a + b)^9$ **1**
9. the coefficient of a^5b^5 in $(a + b)^{10}$ **252**
10. the coefficients of a^7b^4 and of a^4b^7 in $(a + b)^{11}$ **330, 330**
11. the coefficient of a^7b^5 in $(a + b)^{12}$ **792**
12. the coefficient of a^5b^8 in $(a + b)^{13}$ **1287**
13. the coefficients of a^7b^7 and a^4b^{10} in $(a + b)^{14}$ **3432, 1001**

What is the sum of the circled numbers? **7562**

n = 0 → 1
n = 1 → 1 1
n = 2 → 1 2 1
n = 3 → 1 3 3 1
n = 4 → 1 4 6 4 1
n = 5 → 1 5 10 10 5 1
n = 6 → 1 6 15 20 15 6 1
n = 7 → 1 7 21 35 35 21 7 1
n = 8 → 1 8 28 56 70 56 28 8 1
n = 9 → 1 9 36 84 126 126 84 36 9 1
n = 10 → 1 10 45 120 210 252 210 120 45 10 1
n = 11 → 1 11 55 165 330 462 462 330 165 55 11 1
n = 12 → 1 12 66 220 495 792 924 792 495 220 66 12 1
n = 13 → 1 13 78 286 715 1287 1716 1716 1287 715 286 78 13 1
n = 14 → 1 14 91 364 1001 2002 3003 3432 3003 2002 1001 364 91 14 1

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5-8 Activity: Savings Plans

Polynomial Models in the Real World

Polynomials can play a role in determining the amount of money you accumulate in a savings account. Suppose you make an annual deposit of \$1 into an account that pays simple interest compounded annually. The interest rate is represented by a decimal; for example, 4% = 0.04. At the end of the first year, you will have \$(1 + 0.04), at the end of the second year you will have $\$(1 + 0.04)^2$, and so on. If the interest rate is represented by x, at the end of the first year you will have \$(1 + x), at the end of the second year you will have $\$(1 + x)^2$, and so on.

Complete the table below by using x to represent the interest rate. The amount you finish with for one year is the amount you start with the next year. Each year you deposit \$1, so the sum of the three polynomials in the far right column of each row is the total amount in the account after three years.

Year 1 (dollars)		Year 2 (dollars)		Year 3 (dollars)	
Deposit	End	Start	End	Start	End
1	**(1 + x)**	**(1 + x)**	**$(1 + x)^2$**	**$(1 + x)^2$**	**$(1 + x)^3$**

Year 2 (dollars)		Year 3 (dollars)	
Deposit	End	Start	End
1	**(1 + x)**	**(1 + x)**	**$(1 + x)^2$**

Year 3 (dollars)	
Deposit	End
1	**(1 + x)**

1. Expand the sum of the three polynomials in the far right column of each row. Show your work. Label the sum $S_3(x)$. Then compare your results to your classmates' to make sure all the calculations are correct.

   $S_3(x) = x^3 + 4x^2 + 6x + 3$

2. Explain how to change $S_3(x)$ so it gives the total amount in the account over three years for an annual payment of \$1000.

   Multiply each power of $1 + x$ by 1000. Then add to get $1000(x^3 + 4x^2 + 6x + 3)$.

3. Explain how to find polynomials that give the total amounts in the account over four and five years for an annual payment of \$1.

   Simplify $(1 + x)^4 + (1 + x)^3 + (1 + x)^2 + (1 + x)$ to get $S_4(x) = x^4 + 5x^3 + 10x^2 + 10x +$
   Simplify $(1 + x)^5 + (1 + x)^4 + (1 + x)^3 + (1 + x)^2 + (1 + x)$ to get $S_5(x) = x^5 + 6x^4 + 15x^3 + 20x^2 + 15x + 5$.

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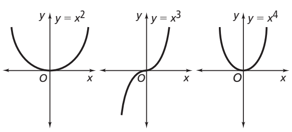
5-9 Game: Parent Functions

Transforming Polynomial Functions

In this game you will work in teams, using the parent functions graphed at the right.

