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12-5 Game: An "Element"-ary Matching Game

Geometric Transformations

This is a game for two players.

- Cut out the squares below. Number the blank sides of the first set of cards 1 through 12, and the label the blank sides of the second set of cards with the letters A through L. Turn the cards face down on a table or desk, with the sorted numbers on your left and the sorted letters on your right.
- To begin play, one student draws a number card and a letter card. If both students agree that the cards match, then the student keeps the cards and repeats the turn. If the students agree that the cards do not match, then both cards are turned over and it is the opponent's turn. If the students disagree, then the student who can prove an error gets a bonus turn. The student with the most cards wins! See Teacher Instructions page.

for $\begin{bmatrix} 0 & -1 \\ 0 & 4 \end{bmatrix}$, the matrix whose addition 6 to x and subtracts 2 from y	the image after dilating $\begin{bmatrix} 5 & 8 \\ 5 & 1 \end{bmatrix}$ by 2	the reflection of $\begin{bmatrix} -3 & -5 \\ 4 & 7 \end{bmatrix}$ across the x-axis	the image after rotating $\begin{bmatrix} 7 & 10 \\ 2 & 0 \end{bmatrix}$ by 270°
the matrix that rotates $\begin{bmatrix} 1 & 4 \\ 5 & -3 \end{bmatrix}$ by 180°	the image after rotating $\begin{bmatrix} -8 & 2 \\ 5 & 7 \end{bmatrix}$ by 360°	for $\begin{bmatrix} -3 & 4 \\ 4 & 0 \end{bmatrix}$, the matrix whose addition 1 from x and adds 5 to y	the matrix that reflects $\begin{bmatrix} 6 & -5 \\ -4 & 8 \end{bmatrix}$ across the line $y = x$
the matrix that reflects $\begin{bmatrix} -1 & 2 \\ 4 & 5 \end{bmatrix}$ across the y-axis	the image after reflecting $\begin{bmatrix} 3 & 5 \\ 1 & -3 \end{bmatrix}$ across the line $y = -x$	the image after dilating $\begin{bmatrix} 0 & 12 \\ 8 & 8 \end{bmatrix}$ by $\frac{1}{4}$	the matrix that rotates $\begin{bmatrix} -7 & 2 \\ 4 & 8 \end{bmatrix}$ by 90°

$\begin{bmatrix} 0 & 3 \\ 2 & 2 \end{bmatrix}$	$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$	$\begin{bmatrix} -8 & 2 \\ 5 & 7 \end{bmatrix}$	$\begin{bmatrix} 6 & 6 \\ -2 & -2 \end{bmatrix}$
$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} -1 & -1 \\ 5 & 5 \end{bmatrix}$	$\begin{bmatrix} 10 & 16 \\ 10 & 2 \end{bmatrix}$	$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$
$\begin{bmatrix} -3 & -5 \\ -4 & -7 \end{bmatrix}$	$\begin{bmatrix} 2 & 0 \\ -7 & -10 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} -1 & 3 \\ -3 & -5 \end{bmatrix}$

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TEACHER INSTRUCTIONS

12-6 Game: Last Will Be First

Vectors

Provide the host with the following questions and answers.

Questions	Answers
the magnitude of the vector $<20, 21>$	29
the dot product of $<4, 7>$ and $<9, -5>$	1
the sum of $<2, 3>$ and $<-1, 4>$	$<1, 7>$
the magnitude of the vector $<3, -4>$	5
the dot product of $<6, -9>$ and $<7, 5>$	−3
the sum of $<1, 5>$ and $<3, -8>$	$<4, -3>$
the magnitude of the vector $<-5, 14>$	14.9
the dot product of $<8, 6>$ and $<-3, 4>$	0
the sum of $<-2, 7>$ and $<5, 0>$	$<3, 7>$
the magnitude of the vector $<5, 6>$	7.8
the dot product of $<-2, 5>$ and $<8, 4>$	4
the sum of $<4, -1>$ and $<3, 2>$	$<7, 1>$
the magnitude of the vector $<-3, -3>$	4.2
the dot product of $<5, 6>$ and $<-3, 4>$	9
the sum of $<8, -5>$ and $<-5, 9>$	$<3, 4>$
the magnitude of the vector $<-8, 15>$	17
the dot product of $<6, -4>$ and $<5, 7>$	2
the sum of $<2, -3>$ and $<4, -1>$	$<6, -4>$
the magnitude of the vector $<-7, -8>$	10.6
the dot product of $<7, 3>$ and $<-4, 6>$	−10
the sum of $<8, -7>$ and $<-6, 5>$	$<2, -2>$
the magnitude of the vector $<-5, -12>$	13
the dot product of $<7, 4>$ and $<-5, 7>$	−7
the sum of $<9, -5>$ and $<-4, 3>$	$<5, -2>$

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12-6 Game: Last Will Be First

Vectors

This is a game for two players.

