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9-5 Puzzle: Sum Find!

Geometric Series

Calculate the sum S_n of each geometric series. In the space provided, write the sum. Then find and circle the spelled-out answer in the word search below.

1. $S_8 = \frac{-1}{17}(1 - 2 + 4 - \cdots - 128)$ **5**
2. $S_6 = \frac{1}{217}(-1 + 5 - 25 + \cdots + 3125)$ **12**
3. $S_8 = \frac{-1}{205}(1 - 3 + 9 - \cdots - 2187)$ **8**
4. $S_8 = \frac{1}{164}(1 + 3 + 9 + \cdots + 2187)$ **20**
5. $S_4 = \frac{-1}{185}(1 - 6 + 36 - 216)$ **1**
6. $S_{12} = \frac{1}{273}(1 + 2 + 4 + \cdots + 2048)$ **15**
7. $S_4 = \frac{1}{30}(-1 + 7 - 49 + 343)$ **10**
8. $S_8 = \frac{1}{85}(1 + 2 + 4 + \cdots + 128)$ **3**
9. $S_6 = \frac{1}{651}(1 + 5 + 25 + \cdots + 3125)$ **6**
10. $S_4 = \frac{1}{52}(-1 + 5 - 25 + 125)$ **2**

Z	F	H	R	T	E	T	H	H	S	Q	I	G	X
K	B	E	Q	Y	N	N	N	N	N	X	N	H	Q
E	N	E	E	T	N	E	V	E	S	E	O	F	S
O	K	R	R	O	V	E	E	E	E	V	M	G	V
G	Y	T	N	E	W	T	E	T	B	T	M	M	U
T	W	E	L	V	E	R	F	T	H	L	X	F	I
O	X	E	P	N	H	I	V	S	R	G	Y	I	S
H	O	D	I	T	F	H	V	B	K	U	I	V	S
C	W	N	W	I	D	T	Y	Z	A	B	O	E	N
E	Y	G	B	Z	W	I	B	G	P	E	X	F	Z
B	C	E	N	Z	Z	U	A	W	O	P	E	A	L

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10-1 Activity: Graphic Organizers

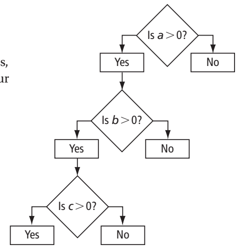
Exploring Conic Sections

Work in pairs for this activity. The three equations below have the general form $ax^2 + by^2 = c$. Can you tell whether the graphs of these equations will be circles, ellipses, or hyperbolas? The key is to examine the values of *a*, *b*, and *c*.

$4x^2 + 9y^2 = 36$ $\quad 4x^2 - 9y^2 = 36$ $\quad 9x^2 - 9y^2 = 36$

A flowchart or a table can help you distinguish among the three equations and their graphs.

The partially completed flowchart at the right shows how you can use a decision-making process, focusing on the values of *a*, *b*, and *c*, to narrow your choices among circles, ellipses, and hyperbolas. The diamond shapes represent a question. The rectangles contain alternative answers.



The table below shows how you can analyze different possibilities for the signs of *a*, *b*, and *c* to narrow your choices among circles, ellipses, and hyperbolas. (Assume that none of the three values—*a*, *b*, or *c*—equals 0.)

Complete the table below by determining if each situation describes a *circle, ellipse, hyperbola,* or *no graph.* Graph test cases as needed.

Sign of *a*	Sign of *b*	Sign of *c*	Graph Type
+	+	+	circle or ellipse If a = b: circle If a ≠ b: ellipse
+	+	−	no graph
+	−	+	hyperbola
−	+	−	hyperbola
−	−	+	no graph
−	−	−	circle or ellipse If a = b: circle If a ≠ b: ellipse

On a separate sheet of paper, make a complete flowchart that helps you determine whether the graph of the equations shown at the top of the page will be a circle, ellipse, or hyperbola. Compare your flowchart with other groups in your class.

Check student's work.

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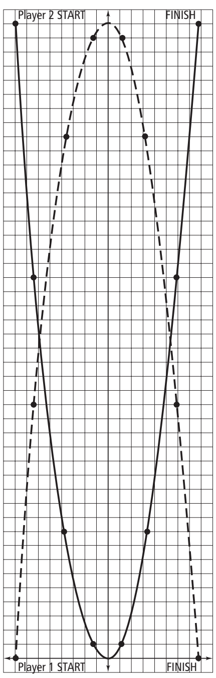
10-2 Game: A Parabolic Race

Parabolas

This is a game for two players. Each player uses one of the parabolic racetracks provided. For each item below, decide whether the parabola described *opens up, down, to the right,* or *to the left.* Write your answer to the right of each item.

- Move from one point to the next for each correct answer.
- Players must agree on the accuracy of each answer.
- The first player to cross the finish line wins. If neither player crosses, then the player who advances farthest wins.



