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8-5 Activity: Graphing Calculator Check

Adding and Subtracting Rational Expressions

This activity can be done in groups of two or three students. Discuss each group's results once everyone is finished.

Example: Use your graphing calculator to add the following rational expressions.

$$\frac{x}{x+4} + \frac{3}{x-3}$$

Step 1 Enter $Y_1 = \frac{x}{x+4}$

Step 2 Enter $Y_2 = \frac{3}{x-3}$

Step 3 Enter $Y_3 = Y_1 + Y_2$.

Turn on the Y_3 function *only*. (You switch a function on or off by moving the cursor over the equals sign and pressing ENTER. A highlighted equals sign means that a function is turned on.) Graph Y_3 (see Figures 1 and 2).

Step 4 Add $\frac{x}{x+4} + \frac{3}{x-3} = \frac{x(x-3)}{(x+4)(x-3)} + \frac{3(x+4)}{(x-3)(x+4)}$

$$= \frac{(x^2 - 3x) + (3x + 12)}{(x+4)(x-3)}$$

$$= \frac{x^2 + 12}{(x+4)(x-3)}.$$

Step 5 On another calculator; have a classmate enter $Y_4 = \frac{x^2+12}{(x+4)(x-3)}$ (see Figures 3 and 4).

Since these graphs coincide, you can conclude the addition was performed correctly.

Figure 1

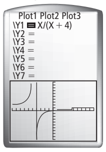


Figure 2

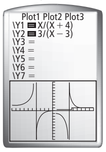


Figure 3

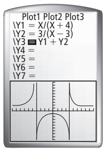
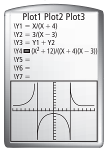


Figure 4



Repeat this process for the following expressions.

1. $\frac{2}{x+3} + \frac{2}{x-3}$ $\frac{4x}{(x-3)(x+3)}$

2. $\frac{2}{x-2} - \frac{2}{x+2}$ $\frac{8}{(x-2)(x+2)}$

3. $\frac{1}{x^2-4x} + \frac{x}{x^2-16}$ $\frac{x^2+x+4}{(x)(x-4)(x+4)}$

4. $\frac{10}{x^2-3x-10} - \frac{10}{x^2+3x-10}$ $\frac{60x}{(x-5)(x+2)(x+5)(x-2)}$

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TEACHER INSTRUCTIONS

8-6 Game: Rational Learning

Solving Rational Equations

Points	*Vocabulary:* Question	Answer
10	Rational equation?	An equation containing rational expressions
20	Least common denominator?	The smallest integer that can be evenly divided by all the denominators
30	Cross products?	For the equation $\frac{a}{b} = \frac{c}{d}$, *ad* and *bc*
40	Solution of rational equation?	A value that, when substituted for the variable, makes the rational equation true
50	Extraneous solution?	A solution of the derived equation, but not of the original equation

Points	*Solution:* Question	Answer
10	$\frac{4}{x} = \frac{x}{9}$ $x = 6$ and $x = -6$	yes; yes
20	$\frac{x}{x-7} = \frac{-3}{x+1}$; $x = 3$ and $x = 4$	yes; no
30	$\frac{8(x-1)}{x-7} = \frac{4}{x-2}$; $x = 4$	no
40	$1 + \frac{x^2}{2x-1} = \frac{14}{2x+4}$; $x = 3$	no
50	$1 + \frac{2}{x-4} = \frac{15}{x^2-4x}$; $x = -3$ and $x = 5$	yes; yes

Points	*Solve:* Question	Answer
10	$\frac{2y}{5} + \frac{2}{6} = \frac{y}{2} - \frac{1}{6}$	5
20	$\frac{x}{x+3} = \frac{4}{x+5}$	−4, 3
30	$x + \frac{10}{x-2} = \frac{x^2+3x}{x-2}$	no solution
40	$\frac{2}{x} + \frac{1}{x+1} = \frac{5}{x^2+x}$	1
50	$\frac{x+3}{x^2+3x-4} = \frac{x+2}{x^2-16}$	−5

Pionts	*Review:* Question	Answer
10	What is the next number in the pattern 1, 4, 7, 10, . . . ?	13
20	What is the next number in the pattern 1, −3, 9, −27, . . . ?	81
30	Solve $5n + 2 = 37$.	7
40	What is the 4th term of the expansion of $(2x + 3y)^6$?	$4320x^3y^3$
50	Solve $-6144 = -3(2^x)$.	11

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8-6 Game: Rational Learning

Solving Rational Equations

This is a game for the entire class. Your teacher will divide the class into two teams.

