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6-8 Activity: Lost in Translation

Graphing Radical Functions

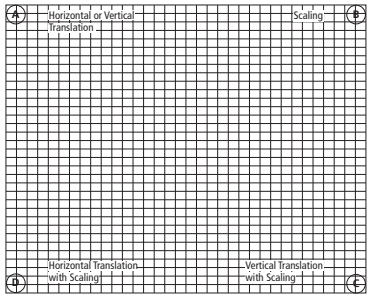
Your teacher will divide the class into teams of three or four students. Each team will be assigned a number.

Use a sheet of graph paper and divide it into four regions of equal size similar to the one shown below. In each of the four regions, draw a set of coordinate axes in order to graph square root functions. Each region represents a transformation of the square root function $y = \sqrt{x}$ as described below.

- Choose a square root function that satisfies the description in Region A of the graph. Write the function on a separate sheet of paper, and then graph the function in Region A.
- Choose a square root function that satisfies the description in Region B of the graph, and follow the same steps as you did for Region A.
- Choose a square root function that satisfies the description in Region C of the graph, and follow the same steps as you did for Region A.
- Choose a square root function that satisfies the description in Region D of the graph, and follow the same steps as you did for Region A.

Once your team has drawn a graph for each of the four regions, write your team number and the team leader's below the grid.

Place your team's grid on a desk or on the board. Teams should examine the grids from the other teams, and write the functions for the four different graphs drawn by each team. As a class, compare and discuss the graphs. **Check students' work.**



Team Number: _____ **Leader:** _____

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7-1 Activity: Financial Considerations

Exploring Exponential Models

You can work on your own or with a partner.

Suppose you received a gift of $10,000 and want to invest it. You visit two banks to see what they have to offer. Bank A is near your home and pays 5% interest compounded annually. Bank B is farther from your home and pays 6% interest compounded annually. You do not think a 1% difference in rates is that significant, but you want to check.

Calculate the amount of interest each plan will earn after one year. Record your answers on the lines provided.

Bank A: $10,000(1.05) − $10,000 = $500 **Bank B:** $10,000(1.06) − $10,000 = $600

You decide to calculate how long it will take each bank to pay $5000 interest.

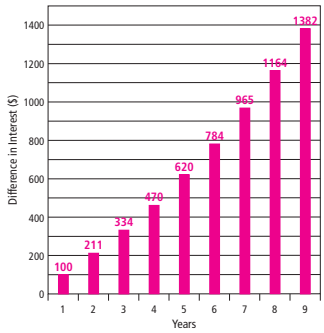
Complete the table below. Round your answers to the nearest dollar. Then record the number of years below the table.

Years	1	2	3	4	5	6	7	8	9
Bank A($)	500	1025	1576	2155	2763	3401	4071	4775	5513
Bank B($)	600	1236	1910	2625	3382	4185	5036	5938	6895

Bank A: about 9 years

Bank B: about 7 years

Complete the bar graph to show the amount by which Bank B will outperform Bank A over nine years. Use estimation to determine the heights of the bars.



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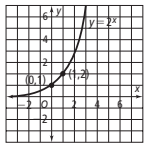
7-2 Game: Transforming Graphs

Properties of Exponential Functions

The graph of $y = 2^x$ crosses the y-axis at (0, 1) and contains (1, 2). If you know the images of these points under a transformation of the parent function, then you know an equation for the function you have.



In this game, you are given the image of (0, 1) and (1, 2) under one or more transformations of the graph of $y = 2^x$. If you can write the correct equation, you earn 5 points.

Option 1

Your teacher can play host and all students can be contestants. Play until all the game items have been answered. The highest score wins.

Option 2

Challenge another student to play the game. Players must agree on the correct answers. Play until all the game items have been answered. The highest score wins.

For each item below, write an equation of the form $y = a(2^{x-h}) + k$.

1. (0, 1) → (0, 0.5) and (1, 2) → (1, 1) — $y = 2^{x-1}$
2. (0, 1) → (0, 2) and (1, 2) → (1, 3) — $y = 2^x + 1$
3. (0, 1) → (0, 2) and (1, 2) → (1, 4) — $y = 2^{x+1}$
4. (0, 1) → (0, 0) and (1, 2) → (1, 1) — $y = 2^x - 1$
5. (0, 1) → (0, 3) and (1, 2) → (1, 6) — $y = 3(2^x)$
6. (0, 1) → (0, 0.4) and (1, 2) → (1, 0.8) — $y = 0.4(2^x)$
7. (0, 1) → (1, 2) and (1, 2) → (2, 3) — $y = 2^{x-1} + 1$
8. (0, 1) → (−1, 0) and (1, 2) → (0, 1) — $y = 2^{x+1} - 1$
9. (0, 1) → (2, 3) and (1, 2) → (3, 4) — $y = 2^{x-2} + 2$
10. (0, 1) → (−2, −2) and (1, 2) → (−1, −1) — $y = 2^{x+2} - 3$
11. (0, 1) → (0, 0.75) and (1, 2) → (1, 1.5) — $y = 3(2^{x-2})$
12. (0, 1) → (1, 5) and (1, 2) → (2, 11) — $y = 3(2^x - 1)$

My Total Score: ☐

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7-3 Puzzle: Evaluating Logs

Logarithmic Functions as Inverses

The puzzle at the bottom of the page has been separated into twelve sections. Each section contains three squares. In each section, there is a number in a circle. This tells you the sum of the two numbers in the section's empty squares. All missing numbers are natural numbers: 1, 2, 3, . . . Additional instructions (A-L) are given for each section of missing numbers.

