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14-1 Puzzle: Math Humor

Trigonometric Identities

Complete this puzzle alone or with a partner.

Question: What did the math teacher have to do so his daughter could buy a used car?

Answer: Match each trigonometric expression on the left with its simplified version on the right. Then write the correct letter to the left of each number.

Not all expressions on the right will be used, and some may be used more than once.

c **1.** $1 - \cos^2\theta$

o **2.** $\sin\theta\cot\theta$

s **3.** $\sec\theta\cos\theta\sin\theta$

i **4.** $\sec^2\theta - 1$

g **5.** $\frac{\cos\theta}{\sec\theta}$

n **6.** $\sec\theta\cos\theta$

t **7.** $\frac{\cos\theta}{\cos\theta\cot\theta}$

h **8.** $\sin\theta + \cos\theta\cot\theta$

e **9.** $\cos\theta\sin\theta(1 + \cot^2\theta)$

l **10.** $\frac{\csc\theta}{\sin\theta}$

o **11.** $\frac{\sec\theta}{\tan^2\theta + 1}$

a **12.** $(1 - \sin^2\theta)(1 + \cot^2\theta)$

n **13.** $\csc^2\theta - \cot^2\theta$

a. $\cot^2\theta$
b. $\cos\theta\sin\theta$
c. $\sin^2\theta$
d. 0
e. $\cot\theta$
f. $\csc\theta\sec\theta$
g. $\cos^2\theta$
h. $\csc\theta$
i. $\tan^2\theta$
j. $\sec\theta$
k. $\sin\theta\tan\theta$
l. $\csc^2\theta$
m. $\csc\theta\cot\theta$
n. 1
o. $\cos\theta$
p. $\sec\theta\tan\theta$
q. $\cos\theta\cot\theta$
r. $\sec^2\theta$
s. $\sin\theta$
t. $\tan\theta$

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14-2 Puzzle: Crossnumber Puzzle

Solving Trig Equations Using Inverses

Solve the following equations to complete the crossnumber puzzle below.

- All ACROSS solutions should be given in degrees and all DOWN solutions should be given in radians.
- Round your answers to the nearest hundredth if necessary .
- Use a leading zero (for example, 0.52) if necessary.
- All decimal points should be put in their own boxes.
- Be sure to give the solution over the indicated interval.
- Two examples for filling in the boxes are shown below.

2	8	.	3	1

0	.	7	9

1. 6	11. 0					13. 4							16. 4			
	.			12. 0		.					5. 5	6	.	3	1	
	10. 2	8	9	.	4	7			15. 2				7			
	0			8		8. 1	5	6	.	4	17. 2		4. 1	14. 3	5	
9. 0			2. 2	4	0				3		.			.		
.			.				3. 2	7	0		2			6. 1	3	5
2			2								9			4		
4			7. 1	6	1	.	5	7								

ACROSS (Degrees)

1. $2\cos\theta = 1; 0° \le \theta \le 180°$ **60**
2. $2\sin\theta + \sqrt{3} = 0; 90° \le \theta \le 270°$ **240**
3. $\sqrt{2}\sin\theta + \sqrt{2} = 0; 90° \le \theta \le 270°$ **270**
4. $5 + 2\tan\theta = -3\tan\theta; 0° \le \theta \le 180°$ **135**
5. $(2\tan\theta - 3)(\tan\theta + 3) = 0; 0° \le \theta \le 90°$ **56.31**
6. $2\sin^2\theta - 1 = 0; 90° \le \theta \le 180°$ **135**
7. $3\tan^2\theta + \tan\theta = 0; 0° < \theta < 180°$ **161.57**
8. $5\sin^2\theta = 2\sin\theta; 90° \le \theta < 180°$ **156.42**
10. $3\cos^2\theta - 7\cos\theta + 2 = 0; 180° \le \theta < 360°$ **289.47**

DOWN (Radians)