1. focus: $F(-2, 1)$; directrix: $y = 3$ **down**
2. vertex: $V(2, 7)$; focus: $F(-2, 7)$ **left**
3. $y = -x^2 + 2x + 5$ **down**
4. $A(0, 3)$, $B(4, 7)$, $C(4, -7)$ **right**
5. $y = 3(x - 2)^2 + 1$ **up**
6. vertex: $V(5, 2)$; directrix: $x = 6$ **left**
7. vertex: $V(0, 7)$; focus: $F(0, 0)$ **down**
8. $x = -5y^2 - 2$ **left**
9. $A(-3, 6)$, $B(0, 1)$, $C(4, 8)$ **up**
10. $x = 2(y - 1)^2 + 2$ **right**
11. focus: $F(2, 8)$; directrix: $x = 0$ **right**
12. vertex: $V(-3, 1)$; directrix: $y = 2$ **down**
13. $y = 0.3x^2 - 2x + 1$ **up**
14. focus: $F(2.2, 2)$; vertex: $V(5, 2)$ **left**
15. $x = -(y + 1)^2 - 3$ **left**
16. $K(3, 6)$, $L(0, 3)$, $M(3, -6)$ **right**

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10-3 Puzzle: Circles In Squares

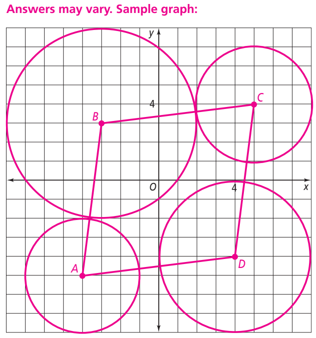
Circles

Here is a puzzle about circles in the coordinate plane. You will need a compass.

Draw circles in the grid that meet all of the conditions described below. Use a compass to lightly draw different combinations of circles until you find a correct answer.

- No circle extends beyond the boundary of the puzzle grid.
- All circles have centers whose coordinates are integers.
- There must be two circles with a radius of 3 units, one circle with a radius of 4 units, and one circle with a radius of 5 units.
- No two circles intersect at more than one point.

Answers may vary. Sample graph:



There are different combinations of circles that can be used to find the solution. Use the space below to write the equation for each circle in your solution.

Answers may vary. Sample: $(x + 3)^2 + (y - 3)^2 = 5^2$; $(x - 4)^2 + (y + 4)^2 = 4^2$; $(x - 5)^2 + (y - 4)^2 = 3^2$; $(x + 4)^2 + (y + 5)^2 = 3^2$

On the grid above, insert a point at the center of each circle. Then form a polygon by drawing segments between the centers of the circles. Classify the polygon by the number of sides. Is the polygon *equilateral, equiangular, regular,* or *none of these*? Explain.

Answers may vary. Sample: Quadrilateral; the polygon is equilateral because all sides are the same length.

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10-4 Game: Eccentricity of Ellipses

Ellipses

Provide the host with the following questions and answers.

Round 1	*Far from, Close to, or Circle?*	Player 1	Player 2
1.	$\frac{x^2}{400}+\frac{y^2}{4}=1$ Answer: far from		
2.	$\frac{x^2}{1}+\frac{y^2}{2^2}=1$ Answer: close to		
3.	$\frac{x^2}{1.3}+\frac{y^2}{1.3}=1$ Answer: circle		
4.	$\frac{x^2}{2.3^2}+\frac{y^2}{3.8^2}=1$ Answer: close to		
5.	$\frac{x^2}{36}+\frac{y^2}{1}=1$ Answer: far from		
6.	$\frac{x^2}{8.73^2}+\frac{y^2}{2.01^2}=1$ Answer: far from		
Round 2	**Lengths of the major and minor axes?**	**Player 1**	**Player 2**
1.	$\frac{x^2}{400}+\frac{y^2}{4}=1$ Answer: 40, 4		
2.	$\frac{x^2}{1}+\frac{y^2}{9}=1$ Answer: 2, 6		
3.	$\frac{x^2}{4}+\frac{y^2}{25}=1$ Answer: 4, 10		
4.	$\frac{x^2}{100}+\frac{y^2}{100}=1$ Answer: 20, 20		

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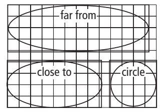
10-4 Game: Eccentricity of Ellipses

Ellipses

This is a game for three students. Decide on a host and two players. The host should receive a sheet of questions and answers from the teacher.



- The three diagrams at the right give an example of an ellipse that is *far from* being shaped like a circle, an ellipse that is *close to* being shaped like a circle, and an actual *circle*. Refer to these figures during each round of the game.
- The host will assign each player three questions in Round 1 and two questions in Round 2. The host may put a time limit on response time if necessary.
- Game note: If $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and $0<|a-b|\le 2$, then the ellipse is *close to* a circle. Otherwise, it is *far from* a circle or a *circle*.