For example, look at the section marked by (A). The sum of the two missing numbers in the empty squares is 11. Complete each equation with numbers that meet the requirements for each given section, and then place them in the puzzle.

A. $\log_2 8 = 3$

B. $\log_5 25 = 2$

C. $\log_3 27 = 3$

D. $\log_5 125 = 3$

E. $\log_{10} 10 = 1$

F. $\log_2 32 = 5$

G. $\log_7 49 = 2$

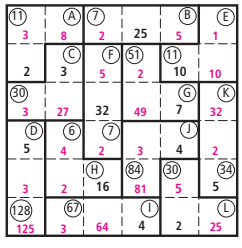
H. $\log_2 16 = 4$

I. $\log_4 64 = 3$

J. $\log_3 81 = 4$

K. $\log_2 32 = 5$

L. $\log_5 25 = 2$



Final Question: Use four letters from the puzzle to find the value of $\log_2 \sqrt{2}$.

"ONE – H A L F"

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7-4 Puzzle: Letter Scramble

Properties of Logarithms

Next to each expression in the left column, write the letter of the expression in the right column whose value is equivalent to it. Some letters will be used more than once. Unscramble the letters to find the four-word phrase that answers the question at the bottom of the page.

1. $\log_2 75$ **N**
2. $\log_2 36$ **I**
3. $\log_2(75 \times 2^a) - a$ **N**
4. $\log_2 ab^5$ **A**
5. $\log_2(36 \times 2^5) - 5$ **I**
6. $\log_2 108$ **P**
7. $\log_2 2^8$ **L**
8. $\log_2(a \times 2^8)$ **O**
9. $\log_2(3a \times 2^9)$ **G**
10. $\log_2 2^{16} - 8$ **L**
11. $\log_2\left(\frac{6^2}{2a}\right) + \log_2(2a)$ **I**
12. $\log_2(a^4b^3)$ **S**
13. $(\log_2 75)(\log_2(2^{a+1}) - a)$ **N**
14. $(\log_2 2)(\log_2 ab^5)$ **A**
15. $\log_2\left(\frac{a^4}{b^3}\right)$ **B**
16. $\log_2 16 - 5$ **C**

A $\log_2 a + 5\log_2 b$

B $4\log_2 a - 3\log_2 b$

C -1

D $\log_2(5a) - 9$

E $5\log_2 2 + \log_2 2^{20}$

F $\log_2(3b)$

G $\log_2(3a) + 9$

I $2(\log_2 2 + \log_2 3)$

K 20

L 8

M $3a$

N $2\log_2 5 + \log_2 3$

O $\log_2 a + 8$

P $3\log_2 3 + \log_2 4$

R $a + b$

S $4\log_2 a + 3\log_2 b$

Where do simple exponents live? I N P L A I N L O G C A B I N S

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7-5 Game: Analyzing Equations

Exponential and Logarithmic Equations

This is a game for the entire class. You will work in teams of three students. Your teacher can serve as the host. Your teacher will decide the order in which the questions are chosen and he or she may also decide on a time limit for each question. No calculators allowed!

- Write your answer next to each question. A correct response is worth three points.
  - **Category 1:** Answer *none, one,* or *two*.
  - **Category 2:** Give two consecutive numbers such as *5 and 6*.
  - **Category 3:** Write *A* or *B*.

	Category 1: How Many Solutions?	Score
1.	$2^x = x + 3$ **two**	
2.	$\log_2 x = x + 2$ **none**	
3.	$2^x = 5$ **one**	
4.	$\log_2 x = 2$ **one**	
5.	$2^x = -5$ **none**	
6.	$\log_2 x = x - 2$ **two**	
7.	$2^x + 2 = 0$ **none**	
8.	$\log_2(x - 2) = 4x$ **none**	

	Category 2: Between Which Two Whole Numbers is x?	Score
1.	$3 \times 2^x = 75$ **4 and 5**	
2.	$2 \times 2^x = 20$ **3 and 4**	
3.	$\log_2 x = 3.1$ **8 and 9**	
4.	$\log_2 x = 0.1$ **1 and 2**	
5.	$5 \times 2^x = 60$ **3 and 4**	
6.	$-3 \times 2^x = -9$ **1 and 2**	

	Category 3: Which Equation Has the Greater Solution?			Score
1.	A. $2^x = 64$	or	B. $3^x = 81$ **A**	
2.	A. $10^x = 1000$	or	B. $5^x = 25$ **A**	
3.	A. $4^x = 1$	or	B. $5^x = 125$ **B**	
4.	A. $3^x = 27$	or	B. $2^x = 32$ **B**	

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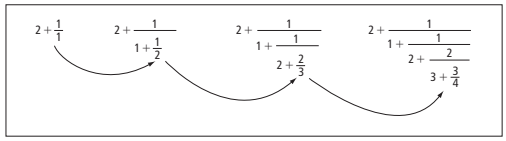
7-6 Activity: An Irrational Pattern

Natural Logarithms

This is an activity for the entire class. It involves a fraction that never ends.