Answer each question below. When your team is done, your teacher will check your answers and tally the scores to see who wins.

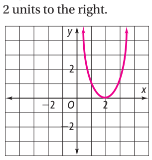


Write the function. (1 point each)

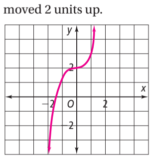
1. The graph of the quadratic parent function is moved 3 units right and 2 units down. **$y = (x - 3)^2 - 2$**
2. The graph of the cubic parent function is vertically stretched by a factor of 3. **$y = 3x^3$**
3. The graph of the fourth degree parent function is moved 3 units to the left. **$y = (x + 3)^4$**
4. The graph of the quadratic parent function is moved 3 units to the left and 2.5 units up. **$y = (x + 3)^2 + 2.5$**

Sketch the graph. (3 points each)

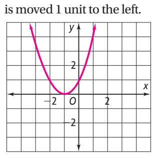
5. The graph of the fourth degree parent function moved 2 units to the right.



6. The graph of the cubic parent function is moved 2 units up.

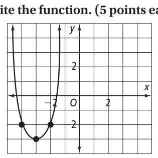


7. The graph of the quadratic parent function is moved 1 unit to the left.



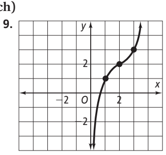
Write the function. (5 points each)

8.



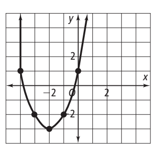
$y = (x + 3)^4 - 3$

9.



$y = (x - 2)^3 + 2$

10.



$y = (x + 2)^2 - 3$

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6-1 Game: The Spiral Boardwalk

Roots and Radical Expressions

This is a game for two or more players. Each player will use a pencil to trace his or her path on the spiral of the game board provided.

- Make cards by cutting out each root expression below. Mix up the cards and place them face down on the table.
- Each player starts at the dot closest to the center of the spiral and moves along the Xs.
- In turn, a player selects an expression from the table.
- The player simplifies the expression.
- If the simplified expression is an integer, the player advances (clockwise) or retreats (counterclockwise) by the number of steps indicated by the integer. A positive integer is an advance. A negative integer is a retreat. Players do not retreat farther than the starting dot. If the simplified expression is not an integer, the player does not move.
- When you have finished with a card, place it back in the pile (face down) and shuffle the deck before your opponent goes.
- Players check each other's answers. If an answer is incorrect, the player loses his or her turn. If players cannot agree on an answer, they should ask the teacher.
- The player to advance the farthest or cross the finish line first wins.

$\sqrt[6]{64}$ **2**	$\sqrt[3]{125}$ **5**	$\sqrt[4]{81}$ **3**	$\sqrt[3]{-27}$ **−3**	$-\sqrt[3]{-64}$ **4**
$\sqrt[3]{8}$ **2**	$\sqrt[5]{-32}$ **−2**	$\sqrt[4]{625}$ **5**	$-\sqrt{16}$ **−4**	$-\sqrt[3]{-27}$ **3**
$\sqrt[3]{64}$ **4**	$\sqrt[10]{2^{10}}$ **2**	$\sqrt[7]{3^7}$ **3**	$-\sqrt[3]{-125}$ **5**	$-\sqrt[4]{16}$ **−2**
$-\sqrt[3]{8}$ **−2**	$\sqrt[8]{3^8}$ **3**	$\sqrt[4]{10}$ **$\sqrt[4]{10}$**	$\frac{\sqrt[4]{16}}{2}$ **1**	$\frac{-\sqrt[3]{-125}}{5}$ **1**
$\sqrt[3]{\frac{16}{2}}$ **2**	$2\sqrt[3]{27}$ **6**	$-2\sqrt[4]{81}$ **−6**	$\sqrt[3]{27} \times \sqrt[4]{-16}$ **−6**	$0.5\sqrt[3]{64}$ **2**

