page 1

11-1 **ELL Support**
Permutations and Combinations

Fundamental Counting Principle

If event M can occur in m ways and is followed by event N that can occur in n ways, then event M followed by event N can occur in $m \cdot n$ ways.

Example 4 different fruits and 6 different vegetables give $4 \cdot 6$ possible fruit and vegetable combinations.

Solve.

1. Hector has 6 computers and 7 printers to choose from. How many possible

   computer-printer combinations can he make? **42**

2. Raymond and Jasmine have 8 sofas and 14 chairs to choose between. How

   many possible sofa-chair combinations can they make? **112**

Number of Permutations

The number of permutations of n items of a set arranged r items at a time is

$$_nP_r = \frac{n!}{(n-r)!} \text{ for } 0 \le r \le n$$

Example

$$_8P_3 = \frac{8!}{(8-3)!} = \frac{8!}{5!} = 336$$

Evaluate each expression.

3. $_7P_4 = $ **840**

4. $_9P_5 = $ **15,120**

Number of Combinations

The number of combinations of n items of a set chosen r items at a time is

$$_nC_r = \frac{n!}{r!(n-r)!} \text{ for } 0 \le r \le n$$

Example

$$_7C_3 = \frac{7!}{3!(7-3)!} = \frac{7!}{3! \cdot 4!} = \frac{7!}{6 \cdot 24} = 35$$

Evaluate each expression.

5. $_8C_5 = $ **56**

6. $_9C_3 = $ **84**

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11-1 **Think About a Plan**
Permutations and Combinations

Consumer Issues A consumer magazine rates televisions by identifying two levels of price, five levels of repair frequency, three levels of features, and two levels of picture quality. How many different ratings are possible?

Understanding the Problem

1. How many levels of price are possible? **2**

2. How many levels of repair frequency are possible? **5**

3. How many levels of features are possible? **3**

4. How many levels of picture quality are possible? **2**

5. What is the problem asking you to determine?
   the number of different ratings that are possible

Planning the Solution

6. What is the Fundamental Counting Principle?
   If an event M can occur in m ways and is followed by event N that can occur in n
   ways, then event M followed by event N can occur in $m \cdot n$ ways .

7. How can the Fundamental Counting Principle help you solve the problem?
   Answers may vary. Sample: Each rating type is an event, and I know the
   possible number of ways each event can occur. So by the Fundamental
   Counting Principle, the total number of ratings is the product of the
   number of ways each event can occur .

Getting an Answer

8. Write an expression for the number of different ratings that are possible. **$2 \cdot 5 \cdot 3 \cdot 2$**

9. How many different ratings are possible? **60**

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11-1 **Practice**   *Form G*
Permutations and Combinations

1. How many 2-letter pairs of 1 vowel and 1 consonant can you make from the English alphabet? Consider "y" to be a consonant. **105**

2. An ice cream shop offers 33 flavors of ice cream and 7 toppings. How many different sundaes can the shop make using 1 flavor and 1 topping? **231**

3. A contest winner gets to choose 1 of 8 possible vacations and bring 1 of 10 friends with her. How many different ways could the contest winner select her prize? **80**

Evaluate each expression.

4. $8!$ **40,320**   5. $\frac{11!}{9!}$ **110**   6. $6!4!$ **17,280**   7. $3(5!)$ **360**

8. $\frac{9!}{2!6!}$ **252**   9. $3(7!)$ **15,120**   10. $\frac{10!}{5!}$ **30,240**   11. $\frac{3!8!}{5!}$ **2016**

12. An art gallery plans to display 7 sculptures in a single row.
    a. How many different arrangements of the sculptures are possible? **5040**
    b. If one sculpture is taken out of the show, how many different arrangements are possible? **720**

Evaluate each expression.

13. $_{12}P_{11}$ **479,001,600**   14. $_{12}P_{10}$ **239,500,800**   15. $_{12}P_5$ **95,040**   16. $_{12}P_1$ **12**

17. $_5P_2$ **20**   18. $_7P_4$ **840**   19. $_8P_6$ **20,160**   20. $_6P_2$ **30**

21. In how many ways can four distinct positions for a relay race be assigned from a team of nine runners? **3024**

Evaluate each expression.

22. $_{12}C_{11}$ **12**   23. $_{12}C_{10}$ **66**   24. $_{12}C_5$ **792**   25. $_{12}C_1$ **12**

26. $_{12}C_{12}$ **1**   27. $_5C_4 + _5C_3$ **15**   28. $\frac{_5C_3}{_5C_2}$ **1**   29. $4(_7C_2)$ **84**

30. Thirty people apply for 10 job openings as welders. How many different groups of people can be hired? **30,045,015**

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11-1 **Practice** (continued)   *Form G*
Permutations and Combinations

For each situation, determine whether to use a permutation or a combination. Then solve the problem.

31. You draw the names of 5 raffle winners from a basket of 50 names. Each person wins the same prize. How many different groups of winners could you draw? **combination: 2,118,760**

32. A paint store offers 15 different shades of blue. How many different ways could you purchase 3 shades of blue? **combination: 455**

33. How many different 5-letter codes can you make from the letters in the word *cipher*? **permutation: 720**

Assume a and b are positive integers. Determine whether each statement is *true* or *false*. If it is true, explain why. If it is false, give a counterexample.

