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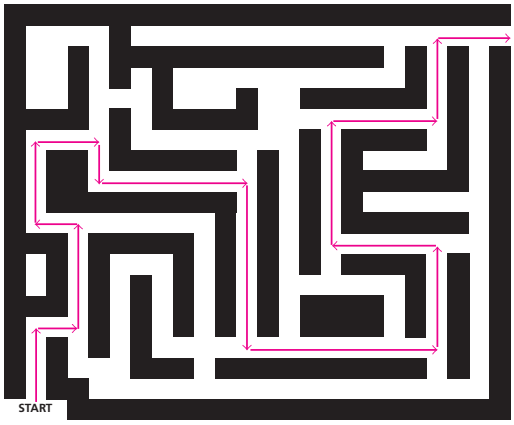
11-8 Puzzle: Right, Then Left, Right?

Binomial Distributions

Bernoulli took a stroll through a garden maze. He used his weighted coin (heads has a probability of 0.3) to get through the maze. Here are the guidelines:

- He will follow the path until he is forced to make a left/right decision. That is, he will not turn off a straight course on his own.
- At a left/right decision, he will repeatedly flip the coin and note the number of heads. His experiments are listed below the maze, in the order he performed them.
- If the probability of getting the number *or* fewer than the number of heads he recorded is less than 25%, he turns left.
- If the probability of getting the number or fewer than the number of heads he recorded is greater than 25%, he turns right.

Trace Bernoulli's path below. Show your work on another sheet of paper.



1. 12 flips, 2 heads ≈16.8%; left
2. 7 flips, 1 heads ≈24.7%; left
3. 8 flips, 2 heads ≈29.6%; right
4. 10 flips, 3 heads ≈26.7%; right
5. 11 flips, 2 heads ≈20.0%; left
6. 10 flips, 2 heads ≈23.3%; left
7. 13 flips, 3 heads ≈21.8%; left
8. 5 flips, 1 heads ≈36.0%; right
9. 7 flips, 2 heads ≈31.8%; right
10. 16 flips, 4 heads ≈20.4%; left
11. 11 flips, 3 heads ≈25.7%; right

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TEACHER INSTRUCTIONS

11-9 Game: Risk and Reward

Normal Distributions

Provide the host with the following questions and answers.

	Vocabulary (Define)	What's the *z*-score?	What's the SAT Score?	How Many Students Scored:	Review
10 pts	mean	Math: 515	Writing: 0	between 399 and 515 on Math?	Find ${}_8P_2$ and ${}_8C_2$.
20 pts	standard deviation	Reading: 390	Math: −1	between 384 and 604 on Writing?	Add: $2+6+10+\cdots+38$
30 pts	*z*-score	Writing: 659	Math: 1.5	between 278 and 614 on Reading?	Solve: $3^{2x+1}=81$
40 pts	normally distributed	Math: 370	Reading: −2.25	greater than 747 on Math?	Find the center and radius of $x^2+y^2-4x+2y=4$.
50 pts	standard normal curve	Reading: 698	Writing: 2.7	greater than 384 on Writing?	Solve: $\begin{cases} 2x+y=5 \\ -3x+4y=31 \end{cases}$

The middle three categories are based on the results of the 2008 SAT scores. About 1,500,000 college-bound students took this test and their normally distributed results are summarized below. Copy the table for each team before starting the game.

2008 SAT Results	Subject	Mean	Standard Deviation
1,500,000 testers	Reading	502	112
	Math	515	116
	Writing	494	110

Source: www.collegeboard.com

Answers to Questions

	Vocabulary	*z*-score?	SAT Score?	Number of Students	Review
10 pts	1. See below.	0	494	510,000	56, 28
20 pts	2. See below.	−1	399	1,020,000	200
30 pts	3. See below.	1.5	689	1,222,500	1.5
40 pts	4. See below.	−1.25	250	37,500	(2, −1), $r=3$
50 pts	5. See below.	1.75	791	1,260,000	(−1, 7)

1. the numerical average
2. a measure of how much the values in a data set vary, $\sigma = \sqrt{\frac{\sum(x-\bar{x})^2}{n}}$
3. a number that tells how many standard deviations away from the mean a particular score is
4. about 68% of data fall within one standard deviation of the mean; about 95% of data fall within two standard deviations of the mean
5. a normal distribution with mean 0 and standard deviation 1

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11-9 Game: Risk and Reward

Normal Distributions

This is a game for five students. One student is the host and the others form two teams.