Use the score card below to record the score and keep track of the questions in each category that have been asked.

Rules

- Decide which team goes first. Teams alternate turns.
- During a turn, a team selects a category from the list below. Your teacher will ask the first available question with the lowest point value. There are five questions in each of the four categories worth 10, 20, 30, 40, or 50 points for a total of 20 questions.
- If the team answers correctly within a reasonable amount of time, the team earns the point value of that question.
- If the team answers incorrectly, the team loses the point value and the other team has the option to answer the question to earn or lose the point value.
- Play continues until all the questions have been used. The team with the highest point total wins.

Points	Category	Team 1	Team 2
10	Vocabulary		
20	Vocabulary		
30	Vocabulary		
40	Vocabulary		
50	Vocabulary		
10	Solution		
20	Solution		
30	Solution		
40	Solution		
50	Solution		
10	Solve		
20	Solve		
30	Solve		
40	Solve		
50	Solve		
10	Review		
20	Review		
30	Review		
40	Review		
50	Review		
	Total		

Categories

- **Vocabulary** Provide a definition for the given vocabulary.
- **Solution** Determine if the given values of *x* are solutions to the rational equation.
- **Solve** Solve the rational equation.
- **Review** Solve a review problem.

See Teacher Instructions page.

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TEACHER INSTRUCTIONS

9-1 Game: De-"Term"-ining the Answer

Mathematical Patterns

Provide the host with the following list of explicit formulas or recursive formulas with initial term and corresponding terms for twenty sequences.

Formula	Next 10 Terms
$a_n = \frac{1}{n}$	$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \frac{1}{9}, \frac{1}{10}$
$a_n = n + 2$	3, 4, 5, 6, 7, 8, 9, 10, 11, 12
$a_n = n - 7$	−6, −5, −4, −3, −2, −1, 0, 1, 2, 3
$a_n = 4n$	4, 8, 12, 16, 20, 24, 28, 32, 36, 40
$a_n = 2n - 1$	1, 3, 5, 7, 9, 11, 13, 15, 17, 19
$a_n = 3n + 5$	8, 11, 14, 17, 20, 23, 26, 29, 32, 35
$a_n = n^3$	1, 8, 27, 64, 125, 216, 343, 512, 729, 1000
$a_n = n^2 + n + 1$	3, 7, 13, 21, 31, 43, 57, 73, 91, 111
$a_{n+1} = a_n + 6; a_1 = -1$	5, 11, 17, 23, 29, 35, 41, 47, 53, 59
$a_{n+1} = a_n + n; a_1 = 1$	2, 4, 7, 11, 16, 22, 29, 37, 46, 56
$a_{n+1} = a_n + 2n; a_1 = 1$	3, 7, 13, 21, 31, 43, 57, 73, 91, 111
$a_{n+1} = a_n + n^2; a_1 = 1$	2, 6, 15, 31, 56, 92, 141, 205, 286, 386
$a_{n+1} = a_n + 2^n; a_1 = 1$	3, 7, 15, 31, 63, 127, 255, 511, 1023, 2047
$a_{n+1} = a_n + n + 4^n; a_1 = 0$	5; 23; 90; 350; 1379; 5481; 21,872; 87,416; 349,569; 1,398,155
$a_{n+1} = a_n + 5(-3)^n; a_1 = 0$	−15; 30; −105; 300; −915; 2730; −8205; 24,600; −73,815; 221,430
$a_{n+1} = 3a_n; a_1 = 2$	6; 18; 54; 162; 486; 1458; 4374; 13,122; 39,366; 118,098
$a_{n+1} = na_n; a_1 = 1$	1; 2; 6; 24; 120; 720; 5040; 40,320; 362,880; 3,628,800
$a_{n+1} = \frac{1}{2}a_n; a_1 = 1$	$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \frac{1}{64}, \frac{1}{128}, \frac{1}{256}, \frac{1}{512}, \frac{1}{1024}$
$a_{n+1} = 3a_n - n; a_1 = 1$	2; 4; 9; 23; 64; 186; 551; 1645; 4926; 14,768
$a_{n+1} = 1 + \frac{1}{a_n}; a_1 = 1$	$2, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}, \frac{13}{8}, \frac{21}{13}, \frac{34}{21}, \frac{55}{34}, \frac{89}{55}, \frac{144}{89}$