The top table has 24 questions involving vectors. The bottom table has the answers. Players alternate turns. During your turn, select up to 7 questions to answer. If you correctly answer all the questions you selected and find the matching answers, you can cross out all of them from the tables. If you make a mistake, you lose your turn and nothing is crossed out. Your opponent will check to verify your answers.

The object of the game is to be the player to cross out the last entries in the tables. So you should carefully consider the number of questions you answer during a turn. If you cross out as many vectors as you can, you may not be the winner.
See Teacher Instructions page.

the magnitude of the vector $<20, 21>$	the dot product of $<4, 7>$ and $<9, -5>$	the sum of $<2, 3>$ and $<-1, 4>$	the magnitude of the vector $<3, -4>$
the dot product of $<6, -9>$ and $<7, 5>$	the sum of $<1, 5>$ and $<3, -8>$	the magnitude of the vector $<-5, 14>$	the dot product of $<8, 6>$ and $<-3, 4>$
the sum of $<-2, 7>$ and $<5, 0>$	the magnitude of the vector $<5, 6>$	the dot product of $<-2, 5>$ and $<8, 4>$	the sum of $<4, -1>$ and $<3, 2>$
the magnitude of the vector $<-3, -3>$	the dot product of $<5, 6>$ and $<-3, 4>$	the sum of $<8, -5>$ and $<-5, 9>$	the magnitude of the vector $<-8, 15>$
the dot product of $<6, -4>$ and $<5, 7>$	the sum of $<2, -3>$ and $<4, -1>$	the magnitude of the vector $<-7, -8>$	the dot product of $<7, 3>$ and $<-4, 6>$
the sum of $<8, -7>$ and $<-6, 5>$	the magnitude of the vector $<-5, -12>$	the dot product of $<7, 4>$ and $<-5, 7>$	the sum of $<9, -5>$ and $<-4, 3>$

4.2	$<3, 7>$	14.9	−7
2	29	−10	$<3, 4>$
17	−3	$<7, 1>$	10.6
13	$<6, -4>$	1	$<5, -2>$
$<1, 7>$	4	$<2, -2>$	5
0	7.8	$<4, -3>$	9

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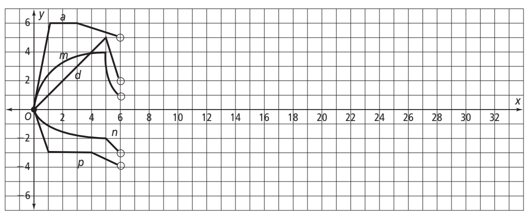
13-1 Game: Repeating Myself

Exploring Periodic Data

The graphs of five periodic functions over one cycle are shown below. The functions are a, m, d, n, and p.



This game is for the whole class. Separate into teams of three or four students.

- Your job is to predict the value of each function based on the graphs. Remember that each function is periodic and one cycle is shown.
- Each correct response earns 3 points.
- The team that earns the most points wins.

1. $a(6) =$ 0
2. $m(5) =$ 4
3. $p(1) =$ −3
4. $n(12) =$ 0
5. $m(12) =$ 0
6. $a(13) =$ 6
7. $m(11) =$ 4
8. $p(13) =$ −3
9. $n(17) =$ −2
10. $d(11) =$ 5
11. $n(18) =$ 0
12. $a(15) =$ 6
13. $p(7) =$ −3
14. $a(18) =$ 0
15. $n(23) =$ −2
16. $m(24) =$ 0
17. $n(24) =$ 0
18. $d(23) =$ 5
19. $a(19) =$ 6
20. $p(10) =$ −3
21. $n(11) =$ −2
22. $d(18) =$ 0
23. $p(28) =$ −3
24. $m(29) =$ 4
25. $n(29) =$ −2
26. $d(30) =$ 0
27. $a(27) =$ 6

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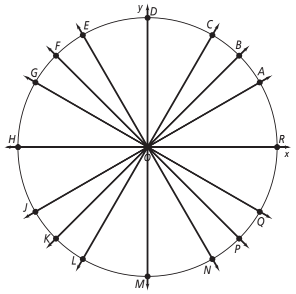
13-2 Puzzle: Spinning Around

Angles and the Unit Circle

The points along the circle shown below represent locations of houses. The diameters of the circle with center *O* determine central angles, whose measures are multiples of 30° and 45°.