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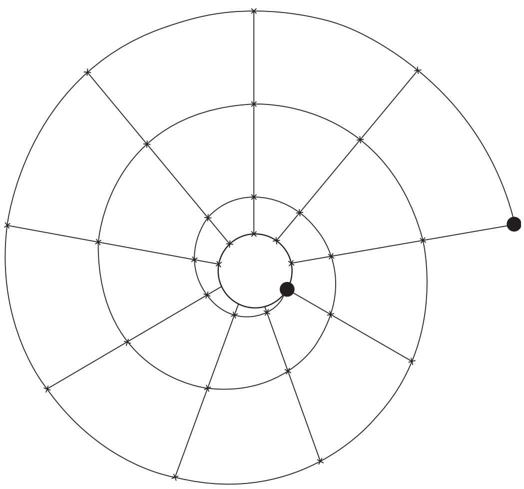
6-1 Game: The Spiral Boardwalk

Roots and Radical Expressions



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6-2 Puzzle: What's Underneath?

Multiplying and Dividing Radical Expressions

Simplify each expression. The simplified expression directs you to a cell in the decoding grid. The exponent for the *y*-variable gives the row and the exponent for the *z*-variable gives the column. For example, the answer to Exercise 1 directs you to the intersection of the fifth row and fifth column where the letter I is located.

In the space below each expression, record its simplified form and its corresponding letter.

Enter the letters in the appropriate spaces at the bottom of the page. Some of the letters are already given.

A	B	C	D	E	F	G
H	I	J	K	L	M	N
O	P	Q	R	S	T	U
V	W	X	A	B	C	D
E	F	G	H	I	J	K
L	M	N	O	P	Q	R
S	T	U	V	W	X	A

1. $\sqrt[4]{z^4y^{20}}\ \sqrt{z^8}$ y^5z^5; I
2. $\sqrt[4]{z^4y^{28}}\ \sqrt{z^0}$ y^7z; S
3. $\sqrt[4]{y^{24}}\ \sqrt{z^2}$ y^6z; L
4. $\sqrt[7]{y^7}\ \sqrt[4]{y^{12}z^{16}}$ y^4z^4; A
5. $\sqrt[3]{y^9z^9}\ \sqrt[4]{y^{12}z^{16}}$ y^6z^7; R
6. $\sqrt[4]{z^4y^4}\ \sqrt{z^8}$ yz^5; E
7. $\sqrt[9]{y^9}\ \sqrt[3]{z^{15}}$ yz^5; E
8. $\sqrt[3]{y^{21}z^3}\ \sqrt[5]{z^5}$ y^7z^2; T
9. $\sqrt{y^8}\ \sqrt{z^6}\ \sqrt{y^6}$ y^7z^3; U
10. $\sqrt{z^2}\ \sqrt[3]{y^3}$ yz; A
11. $\sqrt[8]{z^{16}y^8}\ \sqrt{z^6}$ yz^5; E
12. $\sqrt[5]{y^{35}}\ \sqrt[4]{z^4}$ y^7z; S
13. $\sqrt[5]{y^5}\ \sqrt[3]{z^3y^{12}}\ \sqrt{y^4}$ y^7z; S
14. $\sqrt[7]{z^{21}y^{49}}$ y^7z^3; U
15. $\sqrt[5]{y^5}\ \sqrt[7]{z^7}$ yz; A
16. $\sqrt[4]{y^{20}}\ \sqrt{z^2}$ y^5z; E
17. $\sqrt[3]{z^{12}y^6}\ \sqrt{y^2}$ y^3z^4; R
18. $\sqrt[5]{z^{10}y^{20}}\ \sqrt{y^4z^4}$ y^6z^4; O

Question: WHAT I(1) S(2) BE L(3) OW A(4) T R(5) E(6) E(7) WI T(8) H SQ U(9) A(10) RE LEAV E(11) S(12)?

Answer: S(13) Q U(14) A(15) R E(16) R(17) O(18) O T S

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6-3 Activity: Bringing Closure

Binomial Radical Expressions

Form five teams for this activity.

Each team will investigate one of the expressions shown below. Notice that each of the expressions has the general form $a + b\sqrt{p}$ where a and b are real numbers and p is a prime number. Are numbers of this form closed under multiplication? Let's find out.