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9-1 Game: De-"Term"-ining the Answer

Mathematical Patterns

This is a game for three students. One student is the host and the other two are players.

- Your teacher provides the host with a list of explicit formulas or recursive formulas with an initial term for twenty sequences.
- The host gives a formula to a player and the player must give the first term (if the formula is explicit) or the next term (if the formula is recursive) of the sequence. If the first player is correct, the second player must give the next term in the sequence. Players can use a separate sheet of paper to work out a problem.
- Players continue to alternate giving terms until an error is made or until a player provides ten correct terms.
- If an error is made, the player who gave the last correct term earns a point. If a player provides ten correct terms, neither player earns a point.
- Play resumes with a new sequence.
- Players alternate starting new sequences.
- The player who accumulates the most points wins. See Teacher Instructions page.

Example: The host gives you the explicit formula $a_n = 4n + 1$.

$a_n = 4n + 1$	Write the formula.
$a_1 = 4 \cdot 1 + 1$	You substitute 1 for n.
$= 5$	You simplify and give the term 5.
$a_2 = 4 \cdot 2 + 1$	The other player substitutes 2 for n.
$= 9$	The other player simplifies and gives the term 9.

The first two terms are 5 and 9.

Example: The host gives you the recursive formula $a_{n+1} = 2a_n + 3$, $a_1 = 4$.

$a_{n+1} = 2a_n + 3$	Write the formula.
$a_2 = 2a_1 + 3$	You substitute 1 for n.
$= 2 \cdot 4 + 3$	You substitute 4 for a_1.
$= 11$	You simplify and give the term 11.
$a_3 = 2a_2 + 3$	The other player substitutes 2 for n.
$= 2 \cdot 11 + 3$	The other player substitutes 11 for a_2.
$= 25$	The other player simplifies and gives the term 25.

The next two terms of the sequence are 11 and 25.

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9-2 Game: Four Thought

Arithmetic Sequences

This is a game for two players.

To begin the game, roll a number cube twice. The first roll gives the column of your number on the board, and the second roll gives the row of your number on the board. You get to keep that number by writing it down and crossing it off the board. Then your opponent rolls a number cube twice and keeps another number from the board, but only if that number has not already been taken. Otherwise, the player rolls the number cube twice more. If a player rolls two 1's or two 6's, the player gets a FREE PICK! and may choose any number on the board that has not been taken.

The object of the game is to be the first player to collect four numbers that form an arithmetic sequence. You can recognize arithmetic sequences by looking for a constant difference in any four numbers you have taken from the board.

Example: Four of your numbers are 2, 13, 24, and 35.

This is an arithmetic sequence because consecutive numbers have a common difference d equal to 11.

2 13 24 35
11 11 11

The first player to collect an arithmetic sequence of four numbers wins! Check students' work.

30 FREE PICK!	5	32	3	16	22
10	28	12	34	27	8
35	17	31	6	15	33
21	1	13	24	18	4
7	20	9	26	2	11
19	36	14	29	23	25 FREE PICK!

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9-3 Activity: Finding the Next Term

Geometric Sequences

Work with a partner for this activity.

Select one of the geometric sequences in the top grid and cross it out. Your partner has to locate the next term of that sequence in the bottom grid and cross it out. Take turns selecting the sequence and locating the next term.