34. $a!b! = b!a!$ **True; Commutative Property of Multiplication**

35. $(a^2)! = (a!)^2$ **False; let $a = 2$: $(2^2)! = 24$; $(2!)^2 = 4$**

36. $a \cdot b! = (ab)!$ **False; let $a = 2$ and $b = 3$: $2 \cdot 3! = 12$; $(2 \cdot 3)! = 720$**

37. $(a + 0)! = a!$ **True; Identity Property of Addition**

38. $\frac{a!}{b!} = \left(\frac{a}{b}\right)!$ **False; let $a = 4$ and $b = 2$: $\frac{4!}{2!} = \frac{24}{2} = 12$; $\left(\frac{4}{2}\right)! = 2! = 2$**

39. $a!(b! + c!) = a!b! + a!c!$ **True; Distributive Property**

40. A restaurant offers a fixed-priced meal of 1 appetizer, 1 entrée, 2 sides, and 1 dessert. How many different meals could you choose from 4 appetizers, 5 entrees, 8 sides, and 3 desserts? **1680**

41. **Writing** Explain the difference between a permutation and a combination. **Answers may vary. Sample: If you choose r items from a group of n items and the order in which the items are chosen is important, then it is a permutation. If the order does not matter, then it is a combination.**

42. **Reasoning** Show that for $n = r$, the value of $_nC_r = 1$.
    $_nC_n = \frac{n!}{n!(n-n)!} = \frac{n!}{n!0!} = \frac{n!}{n!(1)} = 1$

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11-1 Practice *Form K*

Permutations and Combinations

Use the Fundamental Counting Principle to solve the following problems.

1. You must make a password for your email account. The password must consist of two letters followed by four digits. How many different passwords are possible? **6,760,000 passwords**

2. Your father is buying a sport coat, a pair of pants, and a tie. Sport coats come in 6 different colors. Pants come in 4 different colors. There are 25 different tie styles to choose from. How many different combinations are possible? **600 combinations**

Evaluate each expression.

3. $6!$ **720**   4. $5!4!$ **2880**   5. $\frac{9!}{7!}$ **72**

Find the number of permutations in the following problems.

6. Your coach has twelve team jerseys numbered from 1 through 12. He plans to give one jersey to each of the twelve members of the basketball team. In how many ways can the jerseys be assigned? **479,001,600 ways**

7. The owner of a car lot is lining up 7 cars in the show-room window. In how many ways can the cars be ordered? **5040 ways**

Evaluate each expression.

8. $_5P_3$ **60**   9. $_8P_5$ **6720**   10. $_{11}P_5$ **55,440**

11. Twelve different types of pizza are being judged in a contest. In how many different ways can the pizzas be judged first, second, third, and fourth? **11,880 ways**

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11-1 Practice (continued) *Form K*

Permutations and Combinations

Evaluate each expression.

12. $_7C_2$ **21**   13. $_9C_5$ **126**   14. $_{12}C_7$ **792**

15. $_8C_6$ **28**   16. $5(_6C_3)$ **100**   17. $_{10}C_7 + _5C_2$ **130**

Decide whether to use a permutation or a combination for each situation. Then solve the problem.

18. An ice cream parlor offers 14 different types of ice cream. In how many different ways can you select 5 types of ice cream to sample? **combination; 2002 ways**

19. Eleven groups entered a science fair competition. In how many ways can the groups finish first, second, and third? **permutation; 990 ways**

20. Your aunt is ordering appetizers for her and her family. The restaurant offers 10 different appetizers. She will select 4 appetizers. How many different combinations of appetizers can your aunt possibly select? **combination; 210 combinations**

21. **Error Analysis** Your friend is shopping for blue jeans. The clothing store offers 18 different types of blue jeans, and your friend will buy 5 different types. Your friend believes that she has 1,028,160 different combinations that she could possibly select. What error did your friend make? How many different combinations could she possibly select? **Your friend used a permutation when she should have used a combination. She could possibly select 8568 different combinations.**

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11-1 Standardized Test Prep

Permutations and Combinations

Multiple Choice

For Exercises 1–5, choose the correct letter.

1. You choose 5 apples from a case of 24 apples. Which best represents the number of ways you can make your selection? **B**
   Ⓐ $_5C_{19}$ Ⓑ $_{24}C_5$ Ⓒ $_5P_{24}$ Ⓓ $_{19}P_5$

2. Which is equivalent to $_7P_3$? **H**
   Ⓕ 28   Ⓖ 35   Ⓗ 210   Ⓘ 840

3. A traveler can choose from three airlines, five hotels, and four rental car companies. How many arrangements of these services are possible? **B**
   Ⓐ 12   Ⓑ 60   Ⓒ 220   Ⓓ 495

4. Which is equivalent to $a!(b!)$? **I**
   Ⓕ $(ab)!$ Ⓖ $(ab!)!$ Ⓗ $ba!$ Ⓘ $b!(a!)$

5. Which is equivalent to $_9C_5$? **A**
   Ⓐ 126   Ⓑ 3024   Ⓒ 15,120   Ⓓ 45,000

Short Response

6. You have a $1 bill, a $5 bill, a $10 bill, a $20 bill, a quarter, a dime, a nickel, and a penny. How many different total amounts can you make by choosing 6 bills and coins? Show your work.

   [2] $_8C_6 = \frac{8!}{6!(8-6)!}$
   $= \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1)(2 \cdot 1)}$
   $= \frac{56}{2}$
   $= 28$
   [1] incorrect or incomplete work shown
   [0] incorrect answer and no work shown OR no answer given

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11-1 Enrichment

Permutations and Combinations

Dinner at a Chinese Restaurant

A typical Chinese restaurant will often feature a Special Dinner, in which the customer has the choice of ordering one appetizer and one entree.