Host: Your teacher will provide you with a separate sheet of questions and answers. Keep track of the score using the table below.

Players: A gameboard with categories and point values is shown below. Use it to keep track of which questions are still available and your score.

Rules: Decide which team goes first. When it is your turn:

- Select a category. The host will start with the least-points available question.
- If you answer correctly within the time assigned by your teacher, you earn the points for that question. Your turn continues and you select a category again.
- If you answer incorrectly, you lose that number of points and your opponent takes over. In addition, your opponent has 10 seconds to provide the correct answer and earn the points from the missed question. Your group or your teacher can decide to change the response time if needed.
- Play continues in this manner until all the questions have been used. The team with the highest point total wins. See Teacher Instructions page.

	Vocabulary (Define)	What's the *z*-score?	What's the SAT Score?	How Many Students Scored:	Review
10 pts					
20 pts					
30 pts					
40 pts					
50 pts					

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12-1 Activity: Board With Matrices

Adding and Subtracting Matrices

For this activity, you will need some markers and highlighters in different colors, and a large poster board or sheet of paper. Your teacher may also provide colored cellophane, glitter, stencils, paste, and other art supplies.

- On your poster board, make a large and clear presentation of how to add and subtract two matrices. Use an example from the textbook or write one of your own.
- Be inventive in your use of artistic devices. For example, in your addition of two matrices, you could draw boxes and other shapes around corresponding matrix elements.

$$\begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 1 & 9 \end{bmatrix}$$

- Or you could choose a color scheme. This means that in your presentation of adding two matrices, you could highlight corresponding pairs of addends in the same color. Or you could write an explanation of each matrix operation in your own words and color-code each step.
- These are only a few suggestions. The use of different colors is only one artistic device; the use of different lettering is another. Choose an artistic device that will help classmates follow the math and see patterns they might otherwise miss. Use whatever artistic device works best for you.

Your teacher can display all of the posters when everyone has finished. Select the poster that is your favorite. If there is enough time left in class, your teacher can start a discussion by calling on students. Be prepared to discuss and explain why you chose your favorite poster. Comment not only on the artistic elements, but also on mathematical accuracy and any other element you think is important.

Check students' work.

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12-2 Activity: Matrix "Eggsperiment"

Matrix Multiplication

For this activity you will need some empty egg cartons, pennies, dimes, and tape.

- Use scissors to divide the cartons into 2-by-1, 2-by-2, and 2-by-3 sections.
- Decide each section's orientation to obtain a matrix. For instance, a 2-by-3 section can give you a 2×3 matrix or 3×2 matrix.
- Place anywhere from 0 to 5 coins in each space for every matrix. Use dimes to represent negative numbers. Leave some of the spaces empty.

Activity Directions

The number of pennies minus the number of dimes in each position of the carton represents the value of that entry in the matrix. Pick a partner and compare your matrices to your classmates' matrices. Decide whether or not you can add your matrix to another matrix or multiply your matrix with another matrix.

Add or multiply your matrix with your partner's to make a new resulting matrix. Decide whether or not a different size matrix or additional coins are required. You can use tape to put smaller matrices together and create larger ones. If time permits, change partners and repeat the activity.

1. Which size matrices could be added to your matrix?
   matrices with the same dimension as your matrix

2. Which size matrices could be multiplied with your matrix? Does the way the carton was oriented make a difference?
   To multiply two matrices, the number of columns in the first matrix must equal the number of rows in the second matrix. Orientation makes a difference; a 2×3 matrix is different from a 3×2 matrix.

3. How do you find the size of a matrix resulting from the multiplication of two matrices? Explain by giving an example.
   An $n \times m$ matrix multiplied by an $m \times p$ matrix gives an $n \times p$ matrix.