The activity ends when all entries in both grids have been crossed out.

2, 6, 18,. . . **54**	$\frac{3}{5}, -\frac{3}{25}, \frac{3}{125}, \ldots$ $\mathbf{-\frac{3}{625}}$	5, −10, 20,. . . **−40**	4, 20, 100,. . . **500**
$\frac{1}{3}, \frac{1}{6}, \frac{1}{12}, \ldots$ $\mathbf{\frac{1}{24}}$	10, 2, 0.4,. . . **0.08**	0.4, 0.12, 0.036,. . . **0.0108**	$\frac{1}{2}, -\frac{1}{8}, \frac{1}{32}, \ldots$ $\mathbf{-\frac{1}{128}}$
$\frac{8}{3}, \frac{8}{15}, \frac{8}{75}, \ldots$ $\mathbf{\frac{8}{375}}$	2, −0.1, 0.005,. . . **−0.00025**	$\frac{1}{7}, \frac{1}{14}, \frac{1}{28}, \ldots$ $\mathbf{\frac{1}{56}}$	1, 10, 100,. . . **1000**
5, −1, 0.2,. . . **−0.04**	1, 5, 25,. . . **125**	$\frac{1}{10}, -\frac{1}{20}, \frac{1}{40}, \ldots$ $\mathbf{-\frac{1}{80}}$	20, 4, 0.8,. . . **0.16**
2, −0.6, 0.18,. . . **−0.054**	8, 24, 72,. . . **216**	$\frac{1}{30}, \frac{1}{60}, \frac{1}{120}, \ldots$ $\mathbf{\frac{1}{240}}$	3, 12, 48,. . . **192**
$\frac{5}{8}, \frac{5}{24}, \frac{5}{72}, \ldots$ $\mathbf{\frac{5}{216}}$	0.1, 0.01, 0.001,. . . **0.0001**	8, −16, 32,. . . **−64**	0.2, 0.04, 0.008,. . . **0.0016**

0.16	$-\frac{1}{128}$	216	−0.00025
1000	0.08	$\frac{1}{240}$	500
$-\frac{3}{625}$	−40	0.0108	54
192	0.0016	$-\frac{1}{80}$	−0.04
$\frac{8}{375}$	$\frac{1}{24}$	−64	$\frac{1}{56}$
0.0001	−0.054	$\frac{5}{216}$	125

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9-4 Activity: Summing a Sequence

Arithmetic Series

The nth term of an arithmetic sequence is given by the formula

$$a_n = a_1 + (n - 1) \cdot d.$$

The sum of the first n terms of an arithmetic series is given by the formula

$$S_n = \left(\frac{n}{2}\right) \cdot (a_1 + a_n).$$

Cut out six markers of any shape. These represent terms of a series. Choose an initial value a_1 for the series and write this on a marker. Now choose a value for d, the difference between successive terms in the sequence. On another marker, write the value of $a_1 + d$. Write the next four terms of the sequence on the remaining markers and lay out all six markers in order.

Now rearrange the markers into pairs so that the first and last terms, the second and fifth terms, and the third and fourth terms are together.

1. What is the sum of each pair? Check students' work.
2. Why does each pair have the same sum? (*Hint*: $a_2 = a_1 + d$ and $a_5 = a_6 - d$. What is $a_2 + a_5$?) $a_2 + a_5 = (a_1 + d) + (a_6 - d) = a$

The sum of the first six terms of a series is the number of pairs multiplied by the sum of each pair.

3. Number of pairs: 3
4. Sum of each pair: Check students' work.
5. $S_6 =$ Check students' work.

What happens if you want to compute the sum of an odd number of terms? Cut out another marker and write the seventh term of your sequence on it.