1. If there are 8 appetizers and 11 entrees, how many different Special Dinners are there? **88**

2. If there are 12 appetizers and 7 entrees, how many different Special Dinners are there? **84**

3. If there are A appetizers and E entrees, how many different Special Dinners are there? **AE**

4. There are 12 appetizers; 4 are soups; 6 contain meat, and 2 do not. In how many different orders can 3 different appetizers be brought to the table? **1320**

5. In how many different orders can 5 different appetizers of the 12 be brought to the table? **95,040**

6. Do Exercises 1–5 involve permutations or combinations? **permutations**

7. Assume that 3 customers arrive and order different appetizers to share from a choice of 12 appetizers.
   a. Does this problem involve permutations or combinations? **combinations**
   b. Why? **order doesn't matter**
   c. In how many possible ways can this be done? **$_{12}C_3 = 220$**

8. Suppose that 5 customers arrive, and each orders a different appetizer to share from a choice of 12 appetizers. In how many ways can this be done? **$_{12}C_5 = 792$**

9. Suppose that 7 customers arrive, and each orders a different appetizer to share from a choice of 12 appetizers.
   a. In how many ways can this be done? **$_{12}C_7 = 792$**
   b. Why is this answer the same as the number of ways that 5 customers can order different appetizers? **$_nC_r = _nC_{n-r}$ or $_{12}C_5 = _{12}C_7$**

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11-1 Reteaching
Permutations and Combinations

If you select some items out of a group and the order of the items in your selection is important, then your selection is a *permutation* of the group.

For example, suppose Ana, Bob, Cal, and Dan enter a local essay contest. Here are some possible ways for the judges to select the first-prize and second-prize winners.

First Prize	Second Prize
Ana	Bob
Dan	Cal
Bob	Ana
Bob	Dan

"Ana, Bob" means Ana is first and Bob is second.

"Bob, Ana" means Bob is first and Ana is second.

The order of the names in the selection is important. The selection "Ana, Bob" is a *permutation* of the group of contestants.

The number of permutations of n items of a set arranged r items at a time is
$$_nP_r = \frac{n!}{(n-r)!}, (0 \le r \le n)$$

Problem

In how many ways can the judges select the first-prize and second-prize winners in the essay contest described above?

Step 1 Is the order of the names in each selection important?
Yes. "Ana, Bob" is not the same as "Bob, Ana." You are looking for the total number of permutations of 2 items each selected from a group of 4 items.

Step 2 Describe n and r.
There are 4 people in the group of contestants. $n = 4$
There are 2 people in each selection of prize winners. $r = 2$

Step 3 Substitute for each variable in the formula.
$$_nP_r = {}_4P_2 = \frac{n!}{(n-r)!} = \frac{4!}{(4-2)!} = \frac{4!}{2!} = \frac{4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1} = 12$$
There are 12 ways for the judges to choose the first-prize and second-prize winners.

Exercises

1. In how many ways can you choose 6 letters for a password from the set A, B, E, L, N, O, S, T, Y? **60,480**

2. In how many ways can a club with 15 members elect a president, vice president, secretary, and treasurer? **32,760**

3. In how many ways can a family of 6 line up in 1 row for a photograph? **720**

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11-1 Reteaching (continued)
Permutations and Combinations

If you select some items out of a group and the order of the items in your selection is NOT important, then your selection is a *combination* of the group.

For example, suppose Ana, Bob, Cal, and Dan enter a local essay contest. Two finalists will go to the statewide essay contest. Here are some ways for the judges to select the contestants who will go to the state contest.

Finalist	Finalist
Ana	Bob
Dan	Cal
Bob	Ana
Bob	Dan

"Ana, Bob" means both Ana and Bob will go.

"Bob, Ana" means both Ana and Bob will go.

The order of the names in the selection is NOT important. The selection "Ana, Bob" is a *combination* of the group of contestants.

The number of combinations of n items of a set chosen r items at a time is
$$_nC_r = \frac{n!}{r!(n-r)!}, (0 \le r \le n)$$

Problem

In how many ways can the judges select the finalists who will go to the state contest?

Step 1 Is the order of the names in each selection important?
No. "Ana, Bob" has the same meaning as "Bob, Ana." You are looking for the total number of combinations of 2 items each selected from a group of 4 items.

Step 2 Describe n and r.
There are 4 people in the group of contestants. $n = 4$
There are 2 people in each selection of contestants going on to state. $r = 2$

Step 3 Substitute for each variable in the formula.
$$_nC_r = {}_4C_2 = \frac{n!}{r!(n-r)!} = \frac{4!}{2!(4-2)!} = \frac{4!}{2!(2!)} = \frac{4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1(2 \cdot 1)} = \frac{12}{2} = 6$$
There are 6 ways for the judges to choose the finalists going to the state contest.

Exercises

4. You have 12 CDs, but only have time to listen to 2 of them. How many combinations of CDs do you have to choose from? **66**

5. Your biology teacher chooses 6 students from a class of 26 to do a special project. How many different groups can your teacher form? **230,230**

6. How many 3-flavor blends can you create from 10 frozen yogurt flavors? **120**

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11-2 ELL Support
Probability

Complete the vocabulary chart by filling in the missing information.