Team A	Team B	Team C	Team D	Team E
$2+\sqrt{2}$	$3-\sqrt{3}$	$1+2\sqrt{5}$	$-1+2\sqrt{7}$	$2+\sqrt{11}$

Complete the table below by raising your radical expression to the 1st, 2nd, and 3rd power.

	Team A	Team B	Team C	Team D	Team E
1st power	$2+\sqrt{2}$	$3-\sqrt{3}$	$1+2\sqrt{5}$	$-1+2\sqrt{7}$	$2+\sqrt{11}$
2nd power	$6+4\sqrt{2}$	$12-6\sqrt{3}$	$21+4\sqrt{5}$	$29-4\sqrt{7}$	$15+4\sqrt{11}$
3rd power	$20+14\sqrt{2}$	$54-30\sqrt{3}$	$61+46\sqrt{5}$	$-85+62\sqrt{7}$	$74+23\sqrt{11}$

1. Look at the answers in your column. What do you notice about the general form of each of them?
   Each answer is a binomial radical expression of the form $a + b\sqrt{p}$.
2. Raise your radical expression to the fourth and fifth powers using the space below. What do you notice about the general form of your answers?
   Each answer is a binomial radical expression of the form $a + b\sqrt{p}$.
3. If you were to raise $1 + 2\sqrt{13}$ to the fifth power, what can you predict about the general form of the answer?
   Answers will have the general form $a + b\sqrt{p}$.
4. Which property is illustrated by your answers to Exercises 1–3? Explain.
   The Closure Property of Multiplication; each time I raised a binomial radical expression to a power (repeated multiplication), the product was also a binomial radical expression. So, the set of numbers $a + b\sqrt{p}$ (where a and b are real numbers and p is a prime number) is closed under multiplication.

Discuss your findings as a class to see if each of the five teams came to the same conclusion. **Check students' work.**

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TEACHER INSTRUCTIONS

6-4 Game: A Rational Quiz

Rational Exponents

Provide the host with the following questions and answers. The answers to the questions in the table on the left are underlined.

	The Greater Exponent?	A	B
1.	$\underline{\left(\sqrt[5]{x^3}\right)^2}$ or $\left(x^2\right)^{\frac{2}{5}}$		
2.	$\underline{\left(\sqrt[4]{x^6}\right)^2}$ or $\left(x^3\right)^{\frac{1}{5}}$		
3.	$\underline{\left(\sqrt[5]{x}\right)^8}$ or $\left(x^2\right)^{\frac{5}{8}}$		
4.	$\underline{\left(\sqrt[5]{x^5}\right)^3}$ or $\left(x^6\right)^{\frac{2}{5}}$		
5.	$\left(\sqrt[6]{x^{10}}\right)^{0.5}$ or $\underline{\left(x^{1.5}\right)^{\frac{2}{3}}}$		
6.	$\underline{\left(\sqrt[3]{x^2}\right)^{2.5}}$ or $\left(x^3\right)^{\frac{2}{7}}$		
7.	$\underline{\left(\sqrt[7]{x^{21}}\right)^{4.5}}$ or $\left(x^{20}\right)^{\frac{1}{3}}$		
8.	$\left(\sqrt[3]{x^3}\right)^{4.3}$ or $\underline{\left(x^9\right)^{\frac{2}{3}}}$		
9.	$\underline{\left(\sqrt[4]{x^6}\right)^6}$ or $\left(x^3\right)^{\frac{8}{3}}$		
10.	$\underline{\left(\sqrt[6]{x^9}\right)^{2.5}}$ or $\left(x^3\right)^{\frac{2}{7}}$		

	Positive or Negative?	A	B
1.	$-(-32)$ Answer: N		
2.	$(-32)^{\frac{6}{5}}$ Answer: P		
3.	$-(27)^{-\frac{2}{3}}$ Answer: N		
4.	$-(-27)^{-\frac{1}{3}}$ Answer: P		
5.	$\left(-(16)^{\frac{1}{4}}\right)^3$ Answer: N		
6.	$\sqrt[3]{(-64)^4}$ Answer: P		
7.	$\left(\sqrt[3]{125}\right)^{\frac{2}{5}}$ Answer: P		
8.	$-\sqrt[5]{-32}$ Answer: P		
9.	$-\left(\sqrt{24}\right)^{-\frac{2}{5}}$ Answer: N		
10.	$-\left(-\sqrt[5]{-2}\right)^{\frac{2}{5}}$ Answer: N		

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6-4 Game: A Rational Quiz

Rational Exponents

This is a game for three students. Decide on a host and two players.