- Starting at point *G*, move along the circle according to these directions: clockwise 60°, counterclockwise 45°, clockwise 135°, counterclockwise 90°, clockwise 60°, counterclockwise 90°, clockwise 180°, clockwise 75°, and counterclockwise 165°

  What point are you at now? **A**

Three students live in three different houses. Follow the clues to find out where each student lives. Use all the clues below.

- The students live along the shorter arc between *G* and the point you found above.
- The three homes are "consecutive" points along the circle.
- Student 1 lives on the circle between Student 2 and Student 3.
- None of the students live at point *C*.
- Student 2 lives closest to point *C*.

Student 1 lives at point **E**.

Student 2 lives at point **D**.

Student 3 lives at point **F**.

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13-3 Activity: Revolutions per Minute

Radian Measure

Circles appear everywhere in everyday life. In earlier work, you learned about central angles and how to measure them in degrees. One way to look at the motion of a point around a circle is to measure the degrees traveled over time. Besides using degrees over time, there are other ways to deal with the motion of a point around a circle.

- Begin by working as a class. Think about and list examples in everyday life where you expect to find a point move around a circle. Be as specific as you can. For example, you could list a compact disc or a bicycle wheel. List specific situations below.

  Answers may vary. Samples: merry-go-round, gyroscope, centrifuge, aircraft propeller, circular saw blade, compact disc, fan blade, some water sprinklers, barrel

- As a group, list various situations in which you hear the term *revolutions per minute* (rpm). It is another way to measure the speed at which a point travels around a circle by counting the number of times a point goes around in a circle over time.

  Answers may vary. Samples: circular saw blade, fan blade, automobile crankshaft

- A point travels around a circle at 5400 revolutions per minute. Through how many degrees does that point travel in one minute? **1,944,000°**

  Now work in small groups to convert measures of circular motion. For example, you can convert speed in miles per hour to revolutions per minute.

- Suppose a bicycle travels at 10 miles per hour. The wheel of the bicycle has a 14-inch radius. Find how many revolutions a point on the wheel travels in one minute. (In other words, find the wheel's rpm.) Show your work.

  $\frac{10\text{ mi}}{1\text{ hr}} = \frac{10 \times 5280\text{ ft}}{60\text{ min}} = \frac{10 \times 5280 \times 12\text{ in.}}{60\text{ min}} = \frac{10{,}560\text{ in.}}{1\text{ min}}$; circumference is 28π in.;

  $\frac{10{,}560\text{ in.}}{1\text{ min}} \times \frac{1\text{ revolution}}{28\pi\text{ in.}} \approx 120$ rpm

- Now generalize for a driver traveling at *m* miles per hour on wheels with radius *x* inches. Use your previous work as a basis to derive a formula that converts *m* miles per hour on a wheel with radius *x* inches to revolutions per minute. Show your work.

  $\frac{m\text{ mi}}{1\text{ hr}} = \frac{m \times 5280\text{ ft}}{60\text{ min}} = \frac{m \times 5280 \times 12\text{ in.}}{60\text{ min}} = \frac{1056m\text{ in.}}{1\text{ min}}$; circumference is $2\pi x$ in.;

  $\frac{1056m\text{ in.}}{1\text{ min}} \times \frac{1\text{ revolution}}{2\pi x\text{ in.}} \approx 168\left(\frac{m}{x}\right)$ rpm

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TEACHER INSTRUCTIONS

13-4 Game: Examining Sines

The Sine Function

Provide the host with the following questions and answers.

Question	Player 1	Player 2
Determine whether the value is positive or negative.		
1. $\sin\frac{5\pi}{6}$ **Answer: positive**		
2. $-\sin\frac{\pi}{4}$ **Answer: negative**		
3. $-\sin\frac{5\pi}{6}$ **Answer: negative**		
4. $-\sin\frac{\pi}{6}$ **Answer: negative**		
Determine which is greater.		
5. $\sin\frac{\pi}{6}$ or $\sin\frac{\pi}{2}$ **Answer:** $\sin\frac{\pi}{2}$		
6. $\sin\frac{5\pi}{3}$ or $\sin\frac{\pi}{2}$ **Answer:** $\sin\frac{\pi}{2}$		
7. $\sin\frac{\pi}{3}$ or $\sin-\frac{\pi}{2}$ **Answer:** $\sin\frac{\pi}{3}$		
8. $-\sin\frac{\pi}{4}$ or $\sin\frac{\pi}{3}$ **Answer:** $\sin\frac{\pi}{3}$		
Determine the quadrant for each ordered pair.		
9. $\left(\frac{\pi}{3}, \sin\frac{\pi}{3}\right)$ **Answer: I**		
10. $\left(-\frac{3\pi}{4}, -\sin\frac{3\pi}{4}\right)$ **Answer: III**		
11. $\left(\frac{5\pi}{4}, \sin\frac{5\pi}{4}\right)$ **Answer: IV**		
12. $\left(-\frac{5\pi}{2}, -\sin\frac{5\pi}{2}\right)$ **Answer: III**		

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13-4 Game: Examining Sines

The Sine Function

This is a game for three students. One student is the host and the other two are players.