Word or Phrase	Definition	Example
experimental probability	**1. the number of times the event occurs divided by the number of trials**	You take 12 marbles from a bag and 3 of them are blue. The experimental probability of pulling a blue marble from the bag is $\frac{3}{12} = 0.25 = 25\%$.
simulation	**2. a *simulation* is a model of an event**	You can simulate guessing on a set of true or false questions by flipping a coin.
sample space	a list of all possible outcomes to an experiment or activity	**3. The sample space for a flip of a coin is heads or tails.**
equally likely sample space	**4. a sample space in which each outcome has the same chance of occurring**	The sample space for randomly selecting a card from a deck of 52 cards includes all of the cards in the deck, and each outcome has an equal chance of occurring.
theoretical probability	If an event A occurs in m out of n equally likely outcomes, then the theoretical probability of A is $\frac{m}{n}$.	**5. The theoretical probability of pulling an Ace from a deck of 52 cards is $\frac{4}{52} = \frac{1}{13}$.**

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11-2 Think About a Plan
Probability

Lottery A lottery has 53 numbers from which five are drawn at random. Each number can only be drawn once. What is the probability of your lottery ticket matching all five numbers in any order?

Know

1. The lottery has [**53**] possible numbers that can be drawn.

2. Each number can be drawn [**1**] time(s).

3. A total of [**5**] numbers will be drawn.

Need

4. To solve the problem I need to find:
the theoretical probability of the numbers on a lottery ticket matching the numbers drawn in any order .

Plan

5. Because order does not matter, the size of the sample space is a [**combination**] .

6. What is the sample space?
all combinations of 53 numbers chosen 5 at a time

7. What is the size of the sample space? **${}_{53}C_5 = 2,869,685$**

8. How many of the events in the sample space represent your ticket? **1**

9. What is the probability of your lottery ticket matching all five numbers in any order? **$\frac{1}{2,869,685} \approx 0.00000035$**

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11-2 Practice *Form G*
Probability

1. A basketball player attempted 24 shots and made 13. Find the experimental probability that the player will make the next shot she attempts. **≈ 0.54 or 54%**

2. A baseball player attempted to steal a base 70 times and was successful 47 times. Find the experimental probability that the player will be successful on his next attempt to steal a base. **≈ 0.67, or 67%**

Graphing Calculator For Exercises 3–4, define a simulation by telling how you represent correct answers, incorrect answers, and the quiz. Use your simulation to find each experimental probability.

3. If you guess the answers at random, what is the probability of getting at least three correct answers on a four-question true-or-false quiz?
Answers may vary. Sample: Let "1" be a correct answer. Let "2" be an incorrect answer. Generate 16 sets of 4 random 1's and 2's; $\frac{5}{16}$ = 0.3125 ≈ 0.31, or 31%.

4. A five-question multiple-choice quiz has four choices for each answer. If you guess the answers at random, what is the probability of getting at least four correct answers? **Answers may vary. Sample: Let "1" be a correct answer. Let "2", "3", and "4" be incorrect answers. Generate 64 sets of 5 random 1's, 2's, 3's, 4's, and 5's; $\frac{1}{64}$ = 0.015625 ≈ 0.02, or 2%.**

A group of five cards are numbered 1–5. You choose one card at random. Find each theoretical probability.

5. P(card is a 2) $\frac{1}{5}$ = 0.20, or 20%
6. P(even number) $\frac{2}{5}$ = 0.40, or 40%
7. P(prime number) $\frac{3}{5}$ = 0.60, or 60%
8. P(less than 5) $\frac{4}{5}$ = 0.80, or 80%

A bucket contains 15 blue pens, 35 black pens, and 40 red pens. You pick one pen at random. Find each theoretical probability.

9. P(black pen) $\frac{35}{90}$ ≈ 0.39, 39%
10. P(blue pen or red pen) $\frac{55}{90}$ ≈ 0.61, 61%
11. P(not a blue pen) $\frac{75}{90}$ ≈ 0.83, 83%
12. P(black pen or not a red pen) $\frac{50}{90}$ ≈ 0.56, 56%

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11-2 Practice (continued) *Form G*
Probability

13. There are 225 juniors and 255 seniors at your school. The school chooses 5 juniors and seniors as Student All-Stars. What is the theoretical probability that exactly 2 of the Student All-Stars will be juniors? **≈ 0.33, or 33%**
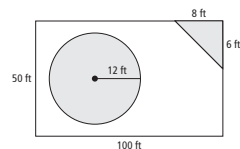
The rectangular yard shown below has a circular pool and a triangular garden. A ball from an adjacent golf course lands at a random point within the yard. Find each theoretical probability.

14. The ball lands in the pool. **≈ 0.09, or 9%**
15. The ball lands in the garden. **≈ 0.005, or 0.5%**
16. The ball lands in the garden or the pool. **≈ 0.095, or 9.5%**
17. The ball does not land in the pool. **≈ 0.91, or 91%**

Five people each flip a coin one time. Find each theoretical probability.

18. P(5 heads) **0.03125, or ≈ 3%**
19. P(exactly 2 tails) **0.3125, or ≈ 31%**
20. P(at least 3 heads) **0.5, or 50%**
21. P(less than 4 tails) **0.8125, or ≈ 81%**

22. The spinner shown at the right has four equal-sized sections. Suppose you spin the spinner two times.
a. What is the sample space? **(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (3, 4), (4, 1), (4, 2), (4, 3), (4, 4)**
b. How many outcomes are there? **16**
c. What is the theoretical probability of getting a sum of 4? **0.1875, or ≈ 19%**

23. If x is a real number and $x = 0$, what is the probability that $\frac{1}{x}$ is undefined? **1**

24. If x is a real number and $x \neq 0$, what is the probability that $\frac{1}{x}$ is undefined? **0**

25. Of the 195 students in the senior class, 104 study Spanish and 86 study French, with 12 studying both Spanish and French. What is the theoretical probability that a student chosen at random is studying Spanish, but not French? **≈ 0.47 or ≈ 47%**

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11-2 Practice *Form K*
Probability

Find each experimental probability.