- The host will take turns asking each player a question from the left column of the table.
- The player determines which expression (when simplified) has the greater exponent, underlines the expression, and states the answer.
- All correct answers are worth 3 points.

Then the host will take turns giving each player a question from the right column of the table.

- The player determines whether the expression is positive or negative, writes either "P" or "N", and states the answer.
- The host will keep score and announce the winner at the end of the game.

See Teacher Instructions page.

	The Greater Exponent?	A	B
1.	$\left(\sqrt[5]{x^3}\right)^2$ or $\left(x^2\right)^{\frac{2}{5}}$		
2.	$\left(\sqrt[4]{x^6}\right)^2$ or $\left(x^3\right)^{\frac{1}{5}}$		
3.	$\left(\sqrt[5]{x}\right)^8$ or $\left(x^2\right)^{\frac{5}{8}}$		
4.	$\left(\sqrt[5]{x^5}\right)^3$ or $\left(x^6\right)^{\frac{2}{5}}$		
5.	$\left(\sqrt[6]{x^{10}}\right)^{0.5}$ or $\left(x^{1.5}\right)^{\frac{2}{3}}$		
6.	$\left(\sqrt[3]{x^2}\right)^{2.5}$ or $\left(x^3\right)^{\frac{2}{7}}$		
7.	$\left(\sqrt[7]{x^{21}}\right)^{4.5}$ or $\left(x^{20}\right)^{\frac{1}{3}}$		
8.	$\left(\sqrt[3]{x^3}\right)^{4.3}$ or $\left(x^9\right)^{\frac{2}{3}}$		
9.	$\left(\sqrt[4]{x^6}\right)^6$ or $\left(x^3\right)^{\frac{8}{3}}$		
10.	$\left(\sqrt[6]{x^9}\right)^{2.5}$ or $\left(x^3\right)^{\frac{2}{7}}$		

	Positive or Negative?	A	B
1.	$-(-32)$		
2.	$(-32)^{\frac{6}{5}}$		
3.	$-(27)^{-\frac{2}{3}}$		
4.	$-(-27)^{-\frac{1}{3}}$		
5.	$\left(-(16)^{\frac{1}{4}}\right)^3$		
6.	$\sqrt[3]{(-64)^4}$		
7.	$\left(\sqrt[3]{125}\right)^{\frac{2}{5}}$		
8.	$-\sqrt[5]{-32}$		
9.	$-\left(\sqrt{24}\right)^{-\frac{2}{5}}$		
10.	$-\left(-\sqrt[5]{-2}\right)^{\frac{2}{5}}$		

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6-5 Puzzle: A Radical Solution

Solving Square Root and Other Radical Equations

You have solved equations that involve square roots of algebraic expressions. Then you checked your solutions to see if they are actual solutions or extraneous solutions.

To find the answer to the question at the bottom of the page:

- Solve each equation. Use the space below to show your work.
- Write solutions below each equation. Identify any extraneous solutions.
- Use only the actual solutions that are positive integers to answer the question.

$x = \sqrt{4x - 2}$

$2 \pm \sqrt{2}$

$x = \sqrt{3 - 2x}$

-3 and 1; extraneous: -3

$x = \sqrt{-2x + 48}$

-8 and 6; extraneous: -8

$x = \sqrt{90 - x}$

-10 and 9; extraneous: -10

$x = \sqrt{117 - 4x}$

-13 and 9; extraneous: -13

$x = \sqrt{6x - 3}$

$3 \pm \sqrt{6}$

$x = \sqrt{2x + 1}$

$1 \pm \sqrt{2}$; extraneous: $1 - \sqrt{2}$

$x = \sqrt{x + 72}$

-8 and 9; extraneous: -8

Actual solutions that are positive integers: **1, 6, and 9**

Question: *In what year did the first astronaut land on the moon?*

The first astronaut landed on the moon in **1969**.