6. Rearrange your markers into pairs so that the first and last terms are together, the second and sixth are together, and so on. There is one extra term that does not fit into a pair. Which term is it? a_4
7. Now what is the sum of each pair? Check students' work.
8. There are still three pairs, but if the sum of each pair is multiplied by the number of pairs, you do not get the right sum. What must you add to this result to get the right answer? a_4
9. The extra term can be thought of as half of a pair. What is two times the extra term? Check students' work.
10. Add $\left(\frac{1}{2}\right) \cdot$ (sum of each pair) to the sum of the first three pairs. What is the result? S_7

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9-5 Puzzle: Sum Find!
Geometric Series

Calculate the sum S_n of each geometric series. In the space provided, write the sum. Then find and circle the spelled-out answer in the word search below.

1. $S_8 = \frac{-1}{17}(1 - 2 + 4 - \cdots - 128)$ **5**
2. $S_6 = \frac{1}{217}(-1 + 5 - 25 + \cdots + 3125)$ **12**
3. $S_8 = \frac{-1}{205}(1 - 3 + 9 - \cdots - 2187)$ **8**
4. $S_8 = \frac{1}{164}(1 + 3 + 9 + \cdots + 2187)$ **20**
5. $S_4 = \frac{-1}{185}(1 - 6 + 36 - 216)$ **1**
6. $S_{12} = \frac{1}{273}(1 + 2 + 4 + \cdots + 2048)$ **15**
7. $S_4 = \frac{1}{30}(-1 + 7 - 49 + 343)$ **10**
8. $S_8 = \frac{1}{85}(1 + 2 + 4 + \cdots + 128)$ **3**
9. $S_6 = \frac{1}{651}(1 + 5 + 25 + \cdots + 3125)$ **6**
10. $S_4 = \frac{1}{52}(-1 + 5 - 25 + 125)$ **2**

Z	F	H	R	T	E	T	H	H	S	Q	I	G	X
K	B	E	Q	Y	N	N	N	N	N	X	N	H	Q
E	N	E	E	T	N	E	V	E	S	E	O	F	S
O	K	R	R	O	V	E	E	E	E	V	M	G	V
G	Y	T	N	E	W	T	E	T	B	T	M	M	U
T	W	E	L	V	E	R	F	T	H	L	X	F	I
O	X	E	P	N	H	I	V	S	R	G	Y	I	S
H	O	D	I	T	F	H	V	B	K	U	I	V	S
C	W	N	W	I	D	T	Y	Z	A	B	O	E	N
E	Y	G	B	Z	W	I	B	G	P	E	X	F	Z
B	C	E	N	Z	Z	U	A	W	O	P	E	A	L

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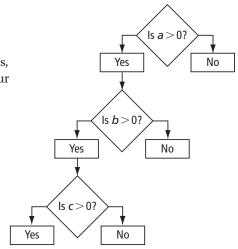
10-1 Activity: Graphic Organizers
Exploring Conic Sections

Work in pairs for this activity. The three equations below have the general form $ax^2 + by^2 = c$. Can you tell whether the graphs of these equations will be circles, ellipses, or hyperbolas? The key is to examine the values of *a*, *b*, and *c*.

$4x^2 + 9y^2 = 36$ $\quad$ $4x^2 - 9y^2 = 36$ $\quad$ $9x^2 - 9y^2 = 36$

A flowchart or a table can help you distinguish among the three equations and their graphs.

The partially completed flowchart at the right shows how you can use a decision-making process, focusing on the values of *a*, *b*, and *c*, to narrow your choices among circles, ellipses, and hyperbolas. The diamond shapes represent a question. The rectangles contain alternative answers.



The table below shows how you can analyze different possibilities for the signs of *a*, *b*, and *c* to narrow your choices among circles, ellipses, and hyperbolas. (Assume that none of the three values—*a*, *b*, or *c*—equals 0.)

Complete the table below by determining if each situation describes a *circle, ellipse, hyperbola,* or *no graph.* Graph test cases as needed.

Sign of *a*	Sign of *b*	Sign of *c*	Graph Type
+	+	+	circle or ellipse If a = b: circle If a ≠ b: ellipse
+	+	–	no graph
+	–	+	hyperbola
–	+	–	hyperbola
–	–	+	no graph
–	–	–	circle or ellipse If a = b: circle If a ≠ b: ellipse

On a separate sheet of paper, make a complete flowchart that helps you determine whether the graph of the equations shown at the top of the page will be a circle, ellipse, or hyperbola. Compare your flowchart with other groups in your class.
Check student's work.