1. A baseball player got a hit in 12 of his last 40 at bats. What is the probability that he will get a hit in his next at bat? **0.3 or 30%**

2. A pitcher struck out 8 of the last 32 batters that he faced. What is the probability that he will strike out the next batter that he faces? **0.25 or 25%**

3. A student rolled a six-sided number cube 60 times. She rolled the number 4 nine times. What is the experimental probability of rolling a 4? **0.15 or 15%**

4. Reasoning There are 50 cars in a used car lot. The experimental probability that a car in the lot has two doors is 0.12. How many cars in the lot have two doors? **6 cars**

Explain how you could simulate each situation. Then use your simulation to find each experimental probability.

5. A quiz consists of 12 true-or-false questions. If you guess the answers at random, what is the probability of getting at least 8 correct answers? **Answers may vary. Sample: Use a coin and allow heads to represent a correct answer and tails to represent an incorrect answer. Flip the coin 12 times to represent the quiz, and record the number of correct answers. Represent the quiz 20 times, and record the number of times there are at least 8 correct answers. The probability is about 20%.**

6. There are 15 multiple-choice questions on a test. Each question has four answer choices, and only one choice is correct. What is the probability of passing the test by guessing at least 7 of the 15 answers correctly? **Answers may vary. Sample: Pick at random from four numbers, and select one of them to represent a correct answer. Pick 15 times to represent one test, and represent the test 20 times. The probability is about 5%.**

7. Writing Explain why simulations are sometimes preferable to conducting actual trials. **Answers may vary. Sample: Sometimes it is impractical to conduct actual trials. For example, a teacher would not ask her students to guess all of the answers on a test. In such cases, it makes sense to use a simulation.**

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11-2 Practice (continued) *Form K*
Probability

Find each of the following theoretical probabilities.

8. Your classmate rolls a fair number cube. What is the theoretical probability that she will roll a number greater than 4? **33%**

9. Shawn rolls a pair of fair number cubes. What is the theoretical probability that he will roll a sum of 3? **about 6%**

10. A box contains 24 green markers, 16 red markers, and 10 blue markers.
a. P(red) **32%**
b. P(green or blue) **68%**
c. P(not green) **52%**

Use combinatorics to find the following theoretical probability.

11. Six of the 32 players on the football team are left-handed. There are 5 starting offensive linemen. What is the theoretical probability that 2 of the starting offensive linemen are left-handed? **about 19%**

Use area to find the following theoretical probabilities.

12. The floor in your friend's house covers 1400 ft². The floor in her bedroom is 14 ft by 10 ft. What is the probability that a randomly selected point on the floor of the house is in your friend's bedroom? **10%**

13. A garden is 15 ft by 12 ft. Tomatoes fill a 5 foot by 4 foot section of the garden. A squirrel leaps from a tree into the garden. What is the theoretical probability that the squirrel will land in the tomato section of the garden? **about 11%**

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11-2 Standardized Test Prep
Probability

Gridded Response

For Exercises 1–3, find each theoretical probability based on one roll of two number cubes. Enter each answer in the grid as a whole percent.
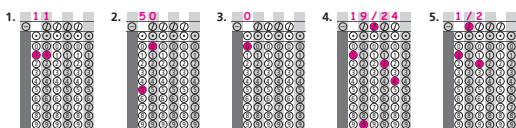
1. $P(\text{sum } 9)$

2. $P(\text{one even, one odd})$

3. $P(\text{sum} > 12)$

For Exercises 4–5, find each theoretical probability based on one marble drawn at random from a bag of 14 red marbles, 10 pink marbles, 18 blue marbles, and 6 gold marbles. Enter each answer in the grid as a fraction in simplest form.

4. $P(\text{not pink})$

5. $P(\text{blue or gold})$

Answers

1. 1 1   2. 5 0   3. 0   4. 1 9 / 2 4   5. 1 / 2

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11-2 Enrichment
Probability

Biologists use a Punnett Square to predict the gene combinations that are possible for an offspring when the genes of the parents are known. Each parent organism carries two genes, or alleles, for a particular trait. For example, a parent might have the genotype Bb for dimples. A capital B represents the dominant trait, which is having dimples. A lower case b represents the recessive trait, which is not having dimples.

1. Both parents contribute one allele to their offspring. For example, if both parents have the genotype Bb, an offspring could inherit the genotype of BB with each parent contributing one dominant B allele. What are the other possible combinations? **Bb, bB, bb**

2. This information can be displayed in a Punnett Square. Each side of the square represents the genotype of one parent. The Punnett Square for the offspring of the parents who both have the genotype Bb is shown at the right. If the dominant allele is present it will be the trait that appears. What is the probability that this offspring will not have dimples? **$\frac{1}{4}$**

	B	b
B	BB	Bb
b	bB	bb

3. Create a Punnett Square to show the possible gene combinations for the offspring if one parent has the genotype Bb and the other has the genotype BB.