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6-6 Activity: Team Compositions

Function Operations

You and your teammates will investigate what happens when a function is applied to itself.

Separate into teams of about four students. Each team should select and investigate one of the functions below. All functions are either linear or quadratic.

$f(x) = 2x + 1$	$f(x) = -3x + 2$	$f(x) = 3x + 5$
$f(x) = 2x^2 + 1$	$f(x) = -3x^2 + 2$	$f(x) = 3x^2 + 5$

1. For the function your team has selected, write an algebraic expression that defines $f(f(x))$. Show your work, and simplify your answer.

Answers may vary. Sample:

If $f(x) = 2x + 1$, then $f(f(x)) = 2(2x + 1) + 1$. This is so because $2x + 1$ replaces x in $f(x)$. Then $f(f(x)) = 2(2x + 1) + 1 = 4x + 2 + 1 = 4x + 3$.

If $f(x) = 2x^2 + 1$, then $f(f(x)) = 2(2x^2 + 1)^2 + 1$. This is so because $2x^2 + 1$ replaces x in $f(x)$. Then $f(f(x)) = 2(2x^2 + 1)^2 + 1 = 8x^4 + 8x^2 + 3$.

2. Talk with other teams and state a conclusion about the composition of linear and quadratic functions applied to themselves. Explain your findings below.

Answers may vary. Sample: The composition of a linear function with itself is another linear function. The composition of a quadratic function is not another quadratic function. It is a function of degree 4.

3. Select a few students to act as reporters. These students should interview team members from other teams. Interviewers should briefly summarize their reports below.

Answers may vary. Sample: Team findings support this overall report. The composition of a linear function with itself is another linear function. Students who explored $f(x) = 2x + 1$, $f(x) = -3x + 2$, or $f(x) = 3x + 5$ found that the result was a linear function. Students who explored $f(x) = 2x^2 + 1$, $f(x) = -3x^2 + 2$, $f(x) = 3x^2 + 5$ found that the result was a polynomial of degree 4, not degree 2. The composition of a quadratic function is not another quadratic function. It is a function of degree 4.

4. Student reporters should give a brief oral presentation of their findings.

Check students' work.

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6-7 Game: Flip-Flop

Inverse Relations and Functions

This is a game for two players. Players take turns answering the questions below. One player answers the odd-numbered questions, and the other player answers the even-numbered questions. Players check each other's answers.

- Players circle the correct inverse relation for the given relation. A correct answer is worth 5 points.
- If a player answers a question incorrectly, he or she has one additional chance to answer it. If the answer is correct, the player earns 3 points.

Correct answer choices are shown in red.

1. $y = 2x - 3$

(A) $y = 3x - 2$ (B) $y = 2 - 3x$ (C) $y = \frac{1}{2}x - 3$ (D) $y = \frac{1}{2}x + \frac{3}{2}$

2. $y = \frac{1}{2}x + 5$

(F) $y = 2x + 10$ (G) $y = 2x - 10$ (H) $y = 2x - 5$ (I) $y = -\frac{1}{2}x - 5$

3. $y = x^2 + 3$

(A) $y = \pm\sqrt{x - 3}$ (B) $y = \sqrt{-x + 3}$ (C) $y = \pm\sqrt{x + 3}$ (D) $y = \pm\sqrt{-x - 3}$

4. $y = -x^2 + 5$

(F) $y = \sqrt{x + 5}$ (G) $y = \pm\sqrt{-x - 5}$ (H) $y = \pm\sqrt{-x + 5}$ (I) $y = \sqrt{x - 5}$

5. $y = \frac{2}{3}x - \frac{1}{2}$

(A) $y = \frac{3}{2}x + \frac{3}{4}$ (B) $y = \frac{2}{3}x + \frac{3}{4}$ (C) $y = \frac{3}{2}x - \frac{3}{4}$ (D) $y = -\frac{2}{3}x - \frac{3}{4}$