4. Is the size of the product of two matrices always different from the original matrices? Explain by giving an example.
   Not necessarily; multiplying two square matrices gives a square matrix of the same size.

5. Is the size of the sum of two matrices always the same as the original matrices?
   yes

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12-3 Puzzle: That's Sum Matrix!

Determinants and Inverses

Match each exercise to its answer. Write the exercise number of the corresponding letter into the empty puzzle grid at the bottom of the page. All of your grid entries are correct if the sum of each column, row, and diagonal is 34.

1. Find the inverse of $\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$. **H**
2. Evaluate $\begin{vmatrix} 3 & 4 \\ 1 & 5 \end{vmatrix}$. **B**
3. Evaluate the determinant of $\begin{bmatrix} 6 & 7 \\ 8 & 9 \end{bmatrix}$. **A**
4. Find $\begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}^{-1}$. **G**
5. Evaluate $\det \begin{bmatrix} 3 & 4 \\ 6 & 8 \end{bmatrix}$. **C**
6. If $M = \begin{bmatrix} 9 & 11 \\ 4 & 5 \end{bmatrix}$, find M^{-1}. **E**
7. Find the inverse of $\begin{bmatrix} 8 & 7 \\ 2 & 3 \end{bmatrix}$. **F**
8. Evaluate $\begin{vmatrix} -1 & 4 \\ 0 & 9 \end{vmatrix}$. **D**

A. -2

B. 11

C. 0

D. -9

E. $\begin{bmatrix} 5 & -11 \\ -4 & 9 \end{bmatrix}$

F. $\begin{bmatrix} 0.3 & -0.7 \\ -0.2 & 0.8 \end{bmatrix}$

G. $\begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix}$

H. $\begin{bmatrix} 0.8 & -0.6 \\ -0.2 & 0.4 \end{bmatrix}$

16	A. 3	B. 2	13
C. 5	10	11	D. 8
9	E. 6	F. 7	12
G. 4	15	14	H. 1

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12-4 Puzzle: A Four-Word and Down-Word Puzzle

Inverse Matrices and Systems

Solve the following linear systems by using inverse matrices. Write the letter of each answer in the table below. Note that you may have to write some answers more than once. The letters of your answers should spell a word across each row and down each column.

1. $x + y = 5$, $x - 2y = -4$ **L**
2. $\begin{bmatrix} 1 & -3 \\ -5 & 16 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 5 \end{bmatrix}$ **I**
3. $2x + 3y = 5$, $x + 2y = 6$ **M**
4. $\begin{bmatrix} 2 & -3 \\ -4 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ **B**
5. $3x - y = 5$, $x + 4y = 6$ **C**
6. $\begin{bmatrix} 2 & -5 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$ **O**
7. $x - 5y = -2$, $3x + 2y = 11$ **K**
8. $\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 11 \\ 2 \end{bmatrix}$ **N**

A. no solution

B. $x = 2, y = 1$

C. $x = 2, y = 1$

I. $x = -1, y = 0$

K. $x = 3, y = 1$

L. $x = 2, y = 3$

M. $x = -8, y = 7$

N. $x = 3, y = 4$

O. $x = 5, y = 2$

1. L	2. I	3. M	4. B
A	R	E	A
5. C	6. O	R	7. K
7. K	8. N	E	E

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TEACHER INSTRUCTIONS

12-5 Game: An "Element"-ary Matching Game

Geometric Transformations

Provide the host with the following questions and answers.

