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11-1 Activity: Word Analysis

Permutations and Combinations

This activity is for groups of two to four students. Parts A and B should be done as a group and Part C should be done individually. Your teacher will determine whether or not you use a calculator.

Part A

1. Find the number of distinct three-letter combinations of the letters that make up the word COMBINE. 35
2. Find the number of distinct three-letter permutations of the letters that make up the word COMBINE. 210
3. Which of these two values is larger? Explain. Support your answer with numbers and actual examples from the word COMBINE.
Permutations; order matters, and for each three-letter combination, there are six distinct orderings: $35 \cdot 6 = 210$; for example: COM → COM, CMO, OCM, OMC, MCO, and MOC.

Part B

4. Find the number of distinct three-letter combinations of the letters that make up the word PERMUTE. 20
5. Find the number of distinct three-letter permutations of the letters that make up the word PERMUTE. 120
6. How does Part B differ from Part A? Explain what you had to do to resolve the difference.
The letter E appears twice in the word PERMUTE. For uniqueness, we can only consider six letters.
7. Write the statement for Part B that parallels $35 \cdot 6 = 210$ from Part A.
$20 \cdot 6 = 120$

Part C

Using the letters in your first, middle, and last names, find the following:

8. The number of distinct four-letter combinations. Check students' work.
9. The number of distinct five-letter permutations. Check students' work.
10. Try to find two English words of four or more letters using the distinct letters in your name. Check students' work.

Bonus: How many of the four-letter combinations from Exercise 8 contain a vowel?
Check students' work.

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11-2 Activity: Colors and Probability

Probability

This activity is best done in groups of two to three students.

Materials

A small container of colored plastic chips Set up a container with no one color less than 1/20 of the total.

Special Instruction

Do not look inside the container until Part 1 has been completed.

Part 1: Experimental Probability

- Select one member of the group to record the colors drawn.
- Thoroughly mix the chips.
- Draw one chip and record its color.
- Replace the chip and thoroughly mix the chips.
- Repeat 50 times.
- Record your results in the space below.
- Use your results to calculate the experimental probabilities of drawing each color.

Results: Check students' work.

Experimental Probabilities: Check students' work.

Part 2: Theoretical Probability

- Examine the contents of the container and record the number of each color below.
- Calculate the theoretical probabilities of drawing each color.

Results: Check students' work.

Theoretical Probabilities: Check students' work.

Part 3: Compare/Contrast

- Use a separate sheet of paper to write a paragraph comparing and contrasting your results from Parts 1 and 2. Was there any difference in your results? Explain. Which type of probability might be more useful? When? What are the advantages and disadvantages of calculating these different types of probabilities? Check students' work.

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TEACHER INSTRUCTIONS

11-3 Game: The Probability Path

Probability of Multiple Events

Provide the host with the following questions and answers (shown in brackets).

What is the probability of the following?