6. $y = 2(3x - 5)$

(F) $y = -x + \frac{5}{3}$ (G) $y = \frac{1}{6}x - \frac{5}{3}$ (H) $y = \frac{1}{6}x + \frac{5}{3}$ (I) $y = -\frac{3}{5}x + 6$

7. $y = \frac{1}{2}x^2 + 1$

(A) $y = \sqrt{2x + 2}$ (B) $y = \pm\sqrt{2x - 2}$ (C) $y = \pm\sqrt{2x - 1}$ (D) $y = \pm\sqrt{x - 2}$

8. $y = 2x^2 - 0.5$

(F) $y = \sqrt{\frac{1}{2}x - \frac{1}{4}}$ (G) $y = \sqrt{-\frac{1}{2}x + \frac{1}{4}}$ (H) $y = \pm\sqrt{x + 4}$ (I) $y = \pm\sqrt{\frac{1}{2}x + \frac{1}{4}}$

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6-8 Activity: Lost in Translation

Graphing Radical Functions

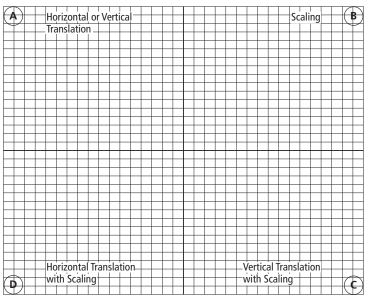
Your teacher will divide the class into teams of three or four students. Each team will be assigned a number.

Use a sheet of graph paper and divide it into four regions of equal size similar to the one shown below. In each of the four regions, draw a set of coordinate axes in order to graph square root functions. Each region represents a transformation of the square root function $y = \sqrt{x}$ as described below.

- Choose a square root function that satisfies the description in Region A of the graph. Write the function on a separate sheet of paper, and then graph the function in Region A.
- Choose a square root function that satisfies the description in Region B of the graph, and follow the same steps as you did for Region A.
- Choose a square root function that satisfies the description in Region C of the graph, and follow the same steps as you did for Region A.
- Choose a square root function that satisfies the description in Region D of the graph, and follow the same steps as you did for Region A.

Once your team has drawn a graph for each of the four regions, write your team number and the team leader's below the grid.

Place your team's grid on a desk or on the board. Teams should examine the grids from the other teams, and write the functions for the four different graphs drawn by each team. As a class, compare and discuss the graphs. **Check students' work.**



Team Number: ______ **Leader:** ______

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7-1 Activity: Financial Considerations

Exploring Exponential Models

You can work on your own or with a partner.

Suppose you received a gift of $10,000 and want to invest it. You visit two banks to see what they have to offer. Bank A is near your home and pays 5% interest compounded annually. Bank B is farther from your home and pays 6% interest compounded annually. You do not think a 1% difference in rates is that significant, but you want to check.

Calculate the amount of interest each plan will earn after one year. Record your answers on the lines provided.

Bank A: **$10,000(1.05) − $10,000 = $500** **Bank B:** **$10,000(1.06) − $10,000 = $600**

You decide to calculate how long it will take each bank to pay $5000 interest.

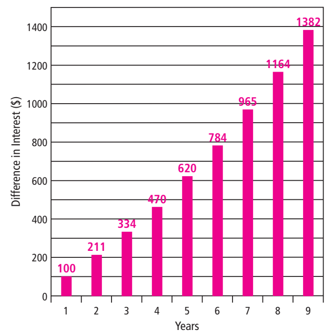
Complete the table below. Round your answers to the nearest dollar. Then record the number of years below the table.

Years	1	2	3	4	5	6	7	8	9
Bank A($)	500	1025	1576	2155	2763	3401	4071	4775	5513
Bank B($)	600	1236	1910	2625	3382	4185	5036	5938	6895

Bank A: about **9** years

Bank B: about **7** years

Complete the bar graph to show the amount by which Bank B will outperform Bank A over nine years. Use estimation to determine the heights of the bars.