Questions	Answers
• for $\begin{bmatrix} 0 & -1 \\ 0 & 4 \end{bmatrix}$, the matrix whose addition adds 6 to x and subtracts 2 from y	$\begin{bmatrix} 6 & 6 \\ -2 & -2 \end{bmatrix}$
• the image after dilating $\begin{bmatrix} 5 & 8 \\ 5 & 1 \end{bmatrix}$ by 2	$\begin{bmatrix} 10 & 16 \\ 10 & 2 \end{bmatrix}$
• the reflection of $\begin{bmatrix} -3 & -5 \\ 4 & 7 \end{bmatrix}$ across the x-axis	$\begin{bmatrix} -3 & -5 \\ -4 & -7 \end{bmatrix}$
• the image after rotating $\begin{bmatrix} 7 & 10 \\ 2 & 0 \end{bmatrix}$ by 270°	$\begin{bmatrix} 2 & 0 \\ -7 & -10 \end{bmatrix}$
• the matrix which rotates $\begin{bmatrix} 1 & 4 \\ 5 & -3 \end{bmatrix}$ by 180°	$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$
• the image after rotating $\begin{bmatrix} -8 & 2 \\ 5 & 7 \end{bmatrix}$ by 360°	$\begin{bmatrix} -8 & 2 \\ 5 & 7 \end{bmatrix}$
• for $\begin{bmatrix} -3 & 4 \\ 4 & 0 \end{bmatrix}$, the matrix whose addition subtracts 1 from x and adds 5 to y	$\begin{bmatrix} -1 & -1 \\ 5 & 5 \end{bmatrix}$
• the matrix which reflects $\begin{bmatrix} 6 & -5 \\ -4 & 8 \end{bmatrix}$ across the line $y = x$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
• the matrix which reflects $\begin{bmatrix} -1 & 2 \\ 4 & 5 \end{bmatrix}$ across the y-axis	$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$
• the image after reflecting $\begin{bmatrix} 3 & 5 \\ 1 & -3 \end{bmatrix}$ across the line $y = -x$	$\begin{bmatrix} -1 & 3 \\ -3 & -5 \end{bmatrix}$
• the image after dilating $\begin{bmatrix} 0 & 12 \\ 8 & 8 \end{bmatrix}$ by $\frac{1}{4}$	$\begin{bmatrix} 0 & 3 \\ 2 & 2 \end{bmatrix}$
• the matrix which rotates $\begin{bmatrix} -7 & 2 \\ 4 & 8 \end{bmatrix}$ by 90°	$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

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12-5 Game: An "Element"-ary Matching Game

Geometric Transformations

This is a game for two players.

- Cut out the squares below. Number the blank sides of the first set of cards 1 through 12, and the label the blank sides of the second set of cards with the letters A through L. Turn the cards face down on a table or desk, with the sorted numbers on your left and the sorted letters on your right.
- To begin play, one student draws a number card and a letter card. If both students agree that the cards match, then the student keeps the cards and repeats the turn. If the students agree that the cards do not match, then both cards are turned over and it is the opponent's turn. If the students disagree, then the student who can prove an error gets a bonus turn. The student with the most cards wins! See Teacher Instructions page.

for $\begin{bmatrix} 0 & -1 \\ 0 & 4 \end{bmatrix}$, the matrix whose addition 6 to x and subtracts 2 from y	the image after dilating $\begin{bmatrix} 5 & 8 \\ 5 & 1 \end{bmatrix}$ by 2	the reflection of $\begin{bmatrix} -3 & -5 \\ 4 & 7 \end{bmatrix}$ across the x-axis	the image after rotating $\begin{bmatrix} 7 & 10 \\ 2 & 0 \end{bmatrix}$ by 270°
the matrix that rotates $\begin{bmatrix} 1 & 4 \\ 5 & -3 \end{bmatrix}$ by 180°	the image after rotating $\begin{bmatrix} -8 & 2 \\ 5 & 7 \end{bmatrix}$ by 360°	for $\begin{bmatrix} -3 & 4 \\ 4 & 0 \end{bmatrix}$, the matrix whose addition 1 from x and adds 5 to y	the matrix that reflects $\begin{bmatrix} 6 & -5 \\ -4 & 8 \end{bmatrix}$ across the line $y = x$
the matrix that reflects $\begin{bmatrix} -1 & 2 \\ 4 & 5 \end{bmatrix}$ across the y-axis	the image after reflecting $\begin{bmatrix} 3 & 5 \\ 1 & -3 \end{bmatrix}$ across the line $y = -x$	the image after dilating $\begin{bmatrix} 0 & 12 \\ 8 & 8 \end{bmatrix}$ by $\frac{1}{4}$	the matrix that rotates $\begin{bmatrix} -7 & 2 \\ 4 & 8 \end{bmatrix}$ by 90°