1. rolling a 2 on your first turn and a 6 on your second turn **[1/36]**
2. rolling a 3 on your first turn and a 3 on your second turn **[1/36]**
3. rolling an even number on your first turn and a 2 on your second turn **[1/12]**
4. rolling a 1 on your first turn and a 3 or 5 on your second turn **[1/18]**
5. landing on a space with an even denominator **[1/2]**
6. landing on a space with an odd denominator **[2/5]**
7. landing on a $\frac{2}{3}$ or a space with an identical neighbor on each side **[7/15]**
8. landing on a $\frac{1}{12}$ or a space with an identical neighbor on each side **[1/3]**
9. from the $\frac{1}{12}$ in the middle, the next move landing on another $\frac{1}{12}$ **[1/3]**
10. moving "forward two spaces" after being on a $\frac{1}{3}$ **[1/6]**
11. landing on $\frac{1}{2}$ on your second move **[1/12]**
12. landing on $\frac{1}{3}$ on your second move **[1/6]**
13. landing on a $\frac{1}{6}$ or a $\frac{1}{3}$ **[7/30]**
14. landing on a $\frac{1}{4}$ or a $\frac{1}{2}$ **[2/15]**
15. the finishing roll being a 3 **[0]**
16. landing on $\frac{1}{12}$ on your second move **[1/9]**
17. landing on a $\frac{1}{12}$ or "forward two spaces" **[7/30]**
18. landing on a $\frac{2}{3}$ or "back two spaces" **[2/5]**
19. landing on a space with an even denominator or an odd denominator **[9/10]**
20. landing on a space with an even denominator or requiring the piece to move two spaces **[3/5]**
21. landing on a space with an odd denominator or requiring the piece to move two spaces **[1/2]**
22. landing on a space with a 1 or a 2 in the numerator **[9/10]**
23. landing on a $\frac{1}{4}$ after being on a corner space and then landing on a $\frac{1}{12}$ **[1/18]**
24. landing on a "forward two spaces" after being on a $\frac{1}{3}$ and then landing on a $\frac{1}{6}$ on the next turn **[1/18]**
25. landing on a $\frac{1}{6}$ on your first two moves **[1/36]**
26. landing on a $\frac{1}{3}$ on your last two moves **[0]**
27. landing on two $\frac{1}{2}$s in a row **[0]**
28. from the $\frac{1}{12}$ in the right corner landing on $\frac{1}{12}$ **[1/2]**
29. rolling a 4 and landing on a $\frac{1}{6}$ at some point in the game **[1/10]**
30. rolling a 3 and landing on a $\frac{1}{6}$ at some point in the game **[1/6]**

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11-3 Game: The Probability Path

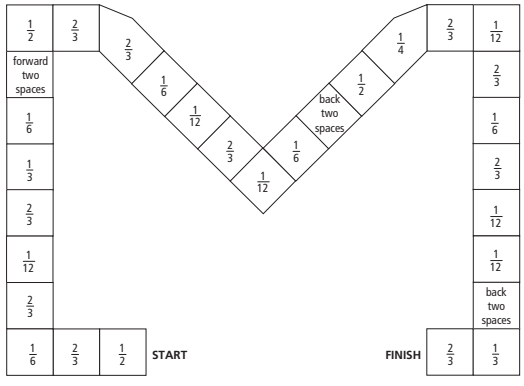
Probability of Multiple Events

This is a game for two teams of one or two players each, with an additional student serving as the host.

Materials: two distinct playing pieces, one for each team; a number cube

Game Play See Teacher Instructions page.

- Determine which team goes first.
- On your turn, you must answer a question asked by the host.
- If you do not answer correctly, your turn is over. If you answer correctly, roll a number cube and move your game piece that number of spaces. Initial the space your piece lands on. If there are directions on the space, follow them. Otherwise, your turn ends.
- Play alternates until a team reaches the finish line. The final roll does not have to land on the last square—it can go beyond it.
- Tie breaker: If all the questions have been used before a team has reached the finish line, add all the values of the spaces your team landed on (and initialed) during the game. The team with the highest total wins.
- If there is enough class time and questions remaining after the first game, select new teams and a new host and continue playing.



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11-4 Game: On One Condition

Conditional Probability

This is a game for two teams of two players. Each team needs nine index cards.

Rules

- Take a few minutes to examine the table below.
- In each round, teams simultaneously write a conditional probability on an index card. Do *not* calculate it at this point.
- Exchange cards and calculate the other team's conditional probability.
- Exchange cards again and calculate your own team's conditional probability, verifying your opponent's result.
- Resolve the difference with your opponent if the probabilities do not match.
- Record your results in the appropriate spaces below.
- The team with the higher-valued probability wins the round.
- A conditional probability can be used only once in the game.
- Do *not* proceed to the probability calculating stage if both teams select the same conditional probability. Both teams must select another conditional probability. This initial one is still available for another round.
- The team that wins the most rounds wins the game.

Number of Fish Stocked in Man-Made Lake

Species	Weight of Fish (lb)					
	0–3.9	4–7.9	8–11.9	12–15.9	16–20	Row Total
Bass	320	250	130	0	0	700
Carp	110	220	310	210	50	900
Catfish	120	340	410	280	50	1200
Trout	150	490	250	110	0	1000
Column Total	700	1300	1100	600	100	3800

	Team 1	Team 2
Round 1	Check students' work.	
Round 2		
Round 3		
Round 4		
Round 5		
Round 6		
Round 7		
Round 8		
Round 9		

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11-5 Puzzle: Can We Set a Data?