2. $4 + 3\tan\theta = 0; 0 \le \theta < \pi$ **2.21**
9. $4\tan^2\theta = \tan\theta; 0 < \theta < \pi$ **0.24**
11. $5\sin\theta = 1; 0 \le \theta \le \pi/2$ **0.20**
12. $3\cos\theta + 2 = 4; 0 \le \theta \le \pi$ **0.84**
13. $(\sin\theta)(\sin\theta + 1) = 0; \pi < \theta < 2\pi$ **4.71**
14. $\cos^2\theta - 1 = 0; \pi/2 \le \theta \le \pi$ **3.14**
15. $3\cos^2\theta + 2\cos\theta = 0; \pi/2 < \theta \le \pi$ **2.30**
16. $\sin^2\theta + 2\sin\theta + 1 = 0; \pi \le \theta \le 2\pi$ **4.71**
17. $4\sin^2\theta - 15\sin\theta + 9 = 0; \pi/2 \le \theta \le \pi$ **2.29**

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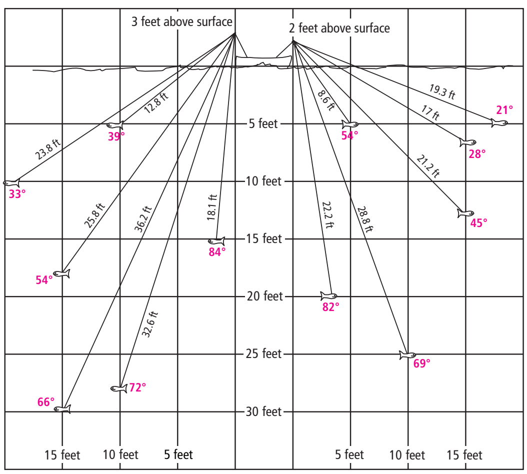
14-3 Game: Gone Fishin'

Right Triangles and Trigonometric Ratios

This game is designed for two teams of one or two students per team.

Rules

- Use a coin toss to determine who goes first.
- One team fishes the left side of the boat *from three feet above the water.*
- The other team fishes the right side of the boat *from two feet above the water*.
- When it is your turn, select a fish to catch on your side of the boat.
- To catch a fish, find the acute angle your line makes with the horizontal.
- Some line lengths are given. Fish that appear to be on integer coordinates are actually on them.
- Write the angle next to the fish to the nearest degree.
- Play alternates until all fish have been caught.
- Your teacher will reveal the correct answers when all teams have finished.
- The team that catches the most fish wins. (*Note:* A tie is possible!)



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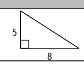
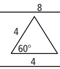
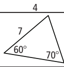
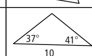
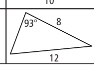
TEACHER INSTRUCTIONS

14-4 Game: The Sine of a Winner

Area and the Law of Sines

Provide the host with the following questions and answers.

	Vocabulary (Define)	Area of Given Triangles	Find the Remaining Sides and Angles	Applications	Review
10 pts	AAS	 5, 8	$a = 3.5$ cm, $b = 3.7$ cm, $c = 5.1$ cm; $A = 44°$, $B = 47°$	See Question 1 below.	Solve for y: $5x = 2y - 7$
20 pts	ASA	 4, 60°, 4	$a = 39$ mi, $b = 59.5$ mi; $A = 38°$, $B = 110°$	See Question 2 below.	Find vertex: $y = 3x^2 - 12x$
30 pts	SSA	 7, 60°, 70°	$b = 75$ in.; $B = 38°$, $C = 64°$	See Question 3 below.	Solve: $\lvert 3x + 2\rvert \le 8$
40 pts	Area Formula for Any Triangle	 37°, 41°, 10	$c = 14$ ft; $A = 52°$, $B = 48°$	See Question 4 below.	Solve: $3x + 2y = 0$, $2x + 3y = 5$
50 pts	Law of Sines	 93°, 8, 12	$a = 86$ m, $b = 91$ m; $A = 69°$	See Question 5 below.	Add: $3 + 7 + 11 + \ldots + 327$