$\begin{bmatrix} 0 & 3 \\ 2 & 2 \end{bmatrix}$	$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$	$\begin{bmatrix} -8 & 2 \\ 5 & 7 \end{bmatrix}$	$\begin{bmatrix} 6 & 6 \\ -2 & -2 \end{bmatrix}$
$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} -1 & -1 \\ 5 & 5 \end{bmatrix}$	$\begin{bmatrix} 10 & 16 \\ 10 & 2 \end{bmatrix}$	$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$
$\begin{bmatrix} -3 & -5 \\ -4 & -7 \end{bmatrix}$	$\begin{bmatrix} 2 & 0 \\ -7 & -10 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} -1 & 3 \\ -3 & -5 \end{bmatrix}$

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TEACHER INSTRUCTIONS

12-6 Game: Last Will Be First

Vectors

Provide the host with the following questions and answers.

Questions	Answers
• the magnitude of the vector $\langle 20, 21 \rangle$	29
• the dot product of $\langle 4, 7 \rangle$ and $\langle 9, -5 \rangle$	1
• the sum of $\langle 2, 3 \rangle$ and $\langle -1, 4 \rangle$	$\langle 1, 7 \rangle$
• the magnitude of the vector $\langle 3, -4 \rangle$	5
• the dot product of $\langle 6, -9 \rangle$ and $\langle 7, 5 \rangle$	−3
• the sum of $\langle 1, 5 \rangle$ and $\langle 3, -8 \rangle$	$\langle 4, -3 \rangle$
• the magnitude of the vector $\langle -5, 14 \rangle$	14.9
• the dot product of $\langle 8, 6 \rangle$ and $\langle -3, 4 \rangle$	0
• the sum of $\langle -2, 7 \rangle$ and $\langle 5, 0 \rangle$	$\langle 3, 7 \rangle$
• the magnitude of the vector $\langle 5, 6 \rangle$	7.8
• the dot product of $\langle -2, 5 \rangle$ and $\langle 8, 4 \rangle$	4
• the sum of $\langle 4, -1 \rangle$ and $\langle 3, 2 \rangle$	$\langle 7, 1 \rangle$
• the magnitude of the vector $\langle -3, -3 \rangle$	4.2
• the dot product of $\langle 5, 6 \rangle$ and $\langle -3, 4 \rangle$	9
• the sum of $\langle 8, -5 \rangle$ and $\langle -5, 9 \rangle$	$\langle 3, 4 \rangle$
• the magnitude of the vector $\langle -8, 15 \rangle$	17
• the dot product of $\langle 6, -4 \rangle$ and $\langle 5, 7 \rangle$	2
• the sum of $\langle 2, -3 \rangle$ and $\langle 4, -1 \rangle$	$\langle 6, -4 \rangle$
• the magnitude of the vector $\langle -7, -8 \rangle$	10.6
• the dot product of $\langle 7, 3 \rangle$ and $\langle -4, 6 \rangle$	−10
• the sum of $\langle 8, -7 \rangle$ and $\langle -6, 5 \rangle$	$\langle 2, -2 \rangle$
• the magnitude of the vector $\langle -5, -12 \rangle$	13
• the dot product of $\langle 7, 4 \rangle$ and $\langle -5, 7 \rangle$	−7
• the sum of $\langle 9, -5 \rangle$ and $\langle -4, 3 \rangle$	$\langle 5, -2 \rangle$

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12-6 Game: Last Will Be First

Vectors

This is a game for two players.

The top table has 24 questions involving vectors. The bottom table has the answers. Players alternate turns. During your turn, select up to 7 questions to answer. If you correctly answer all the questions you selected and find the matching answers, you can cross out all of them from the tables. If you make a mistake, you lose your turn and nothing is crossed out. Your opponent will check to verify your answers.

The object of the game is to be the player to cross out the last entries in the tables. So you should carefully consider the number of questions you answer during a turn. If you cross out as many vectors as you can, you may not be the winner.