Analyzing Data

The table below shows results from the 2008 Summer Olympics in Beijing, China. Use the data to answer the questions below the table, and determine the host countries for the 2012 Summer and 2014 Winter Olympics.

Split Time for First 250 m of Women's Kayak Double (K2), Heat 1									
GER	POL	CZE	FIN	ESP	JPN	SLO	RSA	GBR	
51.05	50.74	50.83	51.50	51.81	52.17	51.41	53.36	53.01	
Men's Handball Scores – Final Day									
CRO	ESP	FRA	ISL	RUS	POL	DEN	KOR		
29	35	28	23	28	29	37	26		
Badminton Mixed Doubles – Medal Round Scores									
CHN	CHN	CHN	INA1	INA1	INA1	KOR	KOR	INA2	INA2
19	21	23	21	17	21	21	21	11	17
Men's Archery Quarterfinals									
UKR	JPN	MAS	RUS	CUB	KOR	MEX	USA		
115	106	104	109	108	108	113	106		
Women's Beach Volleyball – Quarterfinal Scores									
BRA1	USA1	CHN1	AUT	USA2	CHN2	AUS	BRA2		
33	42	42	24	30	42	36	45		
Men's Sailing 470 Finals – Rounded Seconds After Winner (Australia)									
ARG	CRO	FRA	GBR	ITA	JPN	NED	POR	ESP	
24	26	18	6	17	5	24	32	9	
Women's Trampoline – Finals									
CAN1	CAN2	CHN	GEO	GER	RUS	UKR	POR		
37.0	35.5	37.8	36.1	18.9	36.2	36.6	36.9		
Men's Team Sabre – Final Day									
FRA	USA	ITA	RUS	EGY	HUN	BLR	CHN		
45	37	45	44	25	45	45	39		

Summer 2012 Finalists

1. Country with the closest score to the archery mean: RUS, or Russia
2. Country with the median time in sailing: FRA, or France
3. Country with 3 more points than the median of upper part in handball: ESP, or Spain
4. Country with 1.5 fewer points than the median of lower part in volleyball: USA, or the United States of America
5. Country with the lowest time on right whisker of box-and-whisker plot of kayak times: GBR, or Great Britain

Summer 2012 Host Country

6. Country closest to one-third of the mean time in sailing: GBR, or Great Britain

Winter 2014 Finalists

7. Country whose only score in badminton was the mode: KOR, or Korea
8. Country with Sabre score just below the 50th percentile: RUS, or Russia
9. Country with a volleyball score 12.75 points below the mean: AUT, or Austria

Winter 2014 Host Country

10. Country with a trampoline score that was the greatest value less than the median: RUS, or Russia

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11-6 Puzzle: The Usual Suspects

Standard Deviation

Background

On March 14th of last year, a shocking crime was committed. An evil individual set out to crumble the very foundation of Mathtropolis. The villain stated that π is *equal* to 3.14. Fortunately, the Mathtropolis Police Department (MPD) has 10 suspects in custody and they are sure one of them is the culprit.

Goal

Help the MPD determine which of the suspects is guilty. The table shows the suspects' responses to a series of questions whose population mean and standard deviation are given below.

Instructions

- Begin by identifying the five suspects whose responses to Question 1 are more than 1.5 standard deviations from the mean. Suspects B, D, E, G, J
- Of the five, identify the suspects whose responses to Questions 2 and 3 are *both* more than 1.7 standard deviations from the mean. Suspects D, G, J
- Of these three, identify the suspect whose responses to Questions 4, 5, and 6 are all at least 2 standard deviations from the mean. This is the mathematical criminal! Who is it? Suspect G

	A	B	C	D	E	F	G	H	I	J
Q1	14	7	21	28	2	9	5	25	22	27
Q2	0	9	1	8	5	4	2	5	10	1
Q3	77%	96%	89%	66%	59%	32%	100%	92%	86%	60%
Q4	13	1	2	16	12	52	4	13	7	18
Q5	3	4	1	2	2	5	6	3	4	5
Q6	3	5	4	7	3	4	3	5	4	3