1. A telephone pole on a level part of a street casts a shadow 12 ft long. The angle of elevation from the tip of the shadow to the top of the pole is 65°. How tall is the pole?
2. A triangular park has two sides measuring 200 m and 150 m. They intersect at an angle measuring 50°. What is the area of the park?
3. Two players try to recover a fumble. They are 15 yd apart. Their paths will intersect at the ball, forming an angle of 48°. Player 1 is 20 yd from the ball. How far is Player 2 from the ball?
4. Two players try to recover a fumble. The ball is 10 yd from each of them. Their paths will intersect at the ball, forming an angle of 37°. How far apart are they initially?

	Vocab	Area	Sides/Angles	Apps	Review
10 pts	See right.	20	$C = 89°$	25.7 ft	$y = 2.5x + 3.5$
20 pts	See right.	$4\sqrt{3}$	$C = 32°$, $C = 33.6$ mi	11,490.7 m^2	(2, −12)
30 pts	See right.	17.3	$A = 78°$, $a = 119.2$ in, $c = 109.5$ in	15.5 yd	$-\frac{10}{3} \le x \le 2$
40 pts	See right.	20.2	$C = 80°$, $a = 11.2$ ft, $b = 10.6$ ft	6.3 yd	(−2, 3)
50 pts	See right.	34.1	$B = 81.1°$, $C = 29.9°$, $c = 45.9$ m	23.0 ft	13,530

5. The street in Exercise 1 goes up a 10° hill. The angle of elevation of the Sun is 60°, and a pole casts a 15-ft shadow down hill along the road. Find the height of the pole.

10: triangle with two angles and nonincluded side given
20: triangle with two angles and included side given
30: triangle with two sides and nonincluded angle given
40: area $= \frac{1}{2}ab\sin C$
50: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

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14-4 Game: The Sine of a Winner

Area and the Law of Sines

This is a game for three students. One student is the host and the other two are players.

Host: Your teacher will provide you with a separate sheet of questions and answers. Keep track of each player's score below.

Players: The game board shows the available categories and their point values. Use it to keep track of which questions are still available and your score. Round your answers to the nearest tenth when necessary.

Rules: Decide which player goes first. When it is your turn:

- Select a category. The host will always select the least-points-available question.
- If you answer correctly within the time assigned by your teacher, you earn the points for that question. Select a category and go again.
- If you answer incorrectly, you lose that number of points and your opponent takes over. In addition, your opponent has 10 seconds to provide the correct answer and earn the points from the missed question. Your group or your teacher can decide to change the response time if needed.
- Play continues in this manner until all the questions have been used. The player with the highest point total wins. See Teacher Instructions page.

	Vocabulary (Define)	Area of Given Triangles	Find the Remaining Sides and Angles	Applications	Review
10 pts					
20 pts					
30 pts					
40 pts					
50 pts					

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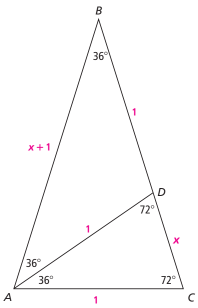
14-5 Activity: Divisors of 360

The Law of Cosines

This activity can be done alone or in small groups.

Background

You have previously learned how to find the exact values of trigonometric functions for certain special angles such as 30°, 45°, 60°, 90°, 120°, and 180°. These are all divisors of 360°. In this activity, you will use the law of cosines to find values of trig functions for 36° and 72°.