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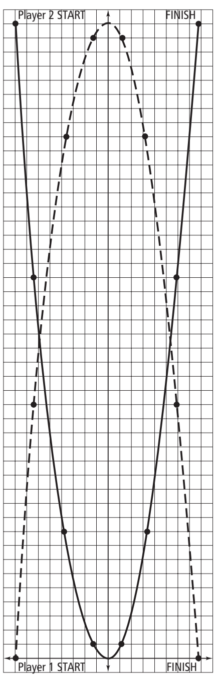
10-2 Game: A Parabolic Race
Parabolas

This is a game for two players. Each player uses one of the parabolic racetracks provided. For each item below, decide whether the parabola described *opens up, down, to the right,* or *to the left.* Write your answer to the right of each item.

- Move from one point to the next for each correct answer.
- Players must agree on the accuracy of each answer.
- The first player to cross the finish line wins. If neither player crosses, then the player who advances farthest wins.

1. focus: $F(-2, 1)$; directrix: $y = 3$ **down**
2. vertex: $V(2, 7)$; focus: $F(-2, 7)$ **left**
3. $y = -x^2 + 2x + 5$ **down**
4. $A(0, 3)$, $B(4, 7)$, $C(4, -7)$ **right**
5. $y = 3(x - 2)^2 + 1$ **up**
6. vertex: $V(5, 2)$; directrix: $x = 6$ **left**
7. vertex: $V(0, 7)$; focus: $F(0, 0)$ **down**
8. $x = -5y^2 - 2$ **left**
9. $A(-3, 6)$, $B(0, 1)$, $C(4, 8)$ **up**
10. $x = 2(y - 1)^2 + 2$ **right**
11. focus: $F(2, 8)$; directrix: $x = 0$ **right**
12. vertex: $V(-3, 1)$; directrix: $y = 2$ **down**
13. $y = 0.3x^2 - 2x + 1$ **up**
14. focus: $F(2.2, 2)$; vertex: $V(5, 2)$ **left**
15. $x = -(y + 1)^2 - 3$ **left**
16. $K(3, 6)$, $L(0, 3)$, $M(3, -6)$ **right**



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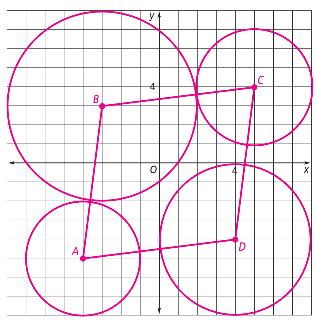
10-3 Puzzle: Circles In Squares
Circles

Here is a puzzle about circles in the coordinate plane. You will need a compass.

Draw circles in the grid that meet all of the conditions described below. Use a compass to lightly draw different combinations of circles until you find a correct answer.

- No circle extends beyond the boundary of the puzzle grid.
- All circles have centers whose coordinates are integers.
- There must be two circles with a radius of 3 units, one circle with a radius of 4 units, and one circle with a radius of 5 units.
- No two circles intersect at more than one point.

Answers may vary. Sample graph:



There are different combinations of circles that can be used to find the solution. Use the space below to write the equation for each circle in your solution.
Answers may vary. Sample: $(x + 3)^2 + (y - 3)^2 = 5^2$; $(x - 4)^2 + (y + 4)^2 = 4^2$; $(x - 5)^2 + (y - 4)^2 = 3^2$; $(x + 4)^2 + (y + 5)^2 = 3^2$

On the grid above, insert a point at the center of each circle. Then form a polygon by drawing segments between the centers of the circles. Classify the polygon by the number of sides. Is the polygon *equilateral, equiangular, regular,* or *none of these*? Explain.
Answers may vary. Sample: Quadrilateral; the polygon is equilateral because all sides are the same length.