Question 1: How many waffles did you eat during this past week?
mean = 17,
standard deviation = 6

Question 2: How many calculators do you and your family own?
mean = 5.1,
standard deviation = 1.3

Question 3: What was your high school algebra average?
mean = 83.2%,
standard deviation = 9.5%

Question 4: How many weeks ago did you last see an action movie?
mean = 10.3,
standard deviation = 2.7

Question 5: How many times a day do you brush your teeth?
mean = 3.9,
standard deviation = 0.8

Question 6: How many digits of π can you recite from memory?
mean = 5.2,
standard deviation = 1.1

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11-7 Activity: Proportions and Samples

Samples and Surveys

This activity is best for groups of three or four students.

Step 1

Decide as a group on a proportion you would like to estimate using sampling. Here are some ideas.

- What proportion of students prefers sweet-tasting food to sour-tasting food?
- What proportion of students drives his or her own car to school?
- What proportion of students has a cell phone at school?

Step 2

Split your group and poll every student in class. Be sure to include yourselves.

Step 3

Analyze the data your group collected. Use your results to answer the following questions.

1. Which proportion are you estimating?
   Check students' work.
2. Which question did your group ask the other students?
   Check students' work.
3. From which population does your sample come?
   Answers will vary. Sample: all students in school; all juniors in school; all seniors in school
4. What is your sample proportion?
   Check students' work.
5. What is your margin of error?
   Check students' work.
6. How many students would you have to poll to get a margin of error of ±5%?
   400
7. Can you identify any bias in your sampling method?
   Answers will vary. Samples: convenience sample; group members did not use the same question.

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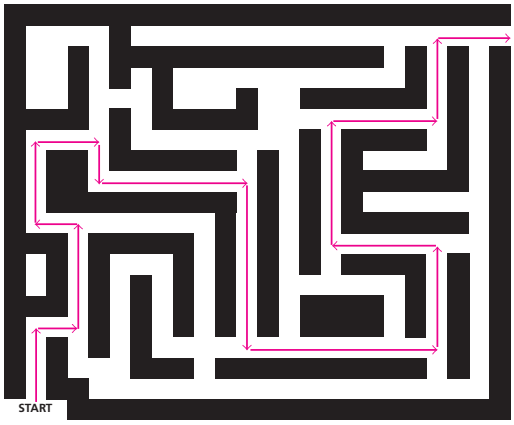
11-8 Puzzle: Right, Then Left, Right?

Binomial Distributions

Bernoulli took a stroll through a garden maze. He used his weighted coin (heads has a probability of 0.3) to get through the maze. Here are the guidelines:

- He will follow the path until he is forced to make a left/right decision. That is, he will not turn off a straight course on his own.
- At a left/right decision, he will repeatedly flip the coin and note the number of heads. His experiments are listed below the maze, in the order he performed them.
- If the probability of getting the number *or* fewer than the number of heads he recorded is less than 25%, he turns left.
- If the probability of getting the number or fewer than the number of heads he recorded is greater than 25%, he turns right.

Trace Bernoulli's path below. Show your work on another sheet of paper.



1. 12 flips, 2 heads ≈16.8%; left
2. 7 flips, 1 heads ≈24.7%; left
3. 8 flips, 2 heads ≈29.6%; right
4. 10 flips, 3 heads ≈26.7%; right
5. 11 flips, 2 heads ≈20.0%; left
6. 10 flips, 2 heads ≈23.3%; left
7. 13 flips, 3 heads ≈21.8%; left
8. 5 flips, 1 heads ≈36.0%; right
9. 7 flips, 2 heads ≈31.8%; right
10. 16 flips, 4 heads ≈20.4%; left
11. 11 flips, 3 heads ≈25.7%; right

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TEACHER INSTRUCTIONS

11-9 Game: Risk and Reward

Normal Distributions

Provide the host with the following questions and answers.