Your teacher will provide the host with a separate sheet of questions and answers. The gameboard below shows categories and questions. Use it to keep track of which questions are available and your score. Players each begin with 50 points. If you are correct, you earn 5 points. If you are incorrect, you lose 5 points. Players alternate turns.

Question	Player 1	Player 2
Determine whether the value is positive or negative.		
1. $\sin\frac{5\pi}{6}$ **See Teacher Instructions page.**		
2. $-\sin\frac{\pi}{4}$		
3. $-\sin\frac{5\pi}{6}$		
4. $-\sin\frac{\pi}{6}$		
Determine which is greater.		
5. $\sin\frac{\pi}{6}$ or $\sin\frac{\pi}{2}$		
6. $\sin\frac{5\pi}{3}$ or $\sin\frac{\pi}{2}$		
7. $\sin\frac{\pi}{3}$ or $\sin-\frac{\pi}{2}$		
8. $-\sin\frac{\pi}{4}$ or $\sin\frac{\pi}{3}$		
Determine the quadrant for each ordered pair.		
9. $\left(\frac{\pi}{3}, \sin\frac{\pi}{3}\right)$		
10. $\left(-\frac{3\pi}{4}, -\sin\frac{3\pi}{4}\right)$		
11. $\left(\frac{5\pi}{4}, \sin\frac{5\pi}{4}\right)$		
12. $\left(-\frac{5\pi}{2}, -\sin\frac{5\pi}{2}\right)$		

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13-5 Puzzle: What's My Path?

The Cosine Function

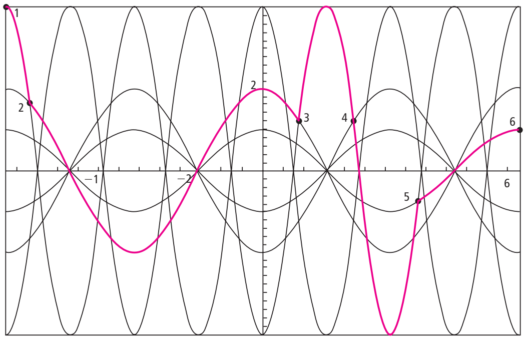
The diagram below shows the graphs of six functions all involving the cosine function. The functions are listed at the right. Each function is graphed over the interval from -2π to 2π.

$y = \cos x$	$y = 2\cos x$	$y = 4\cos 2x$
$y = -\cos x$	$y = -2\cos x$	$y = -4\cos 2x$

On the puzzle, there are six dots numbered from 1 to 6.

- Find a continuous path from Point 1 to Point 6, passing through all of the numbered points in order. Each piece of the path between consecutive points is a piece of one of the six graphs. Identify each piece according to which of those graphs it lies on.

(*Hint*: Give each equation a letter label. Then write that letter on the matching graph. That way, you can keep track of your paths.)



From Point 1 to Point 2: $y = 4\cos 2x$

From Point 2 to Point 3: $y = 2\cos x$

From Point 3 to Point 4: $y = -4\cos 2x$

From Point 4 to Point 5: $y = -4\cos 2x$

From Point 5 to Point 6: $y = \cos x$

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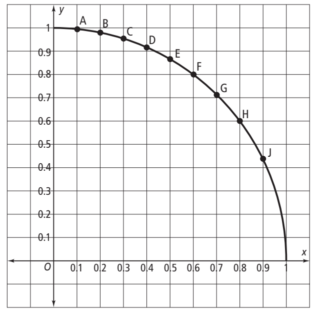
13-6 Activity: Steeper and Steeper

The Tangent Function

The diagram below shows the unit circle in the first quadrant. Notice that the gridlines of the x- and y-axes are marked off every tenth of a unit.



There is a relationship between the tangent and the steepness of a line through the origin. For this activity, work in groups to find out what this relationship is.