See Teacher Instructions page.

the magnitude of the vector $\langle 20, 21 \rangle$	the dot product of $\langle 4, 7 \rangle$ and $\langle 9, -5 \rangle$	the sum of $\langle 2, 3 \rangle$ and $\langle -1, 4 \rangle$	the magnitude of the vector $\langle 3, -4 \rangle$
the dot product of $\langle 6, -9 \rangle$ and $\langle 7, 5 \rangle$	the sum of $\langle 1, 5 \rangle$ and $\langle 3, -8 \rangle$	the magnitude of the vector $\langle -5, 14 \rangle$	the dot product of $\langle 8, 6 \rangle$ and $\langle -3, 4 \rangle$
the sum of $\langle -2, 7 \rangle$ and $\langle 5, 0 \rangle$	the magnitude of the vector $\langle 5, 6 \rangle$	the dot product of $\langle -2, 5 \rangle$ and $\langle 8, 4 \rangle$	the sum of $\langle 4, -1 \rangle$ and $\langle 3, 2 \rangle$
the magnitude of the vector $\langle -3, -3 \rangle$	the dot product of $\langle 5, 6 \rangle$ and $\langle -3, 4 \rangle$	the sum of $\langle 8, -5 \rangle$ and $\langle -5, 9 \rangle$	the magnitude of the vector $\langle -8, 15 \rangle$
the dot product of $\langle 6, -4 \rangle$ and $\langle 5, 7 \rangle$	the sum of $\langle 2, -3 \rangle$ and $\langle 4, -1 \rangle$	the magnitude of the vector $\langle -7, -8 \rangle$	the dot product of $\langle 7, 3 \rangle$ and $\langle -4, 6 \rangle$
the sum of $\langle 8, -7 \rangle$ and $\langle -6, 5 \rangle$	the magnitude of the vector $\langle -5, -12 \rangle$	the dot product of $\langle 7, 4 \rangle$ and $\langle -5, 7 \rangle$	the sum of $\langle 9, -5 \rangle$ and $\langle -4, 3 \rangle$

4.2	$\langle 3, 7 \rangle$	14.9	−7
2	29	−10	$\langle 3, 4 \rangle$
17	−3	$\langle 7, 1 \rangle$	10.6
13	$\langle 6, -4 \rangle$	1	$\langle 5, -2 \rangle$
$\langle 1, 7 \rangle$	4	$\langle 2, -2 \rangle$	5
0	7.8	$\langle 4, -3 \rangle$	9

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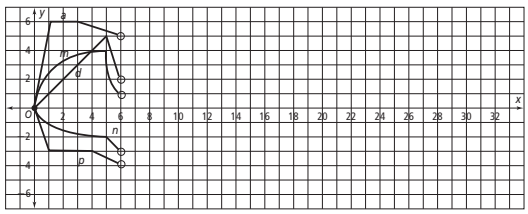
13-1 Game: Repeating Myself

Exploring Periodic Data

The graphs of five periodic functions over one cycle are shown below. The functions are a, m, d, n, and p.



This game is for the whole class. Separate into teams of three or four students.

- Your job is to predict the value of each function based on the graphs. Remember that each function is periodic and one cycle is shown.
- Each correct response earns 3 points.
- The team that earns the most points wins.

1. $a(6) =$ 0
2. $m(5) =$ 4
3. $p(1) =$ −3
4. $n(12) =$ 0
5. $m(12) =$ 0
6. $a(13) =$ 6
7. $m(11) =$ 4
8. $p(13) =$ −3
9. $n(17) =$ −2
10. $d(11) =$ 5
11. $n(18) =$ 0
12. $a(15) =$ 6
13. $p(7) =$ −3
14. $a(18) =$ 0
15. $n(23) =$ −2
16. $m(24) =$ 0
17. $n(24) =$ 0
18. $d(23) =$ 5
19. $a(19) =$ 6
20. $p(10) =$ −3
21. $n(11) =$ −2
22. $d(18) =$ 0
23. $p(28) =$ −3
24. $m(29) =$ 4
25. $n(29) =$ −2
26. $d(30) =$ 0
27. $a(27) =$ 6