	B	b
B	BB	Bb
B	BB	Bb

4. What is the probability that the offspring of parents with Bb and BB will have dimples? **1**

5. You can show more complicated crosses when you consider two or more genes that are independent of each other. For example, pea pods can either be round (R) or wrinkled (r), yellow (Y) or green (y). What are the possible combinations of shape and color? **RY, Ry, rY, ry**

6. Create a Punnett Square to show the possible gene combinations for the pea pods.

	RY	Ry	rY	ry
Ry	$RRYY$	$RRYy$	$RrYY$	$RrYy$
Ry	$RRYy$	$RRyy$	$RrYy$	$Rryy$
rY	$RrYY$	$RrYy$	$rrYY$	$rrYy$
ry	$RrYy$	$Rryy$	$rrYy$	$rryy$

7. What is the probability that a pea pod with both parents $RrYy$ will be round and yellow? In other words, what is the probability that there is an R and a Y present? **$\frac{9}{16}$**

8. What is the probability that a pea pod with both parents $RrYy$ will be wrinkled and yellow? **$\frac{3}{16}$**

9. What is the probability that a pea pod with both parents $RrYy$ will be wrinkled and green? **$\frac{1}{16}$**

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11-2 Reteaching
Probability

Probability is a measure of how likely a specific event is to occur. To find the *experimental probability* of a specific event, you conduct an experiment, or simulation of the experiment, multiple times. Each time you run the experiment or simulation, you are conducting a *trial*. You count the number of times the event you are looking for occurs. The experimental probability is the ratio of the number of times the event occurs to the number of trials.

$$P(\text{event}) = \frac{\text{\# of times the event occurs}}{\text{\# of trials}}$$

Problem

You toss a coin 12 times and record each result: H, T, T, H, H, H, T, H, T, H, T, H. What is the experimental probability of tails?

Step 1 Determine the total number of trials, the event you are looking for, and the number of times the event occurred during the trials.
One toss of the coin is one trial. You did 12 trials.
The event you are looking for is tails. Tails occurred 5 times.

Step 2 Use the formula for experimental probability.
$$P(\text{tails}) = \frac{\text{\# of tails}}{\text{\# of tosses}} = \frac{5}{12} \approx 0.42, \text{ or } 42\%$$
The experimental probability of tails is about 0.42.

Exercises

In a telephone survey of 150 households, 75 people answered "yes" to a particular question, 50 answered "no," and 25 were "not sure." Find each experimental probability.

1. $P(\text{yes})$ **$\frac{1}{2}$, or 50%**

2. $P(\text{no})$ **$\frac{1}{3} \approx 0.33$, or 33%**

3. $P(\text{not sure})$ **$\frac{1}{6} \approx 0.17$, or 17%**

4. $P(\text{not yes})$ **$\frac{1}{2}$, or 50%**

5. $P(\text{yes or no})$ **$\frac{5}{6} \approx 0.83$, or 83%**

6. $P(\text{yes and not sure})$ **0, or 0%**

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11-2 Reteaching (continued)
Probability

To find the *theoretical probability* of a specific event, count the number of desired events that occur in all outcomes. Then determine the number of all possible outcomes. The theoretical probability is the ratio of these two numbers.

$$P(\text{event}) = \frac{\text{\# of outcomes with event}}{\text{\# of outcomes}}$$

Problem

What is the theoretical probability of rolling a fair number cube and getting either a 4 or a 5?

Step 1 Determine the total number of possible outcomes and the number of outcomes in which the event you are looking for occurs.
There are six possible outcomes when you roll a number cube: 1, 2, 3, 4, 5, 6.
The event you are looking for occurs in two outcomes: 4 or 5.

Step 2 Use the formula for theoretical probability.
$$P(4 \text{ or } 5) = \frac{\text{\# of ways to roll a 4 or 5}}{\text{\# of possible rolls}} = \frac{2}{6} = \frac{1}{3} \approx 0.33, \text{ or } 33\%$$

The theoretical probability of getting a 4 or a 5 when you roll a number cube is $\frac{1}{3}$.

Exercises

Use the spinner at the right to find each theoretical probability.

7. $P(\text{the number is even})$
$\frac{5}{12} \approx 0.42$, or 42%

8. $P(5)$
$\frac{1}{12} \approx 0.08$, or 8%

9. $P(\text{the number is prime})$
$\frac{5}{12} \approx 0.42$, or 42%

10. $P(\text{multiple of 3})$
$\frac{1}{3} \approx 0.33$, or 33%

11. $P(\text{the number is less than 6})$
$\frac{1}{3} \approx 0.33$, or 33%

12. $P(\text{not 5 and not 7})$
$\frac{5}{6} \approx 0.83$, or 83%

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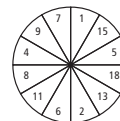
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11-3 ELL Support
Probability of Multiple Events

For Exercises 1–5, draw a line from each word or phrase in Column A to the matching item in Column B.

Column A	Column B
1. dependent events	**A.** the occurrence of one event does NOT affect how a second event can occur
2. probability of independent events	**B.** two events that cannot occur at the same time
3. independent events	**C.** $P(A) + P(B)$
4. probability of mutually exclusive events	**D.** the occurrence of one event affects how a second event can occur
5. mutually exclusive events	**E.** $P(A) \cdot P(B)$

For Exercises 6–8, draw a line from each item in Column A to the matching term in Column B.

Column A	Column B
6. Flip a coin. Then roll a number cube.	**A.** dependent events
7. snow and 80°F weather	**B.** independent events
8. Pick a piece from a set of chess pieces. Then pick a second piece.	**C.** mutually exclusive events

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11-3 Think About a Plan
Probability of Multiple Events

Marbles A jar contains four blue marbles and two red marbles. Suppose you choose a marble at random, and do not replace it. Then you choose a second marble. Find the probability that you select a blue marble and then a red marble.