- Study the lines passing through point O and each of points A, B, C, D, E, F, G, H, and J.
- Use the definition of slope and your estimation skills to report on the steepness of these lines.
- Then measure the angle made by each line with the positive x-axis and use your calculator to find the value of the tangent.
- Last, write a relationship between the steepness of the lines and tangent of the angle the lines made with the positive x-axis.

Answers may vary. Sample: The steeper (less steep) the line, the greater (smaller) the tangent. Moreover, the tangent of the angle made by the line and the positive *x*-axis is equal to the slope (steepness) of the line.

As a class, discuss how you might use the value of the tangent to gauge the steepness of a line. Summarize your discussion below.

Answers may vary. Sample: If the value of the tangent is 1, the line has slope 1. If the value of the tangent is less than 1, the line is not so steep. If the value of the tangent is greater than 1, the line is steeper.

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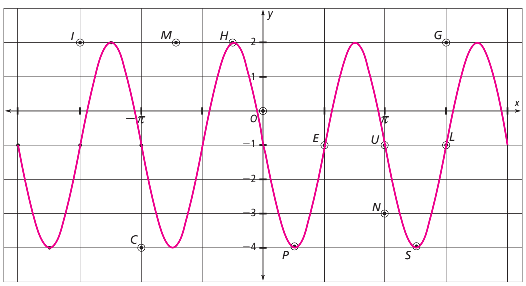
13-7 Puzzle: Rescue!

Translating Sine and Cosine Functions

A rescue ship is sailing along a course modeled by a function of the form $y = a \sin b(x - c) + d$. The diagram below shows five locations (the uncircled points) that the ship has passed on its course. You can predict its exact course if you can find the values of a, b, c, and d. Once you know an equation that describes the course, you can solve the puzzle.



1. Find a, b, c, and d. $a = 3$, $b = 2$, $c = \frac{\pi}{2}$, and $d = -1$
2. Write an equation to model the ship's course. $y = 3\sin\left[2\left(x - \frac{\pi}{2}\right)\right] - 1$
3. Graph the ship's course on the diagram above.
4. Exactly six points that have circles around them lie along the course of the ship. List the corresponding letters. **H, P, E, U, S, and L**
5. Now arrange the letters to spell a two-word message. **HELP US**
6. The rescue ship sends out a message. It consists of the six letters corresponding to points not on the ship's path. Unscramble those letters to find the message. **COMING**

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13-8 Game: Trig Relay

Reciprocal Trigonometric Functions

In this game, teams of seven students play a relay. Your teacher is the host.

- Form teams of seven students. When instructed, the first member of each team takes Item 1. When the host says stop, the first member of your team passes the game sheet to the next member in your team. That player takes Item 2.
- Play continues until each member has received an item. The team with the most items correct is the winning team.

1. Which pair below could give this pair of numbers, in order, for some x? $\left(5, \frac{1}{5}\right)$ **B**

   (A) $(\sin x, \csc x)$ (B) $(\csc x, \sin x)$ (C) $(\sin x, \cos x)$ (D) $(\sec x, \csc x)$

2. Which pair below could give this pair of numbers, in order, for some x? $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ **H**

   (F) $(\tan x, \cot x)$ (G) $(\sin x, \csc x)$ (H) $(\cos x, \sin x)$ (I) $(\csc x, \sin x)$

3. Which pair below could give this pair of numbers, in order, for some x? $(1, 1)$ **A**

   (A) $(\tan x, \cot x)$ (B) $(\sin x, \cos x)$ (C) $(\sin x, \tan x)$ (D) $(\cos x, \cot x)$

4. Which pair below could give this pair of numbers, in order, for some x? $\left(0.4, \frac{5}{2}\right)$ **I**

   (F) $(\tan x, \sin x)$ (G) $(\sin x, \cos x)$ (H) $(\cos x, \sin x)$ (I) $(\tan x, \cot x)$

5. Which pair below could give this pair of numbers, in order, for some x? $\left(\frac{2}{3}, \frac{3}{2}\right)$ **A**

   (A) $(\cot x, \tan x)$ (B) $(\sin x, \cos x)$ (C) $(\sec x, \cos x)$ (D) $(\cos x, \sin x)$

6. Which pair below could give this pair of numbers, in order, for some x? $\left(-3, -\frac{1}{3}\right)$ **F**

   (F) $(\sec x, \cos x)$ (G) $(\sin x, -\sin x)$ (H) $(\cos x, \cot x)$ (I) $(\sin x, -\csc x)$

7. Which pair below could give this pair of numbers, in order, for some x? $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ **B**

   (A) $(\sin x, -\csc x)$ (B) $(\sin x, \cos x)$ (C) $(\sec x, \sin x)$ (D) $(\cos x, \sec x)$