Step 1: Label the following in the figure above.
$AC = 1$ and $CD = x$

Step 2: Find and label the following lengths: AD, BD, and AB.
Note: $\triangle ABC$, $\triangle ABD$, and $\triangle CAD$ are all isosceles triangles. **1; 1; $x + 1$**

Step 3: Solve for x.
Note: $\triangle ABC$ and $\triangle CAD$ are similar triangles; therefore the ratios of leg-to-base in the two triangles are equal. Use this to set up an equation in the variable x. $x = \frac{-1 + \sqrt{5}}{2}$

Step 4: Apply the Law of Cosines to $\triangle CAD$ to find the exact value for cos 36°. $\cos 36° = \frac{1 + \sqrt{5}}{4}$

Step 5: Apply the Law of Cosines to $\triangle CAD$ to find the exact value for cos 72°. $\cos 72° = \frac{-1 + \sqrt{5}}{4}$

Step 6: Use your calculator to verify your results from Steps 4 and 5. $\cos 36° \approx 0.809$; $\cos 72° \approx 0.309$

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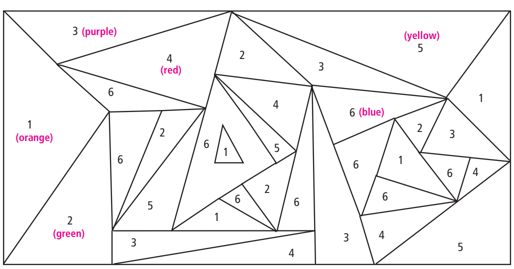
14-6 Activity: Color Me Crazy

Angle Identities

This activity may be done alone or in small groups. You will need colored pencils, markers, or crayons.

Suppose the three primary colors match with specific angles: red → 30°; yellow → 45°; and blue → 60°. *Note:* A secondary color is made by mixing two primary colors.

1. a. Evaluate sin 75° using the primary angles above and a *sum* identity. $\frac{\sqrt{2} + \sqrt{6}}{4}$
   b. Evaluate cos 75° using the primary angles above and a *sum* identity. $\frac{\sqrt{6} - \sqrt{2}}{4}$
   c. Since a *sum* identity was used, what secondary color matches with 75°? **orange**
2. a. Evaluate sin 105° using the primary angles above and a *sum* identity. $\frac{\sqrt{2} + \sqrt{6}}{4}$
   b. Use your result from Part a. to evaluate cos 105°. $\frac{-\sqrt{2 - \sqrt{3}}}{2}$
   c. Since a *sum* identity was used, what secondary color matches with 105°? **green**
3. a. Evaluate sin 90° using two different primary angles and a *sum* identity. **1**
   b. Evaluate cos 90° using your method of preference. **0**
   c. Since a *sum* identity was used, what secondary color matches with 90°? **purple**
   d. Do your results agree with what you already know about 90°? **yes**
4. Color the following triangles by number. Let 1, 2, and 3 match with the colors you found in Exercises 1–3 and let 4 → red, 5 → yellow, and 6 → blue. **Check students' work.**



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TEACHER INSTRUCTIONS

14-7 Game: Verify That Identity

Double Angle and Half Angle Identities

Provide the judges with the following sample answers.

1. $\sec\theta\cot\theta = \csc\theta$

$$\sec\theta\cot\theta = \frac{1}{\cos\theta}\cdot\frac{\cos\theta}{\sin\theta} = \frac{1}{\sin\theta} = \csc\theta$$

2. $\tan(x + \pi) = \tan x$

$$\tan(x + \pi) = \frac{\tan x + \tan\pi}{1 - \tan x\tan\pi} = \frac{\tan x + 0}{1 - \tan x \cdot 0} = \frac{\tan x}{1} = \tan x$$

3. $\sin(2\theta) = 2\sin\theta\cos\theta$

$$\sin(2\theta) = \sin(\theta + \theta) = \sin\theta\cos\theta + \sin\theta\cos\theta = 2\sin\theta\cos\theta$$

4. $\cos^2\theta(1 + \tan^2\theta) = 1$

$$\cos^2\theta(1 + \tan^2\theta) = \cos^2\theta(\sec^2\theta) = \cos^2\theta\cdot\frac{1}{\cos^2\theta} = 1$$

5. $(\sin\theta + \cos\theta)^2 = 1 + \sin 2\theta$

$$(\sin\theta + \cos\theta)^2 = \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 1 + \sin 2\theta$$