Your teacher or a student can lead the activity. Begin by writing the following fractional expressions on the board.



- Write the fourth expression in the fraction pattern on the board and in the space below.

$$2+\cfrac{1}{1+\cfrac{1}{2+\cfrac{2}{3+\cfrac{3}{4+\frac{4}{5}}}}}$$

- Write the fifth expression in the space below. Use parentheses to group numbers in denominators. *Hint:* Start from the bottommost part of $(4 + 4/5)$. Build the fraction from denominators to numerators.

$$2+\cfrac{1}{1+\cfrac{1}{2+\cfrac{2}{3+\cfrac{3}{4+\cfrac{4}{5}}}}}$$

$$5+\frac{5}{6}$$

- Evaluate the fifth expression on a calculator. Write your answer in the space below.
  2.71845; *e*
- Which two familiar numbers do these expressions give? Write your answers in the space between the two expressions. **$\sqrt{2}$ and π**

$$1+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\frac{1}{2}}}}$$

$$3+\cfrac{1}{6+\cfrac{9}{6+\cfrac{25}{6+\frac{49}{6+\frac{81}{121}}}}}$$

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8-1 Puzzle: Constant of Variation

Inverse Variation

Answer the following questions about inverse and combined variation.

Each ordered pair is from an inverse variation. Find the constant of variation.

1. $(2, 1)$ **2**
2. $(-1, 5)$ **-5**
3. $(0.4, 0.5)$ **0.2**
4. $(-5.2, -0.25)$ **1.3**
5. $\left(2, -\frac{1}{3}\right)$ **$-\frac{2}{3}$**
6. $\left(\frac{1}{2}, \frac{7}{5}\right)$ **0.7**

Suppose that x and y vary inversely. Find the constant of variation.

7. $x = 6$ when $y = \frac{1}{2}$ **3**
8. $x = -3$ when $y = 2$ **-6**
9. $x = 0.5$ when $y = -2.2$ **-1.1**
10. $x = 0.2$ when $y = 2$ **0.4**
11. $x = \frac{2}{3}$ when $y = \frac{2}{3}$ **$\frac{4}{9}$**
12. $x = \frac{9}{10}$ when $y = -\frac{2}{3}$ **$-\frac{3}{5}$**

Each pair of values is from an inverse variation. Find the missing value.

13. $(2, 6), (-4, y)$ **-3**
14. $(9, -2), (x, -3)$ **6**
15. $(7, 0.2), (5, y)$ **0.28**
16. $\left(\frac{4}{3}, \frac{2}{3}\right), \left(x, \frac{5}{9}\right)$ **1.6**

For the following, find z when $x = 2$ and $y = 10$.

17. z varies jointly with x and y. When $x = -8$ and $y = -3$, $z = 6$. **5**
18. z varies directly with x and inversely with y. When $x = 4$ and $y = 20$, $z = -1$. **-1**
19. z varies directly with the square of y and inversely with x. When $x = 0.6$ and $y = 0.3$, $z = 0.09$. **30**

The numerical solutions correspond to letters according to the table below.

-6	-6.28	-5	-3	-1.1	-1	$-\frac{2}{3}$	$-\frac{3}{5}$	0.2	$\frac{1}{4}$	0.28	0.4	$\frac{4}{9}$
A	B	C	D	E	F	G	H	I	J	K	L	M

0.7	1.3	1.4	1.5	1.6	2	3	4	5	6	10	30	50
N	O	P	Q	R	S	T	U	V	W	X	Y	Z

The numbers below the spaces correspond to the exercise numbers. Write the letter corresponding to the exercise solution in each space. The resulting quotation is by mathematician and philosopher Bertrand Russell.

T 7 H 12 E 9 — D 13 E 9 G 5 R 16 E 9 E 9 — O 4 F 18 — O 4 N 6 E 9 ' S 1

E 9 M 11 O 4 T 7 I 3 O 4 N 6 S 1 — V 17 A 8 R 16 I 3 E 9 S 1

I 3 N 6 V 17 E 9 R 16 — S 1 E 9 L 10 Y 19 — W 14 I 3 T 7 H 12

O 4 N 6 E 9 ' S 1 — K 15 N 6 O 4 W 14 L 10 E 9 D 13 G 5 E 9 — O 4 F 18

T 7 H 12 E 9 — F 18 A 8 C 2 T 7 S 1