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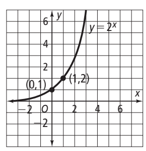
7-2 Game: Transforming Graphs

Properties of Exponential Functions



The graph of $y = 2^x$ crosses the y-axis at (0, 1) and contains (1, 2). If you know the images of these points under a transformation of the parent function, then you know an equation for the function you have.

In this game, you are given the image of (0, 1) and (1, 2) under one or more transformations of the graph of $y = 2^x$. If you can write the correct equation, you earn 5 points.

Option 1

Your teacher can play host and all students can be contestants. Play until all the game items have been answered. The highest score wins.

Option 2

Challenge another student to play the game. Players must agree on the correct answers. Play until all the game items have been answered. The highest score wins.

For each item below, write an equation of the form $y = a(2^{x-h}) + k$.

1. $(0, 1) \rightarrow (0, 0.5)$ and $(1, 2) \rightarrow (1, 1)$ — $y = 2^{x-1}$
2. $(0, 1) \rightarrow (0, 2)$ and $(1, 2) \rightarrow (1, 3)$ — $y = 2^x + 1$
3. $(0, 1) \rightarrow (0, 2)$ and $(1, 2) \rightarrow (1, 4)$ — $y = 2^{x+1}$
4. $(0, 1) \rightarrow (0, 0)$ and $(1, 2) \rightarrow (1, 1)$ — $y = 2^x - 1$
5. $(0, 1) \rightarrow (0, 3)$ and $(1, 2) \rightarrow (1, 6)$ — $y = 3(2^x)$
6. $(0, 1) \rightarrow (0, 0.4)$ and $(1, 2) \rightarrow (1, 0.8)$ — $y = 0.4(2^x)$
7. $(0, 1) \rightarrow (1, 2)$ and $(1, 2) \rightarrow (2, 3)$ — $y = 2^{x-1} + 1$
8. $(0, 1) \rightarrow (-1, 0)$ and $(1, 2) \rightarrow (0, 1)$ — $y = 2^{x+1} - 1$
9. $(0, 1) \rightarrow (2, 3)$ and $(1, 2) \rightarrow (3, 4)$ — $y = 2^{x-2} + 2$
10. $(0, 1) \rightarrow (-2, -2)$ and $(1, 2) \rightarrow (-1, -1)$ — $y = 2^{x+2} - 3$
11. $(0, 1) \rightarrow (0, 0.75)$ and $(1, 2) \rightarrow (1, 1.5)$ — $y = 3(2^{x-2})$
12. $(0, 1) \rightarrow (1, 5)$ and $(1, 2) \rightarrow (2, 11)$ — $y = 3(2^x - 1)$

My Total Score: ☐

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7-3 Puzzle: Evaluating Logs

Logarithmic Functions as Inverses

The puzzle at the bottom of the page has been separated into twelve sections. Each section contains three squares. In each section, there is a number in a circle. This tells you the sum of the two numbers in the section's empty squares. All missing numbers are natural numbers: 1, 2, 3, . . . Additional instructions (A–L) are given for each section of missing numbers.

For example, look at the section marked by Ⓐ. The sum of the two missing numbers in the empty squares is 11. Complete each equation with numbers that meet the requirements for each given section, and then place them in the puzzle.

A. $\log_2$ **8** = **3**

B. $\log_{\mathbf{5}} 25$ = **2**

C. $\log_3$ **27** = **3**

D. $\log_5$ **125** = **3**

E. $\log_{\mathbf{10}} 10$ = **1**

F. $\log_{\mathbf{2}} 32$ = **5**

G. $\log_7$ **49** = **2**

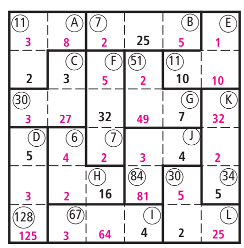
H. $\log_{\mathbf{2}} 16$ = **4**

I. $\log_4$ **64** = **3**

J. $\log_{\mathbf{3}}$ **81** = 4

K. $\log_{\mathbf{2}}$ **32** = 5

L. $\log_{\mathbf{5}} 25 = 2$



Final Question: Use four letters from the puzzle to find the value of $\log_2 \sqrt{2}$.

"O N E – **H** **A** **L** **F**"