6. $\cos^2\left(\frac{\theta}{2}\right) - \sin^2\left(\frac{\theta}{2}\right) = \cos\theta$

$$\cos^2\left(\frac{\theta}{2}\right) - \sin^2\left(\frac{\theta}{2}\right) = \frac{1 + \cos\theta}{2} - \frac{1 - \cos\theta}{2} = \frac{1 + \cos\theta - 1 + \cos\theta}{2} = \frac{2\cos\theta}{2} = \cos\theta$$

7. $\frac{\cos\theta}{1 + \sin\theta} - \frac{1 - \sin\theta}{\cos\theta} = 0$

$$\frac{\cos\theta}{1 + \sin\theta} - \frac{1 - \sin\theta}{\cos\theta} = \frac{\cos\theta}{1 + \sin\theta}\cdot\frac{\cos\theta}{\cos\theta} - \frac{1 - \sin\theta}{\cos\theta}\cdot\frac{1 + \sin\theta}{1 + \sin\theta} = \frac{\cos^2\theta}{\cos\theta(1 + \sin\theta)} - \frac{1 - \sin^2\theta}{\cos\theta(1 + \sin\theta)} = \frac{\cos^2\theta - 1 + \sin^2\theta}{\cos\theta(1 + \sin\theta)} = \frac{1 - 1}{\cos\theta(1 + \sin\theta)} = 0$$

8. $\frac{2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta} = \tan(2\theta)$

$$\frac{2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta} = \frac{\sin 2\theta}{\cos 2\theta} = \tan 2\theta$$

9. $\tan A - \tan B = \frac{\sin(A - B)}{\cos A\cos B}$

$$\frac{\sin(A - B)}{\cos A\cos B} = \frac{\sin A\cos B - \cos A\sin B}{\cos A\cos B} = \frac{\sin A\cos B}{\cos A\cos B} - \frac{\cos A\sin B}{\cos A\cos B} = \frac{\sin A}{\cos A} - \frac{\sin B}{\cos B} = \tan A - \tan B$$

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14-7 Game: Verify That Identity

Double Angle and Half Angle Identities

This is a game for six students. Two students are the judges and the other four students are divided in two teams. Your teacher will provide the judges with sample answers.

Judges: Your responsibility is to make sure the identity verification a team gives is correct.

Players: Your task is to verify all the identities below. Both teams should verify the identity in play in as few steps as possible.

Rules: Determine which team starts. After both teams have had time to work on the identity in play:

- The beginning team bids how many steps they need to verify the identity.
- The opposing team claims they can verify it in fewer steps or they pass.
- Bidding alternates until a team passes.
- The lowest-bidding team must verify the identity in the number of steps bid, to the satisfaction of the judges.
- If both judges accept the verification, the team claims the identity.
- If both judges do not accept the verification, the opposing team has an opportunity to verify the identity at their last bid. If they never bid, they can take as many steps as they need.
- If the judges do not agree, see your teacher for the tie-breaking opinion.
- The opening bid alternates on subsequent identities.
- Play continues in this manner until all the identities have been used. The team with the most verified identities wins. **See Teacher Instructions page.**

1. $\sec\theta\cot\theta = \csc\theta$
2. $\tan(x + \pi) = \tan x$
3. $\sin(2\theta) = 2\sin\theta\cos\theta$
4. $\cos^2\theta(1 + \tan^2\theta) = 1$
5. $(\sin\theta + \cos\theta)^2 = 1 + \sin 2\theta$
6. $\cos^2\left(\frac{\theta}{2}\right) - \sin^2\left(\frac{\theta}{2}\right) = \cos\theta$
7. $\frac{\cos\theta}{1 + \sin\theta} - \frac{1 - \sin\theta}{\cos\theta} = 0$
8. $\frac{2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta} = \tan(2\theta)$
9. $\tan A - \tan B = \frac{\sin(A - B)}{\cos A\cos B}$