Understanding the Problem

1. How many marbles are blue? **4**

2. How many marbles are red? **2**

3. How many marbles are in the jar? **6**

4. What is the problem asking you to determine?
the probability that you select a blue marble and then a red marble

Planning the Solution

5. What is the probability that you choose a blue marble from the jar? $\frac{4}{6} = \frac{2}{3}$

6. Assuming you choose a blue marble and do not replace it, how many marbles of each color remain in the jar? What is the total number of marbles in the jar?
3 blue marbles and 2 red marbles; 5 marbles

7. What is the probability that you now choose a red marble from the jar? $\frac{2}{5}$

8. How can you find the probability that you select a blue marble and then a red marble? **Answers may vary. Sample: The probability is the product of the probabilities for each event**.

Getting an Answer

9. What is the probability that you select a blue marble and then a red marble? $\frac{2}{3} \cdot \frac{2}{5} = \frac{4}{15}$

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11-3 Practice *Form G*
Probability of Multiple Events

Classify each pair of events as *dependent* or *independent*.

1. A member of the junior class is selected; one of her pets is selected. **dependent**

2. A member of the junior class is selected as junior class president; a freshman is selected as freshman class president. **independent**

3. An odd-numbered problem is assigned for homework; an even-numbered problem is picked for a test. **independent**

4. The sum of two rolls of a number cube is 6; the product of the same two rolls is 8. **dependent**

Q and R are independent events. Find $P(Q$ and $R)$.

5. $P(Q) = \frac{1}{8}$, $P(R) = \frac{2}{5}$ $\frac{1}{20}$

6. $P(Q) = 0.8$, $P(R) = 0.2$ **0.16**

7. $P(Q) = \frac{1}{4}$, $P(R) = \frac{1}{5}$ $\frac{1}{20}$

8. $P(Q) = \frac{3}{4}$, $P(R) = \frac{2}{3}$ $\frac{1}{2}$

9. Suppose you have seven CDs in a box. Four are rock, one is jazz, and two are country. Today you choose one CD without looking, play it, and put it back in the box. Tomorrow, you do the same thing. What is the probability that you choose a country CD both days? $\frac{4}{49}$

You randomly select an integer from 1 to 100. State whether the events are mutually exclusive. Explain your reasoning.

10. The integer is less than 40; the integer is greater than 50.
Yes; all integers 1–39 are less than 50.

11. The integer is odd; the integer is a multiple of 4.
Yes; all multiples of 4 are even.

12. The integer is less than 50; the integer is greater than 40.
No; the integers 41–49 are less than 50 and greater than 40.

M and N are mutually exclusive events. Find $P(M$ or $N)$.

13. $P(M) = \frac{3}{4}$, $P(N) = \frac{1}{6}$ $\frac{11}{12}$

14. $P(M) = 10\%$, $P(N) = 45\%$ **55%**

15. $P(M) = 20\%$, $P(N) = 18\%$ **38%**

16. $P(M) = \frac{1}{10}$, $P(N) = \frac{3}{5}$ **70%**

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**11-3 Practice** (continued) *Form G*
Probability of Multiple Events

17. Exactly 62% of the students in your school are under 17 years old. In addition, 4% of the students are over 18. What is the probability that a student chosen at random is under 17 or over 18? **66%**

A fair number cube is tossed. Find each probability.

18. $P(\text{even or } 3)$ $\frac{2}{3}$

19. $P(\text{less than 2 or even})$ $\frac{2}{3}$

20. $P(\text{prime or } 4)$ $\frac{2}{3}$

21. You randomly choose a natural number from 1 to 10. What is the probability that you choose a multiple of 2 or 3? $\frac{7}{10}$

The graph at the right shows the types of bicycles in a bicycle rack. Find each probability.

Bicycles at Your School

5% 11%
14%
28%
42%

1-speed
3-speed
5-speed
10-speed
Other

22. A bicycle is a 1-speed. **11%**

23. A bicycle is a 3-speed or a 5-speed. **70%**

24. A bicycle is not a 10-speed. **86%**

25. A bicycle is not a 1-, 3-, or 10-speed. **47%**
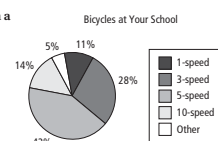
You have a drawer with five pairs of white socks, three pairs of black socks, and one pair of red socks. You choose one pair of socks at random each morning, starting on Monday. You do not put the socks you choose back in the drawer. Find the probability of each event.

26. You select black socks on Monday and white socks on Tuesday. $\frac{5}{24}$

27. You select red socks on Monday and black socks on Tuesday. $\frac{1}{24}$

28. You select white socks on Monday and Tuesday. $\frac{5}{18}$

29. You select red socks on Monday. $\frac{1}{9}$

30. Only 93% of the airplane parts being examined pass inspection. What is the probability that all of the next 5 parts examined will pass inspection? **≈ 69.6%**

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11-3 **Practice** *Form K*
Probability of Multiple Events

Classify each pair of events as *dependent* or *independent*.

1. Roll a number cube. Then roll it again.
independent

2. Pull a card from a deck of playing cards. Then pull a second card. **dependent**

3. Randomly choose a student from your class. Then choose another student.
dependent

4. Flip a coin. Then spin a spinner.
independent

Use the table shown below to answer the following questions.