	Vocabulary (Define)	What's the *z*-score?	What's the SAT Score?	How Many Students Scored:	Review
10 pts	mean	Math: 515	Writing: 0	between 399 and 515 on Math?	Find ${}_8P_2$ and ${}_8C_2$.
20 pts	standard deviation	Reading: 390	Math: −1	between 384 and 604 on Writing?	Add: $2+6+10+\cdots+38$
30 pts	*z*-score	Writing: 659	Math: 1.5	between 278 and 614 on Reading?	Solve: $3^{2x+1}=81$
40 pts	normally distributed	Math: 370	Reading: −2.25	greater than 747 on Math?	Find the center and radius of $x^2+y^2-4x+2y=4$.
50 pts	standard normal curve	Reading: 698	Writing: 2.7	greater than 384 on Writing?	Solve: $\begin{cases} 2x+y=5 \\ -3x+4y=31 \end{cases}$

The middle three categories are based on the results of the 2008 SAT scores. About 1,500,000 college-bound students took this test and their normally distributed results are summarized below. Copy the table for each team before starting the game.

2008 SAT Results	Subject	Mean	Standard Deviation
1,500,000 testers	Reading	502	112
	Math	515	116
	Writing	494	110

Source: www.collegeboard.com

Answers to Questions

	Vocabulary	*z*-score?	SAT Score?	Number of Students	Review
10 pts	1. See below.	0	494	510,000	56, 28
20 pts	2. See below.	−1	399	1,020,000	200
30 pts	3. See below.	1.5	689	1,222,500	1.5
40 pts	4. See below.	−1.25	250	37,500	(2, −1), $r=3$
50 pts	5. See below.	1.75	791	1,260,000	(−1, 7)

1. the numerical average
2. a measure of how much the values in a data set vary, $\sigma = \sqrt{\frac{\sum(x-\bar{x})^2}{n}}$
3. a number that tells how many standard deviations away from the mean a particular score is
4. about 68% of data fall within one standard deviation of the mean; about 95% of data fall within two standard deviations of the mean
5. a normal distribution with mean 0 and standard deviation 1

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11-9 Game: Risk and Reward

Normal Distributions

This is a game for five students. One student is the host and the others form two teams.

Host: Your teacher will provide you with a separate sheet of questions and answers. Keep track of the score using the table below.

Players: A gameboard with categories and point values is shown below. Use it to keep track of which questions are still available and your score.

Rules: Decide which team goes first. When it is your turn:

- Select a category. The host will start with the least-points available question.
- If you answer correctly within the time assigned by your teacher, you earn the points for that question. Your turn continues and you select a category again.
- If you answer incorrectly, you lose that number of points and your opponent takes over. In addition, your opponent has 10 seconds to provide the correct answer and earn the points from the missed question. Your group or your teacher can decide to change the response time if needed.
- Play continues in this manner until all the questions have been used. The team with the highest point total wins. See Teacher Instructions page.

	Vocabulary (Define)	What's the *z*-score?	What's the SAT Score?	How Many Students Scored:	Review
10 pts					
20 pts					
30 pts					
40 pts					
50 pts					

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12-1 Activity: Board With Matrices

Adding and Subtracting Matrices

For this activity, you will need some markers and highlighters in different colors, and a large poster board or sheet of paper. Your teacher may also provide colored cellophane, glitter, stencils, paste, and other art supplies.

- On your poster board, make a large and clear presentation of how to add and subtract two matrices. Use an example from the textbook or write one of your own.
- Be inventive in your use of artistic devices. For example, in your addition of two matrices, you could draw boxes and other shapes around corresponding matrix elements.

$$\begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 1 & 9 \end{bmatrix}$$

- Or you could choose a color scheme. This means that in your presentation of adding two matrices, you could highlight corresponding pairs of addends in the same color. Or you could write an explanation of each matrix operation in your own words and color-code each step.
- These are only a few suggestions. The use of different colors is only one artistic device; the use of different lettering is another. Choose an artistic device that will help classmates follow the math and see patterns they might otherwise miss. Use whatever artistic device works best for you.

Your teacher can display all of the posters when everyone has finished. Select the poster that is your favorite. If there is enough time left in class, your teacher can start a discussion by calling on students. Be prepared to discuss and explain why you chose your favorite poster. Comment not only on the artistic elements, but also on mathematical accuracy and any other element you think is important.

Check students' work.