Round 1	*Far from, Close to, or Circle?*	Player 1	Player 2
1.	$\frac{x^2}{400}+\frac{y^2}{4}=1$ See Teacher Instruction page.		
2.	$\frac{x^2}{1}+\frac{y^2}{2^2}=1$		
3.	$\frac{x^2}{1.3}+\frac{y^2}{1.3}=1$		
4.	$\frac{x^2}{2.3^2}+\frac{y^2}{3.8^2}=1$		
5.	$\frac{x^2}{36}+\frac{y^2}{1}=1$		
6.	$\frac{x^2}{8.73^2}+\frac{y^2}{2.01^2}=1$		
Round 2	**Lengths of the major and minor axes?**	**Player 1**	**Player 2**
1.	$\frac{x^2}{400}+\frac{y^2}{4}=1$		
2.	$\frac{x^2}{1}+\frac{y^2}{9}=1$		
3.	$\frac{x^2}{4}+\frac{y^2}{25}=1$		
4.	$\frac{x^2}{100}+\frac{y^2}{100}=1$		

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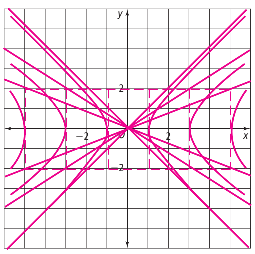
10-5 Activity: Closer and Closer . . .

Hyperbolas

Work with a partner. Select one of the three equations and sketch the graph of it on the grid shown.

- Draw the axes and foci.
- Sketch the asymptotes.
- Write the equations for the asymptotes of your hyperbola below.



A. $\frac{x^2}{5^2}-\frac{y^2}{2^2}=1$ $y=\pm\frac{2}{5}x$

B. $\frac{x^2}{3^2}-\frac{y^2}{2^2}=1$ $y=\pm\frac{2}{3}x$

C. $\frac{x^2}{1^2}-\frac{y^2}{1^2}=1$ $y=\pm x$

Use a graphing calculator to graph your hyperbola and its asymptotes. Then complete the information for your hyperbola below. The answers to the first row are shown. Record only positive values.

x	A		*x*	B		*x*	C	
x	*y*-value curve	*y*-value asymptote	*x*	*y*-value curve	*y*-value asymptote	*x*	*y*-value curve	*y*-value asymptote
50	19.90	20	50	33.27	33.33	50	49.99	50
100	39.95	40	100	66.64	66.67	100	99.995	100
200	79.97	80	200	133.32	133.33	200	199.997	200
500	199.99	200	500	333.327	333.333	500	499.999	500
1000	399.995	400	1000	666.664	666.667	1000	999.999	1000

Is there any value of x for which the curve touches the asymptote? Explain.

No; for every value of x, for example when $x > 0$, the asymptote in the first quadrant is higher up than the curve. But, the distance between them for a given value of x gets smaller and smaller.

What will be the smallest distance between the curve and the asymptote? Where will this distance be found? Explain.

Zero; the distance between the curve and the asymptote becomes zero as x approaches infinity.

As a class, discuss your findings for each of the three given hyperbolas.

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10-6 Puzzle: Fit 'Em In

Translating Conic Sections

To solve this puzzle, you may want to solve the conic section formulas for y.

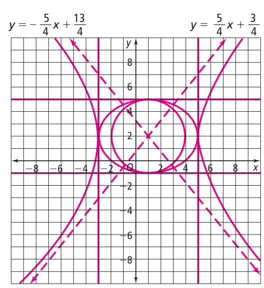
Circle	Ellipse	Hyperbola
$x^2+y^2=r^2$	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$
$y=\pm\sqrt{r^2-x^2}$	$y=\pm\sqrt{b^2\left(1-\frac{x^2}{a^2}\right)}$	$y=\pm\sqrt{b^2\left(\frac{x^2}{a^2}-1\right)}$

Here is the puzzle. Find an equation in standard form for a hyperbola, an ellipse, and a circle such that the following conditions are met. Sketch the graph of the conic sections on the coordinate grid below.

- All three conic sections have the common center $P(1, 2)$.
- The hyperbola has asymptotes $y-2=\frac{5}{4}(x-1)$ and $y-2=-\frac{5}{4}(x-1)$.
- The hyperbola is externally tangent to rectangle R at exactly two points. These points are the midpoints of the vertical sides of R. (Rectangle R is defined below.)
- The ellipse is internally tangent to rectangle R along a point on each of its sides.
- The circle is internally tangent to rectangle R in exactly two points. These points lie along a pair of opposite sides of R.
- Rectangle R is defined by the lines $x=-3$, $x=5$, $y=-1$, and $y=5$.

Hint: Suppose that the common center is the origin $O(0, 0)$. Then use what you know about translations to find the puzzle solution.

Write equations below for each conic section that makes up the puzzle solution.



Circle: $(x-1)^2+(y-2)^2=3^2$

Ellipse: $\frac{(x-1)^2}{4^2}+\frac{(y-2)^2}{3^2}=1$

Hyperbola: $\frac{(x-1)^2}{4^2}-\frac{(y-2)^2}{5^2}=1$