Movie Collection		
	Video	DVD
Action	12	26
Comedy	14	8
Drama	4	16

5. You randomly pick a video and a DVD. What is the probability that you pick an action video and a comedy DVD? **about 6%**

6. Your friend randomly picks a video and a DVD. What is the probability that she picks a comedy video and an action DVD? **about 24%**

7. What is the probability of randomly picking a drama video and a comedy DVD? **about 2%**

8. Writing Explain the difference between independent events and dependent events.
Answers may vary. Sample: Events are dependent when the occurrence of one event affects how a second event can occur. Events are independent when the occurrence of one event does not affect how the other event can occur.

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11-3 **Practice** (continued) *Form K*
Probability of Multiple Events

Two fair number cubes are rolled. State whether the following events are mutually exclusive.

9. The sum is odd. The sum is less than 5. **not mutually exclusive**

10. The difference is 1. The sum is even. **mutually exclusive**

11. The sum is a multiple of 4. The sum is odd. **mutually exclusive**

Find the probability for the following mutually exclusive events.

12. Students can either participate in track and field or play baseball. About 13% of students participate in track and field. About 8% play baseball. What is the probability that a student chosen at random either participates in track and field or plays baseball? **about 21%**

13. About $\frac{1}{5}$ of a town's population has black hair. About $\frac{2}{7}$ of the population has blonde hair. What is the probability that a person chosen at random from this town will have either black hair or blonde hair? **about 49%**

Use the diagram at the right to answer the following questions.

14. Suppose you randomly select a shape from this circle. What is the probability that the shape is black or has five points? **70%**

15. What is the probability of randomly selecting a shape that is black or has four points? **80%**

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11-3 **Standardized Test Prep**
Probability of Multiple Events

Multiple Choice

For Exercises 1–4, choose the correct letter.

A store display shows two red shirts, one blue shirt, and three shirts with red and white stripes. The display also shows two pairs of blue jeans, one pair of white pants, and one pair of white shorts.

1. What is the probability of randomly selecting an item with white or red on it? **D**

Ⓐ $\frac{1}{4}$ Ⓑ $\frac{3}{10}$ Ⓒ $\frac{1}{2}$ Ⓓ $\frac{7}{10}$

2. What is the probability of randomly selecting two items and getting a pair of blue jeans, putting them back in the display, and then randomly selecting a blue shirt? **F**

Ⓕ $\frac{1}{50}$ Ⓖ $\frac{1}{45}$ Ⓗ $\frac{2}{10}$ Ⓘ $\frac{3}{10}$

3. What is the probability of randomly selecting a complete outfit (one shirt and one pair of jeans, pants, or shorts) on two picks? **C**

Ⓐ $\frac{1}{24}$ Ⓑ $\frac{1}{5}$ Ⓒ $\frac{6}{25}$ Ⓓ $\frac{4}{15}$

4. What is the probability of selecting an item with red or blue on it? **I**

Ⓕ $\frac{3}{20}$ Ⓖ $\frac{3}{10}$ Ⓗ $\frac{3}{5}$ Ⓘ $\frac{4}{5}$

Short Response

5. There is a 50% chance of thunderstorms on Monday, a 50% chance on Tuesday, and a 50% chance on Wednesday. Assume these are independent events. What is the probability that there will be thunderstorms on Monday, Tuesday, and Wednesday? Show your work.
[2] P(M and T and W) = P(M) · P(T) · P(W)
= 0.50 · 0.50 · 0.50
= 0.125
There is a 12.5% probability of thunderstorms on Monday, Tuesday, and Wednesday.
[1] incorrect or incomplete work shown
[0] incorrect answer and no work shown OR no answer given

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11-3 **Enrichment**
Probability of Multiple Events

Tree Diagrams

Complex problems involving probability are often easier to visualize and solve using tree diagrams. For example, suppose that Alice, Bob, and Carol are running for president of the Math Club. Alice has a 0.45 probability of being elected, while Bob has 0.35 probability and Carol has a 0.2 probability of being elected. Sue and Ted are the candidates for vice president. If Alice becomes president, the probability is 0.7 that she will choose Sue as her vice president. If Bob becomes president, the probability is 0.4 that he will choose Sue as his vice president, while if Carol becomes president, the probability is 0.6 that she will choose Sue.

Examine the following tree diagram that represents the given information.

Math Club Presidential Election
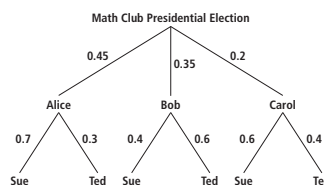
The probability that Alice will be elected president and choose Sue as her vice president can be found by multiplying the probabilities found along the "branches" of the tree. Thus, the Alice-Sue combination has a probability of $0.45 \times 0.7 = 0.315$ of occurring.

1. What is the probability of the Bob-Sue combination? **0.14** of Carol-Sue? **0.12**

Use a tree diagram to compute the probabilities of each of the following events.

2. The probability that a random student at Elmville College is a freshman is 0.3; a sophomore, 0.25; and a junior or senior, 0.45. The probability that a freshman will major in engineering is 0.15; a sophomore, 0.2; and a junior or senior, 0.3. What is the probability that a random student at this college majors in engineering? **0.23**

3. The Adams' restaurant specializes in beef, chicken, and seafood. Of its customers, 32% order beef and 41% order chicken. Of those who order beef, 73% also order dessert. Of those who order chicken, 62% also order dessert. If 65% of the customers order dessert, what is the probability that someone who orders seafood will also order dessert? **about 0.60**

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11-3 **Reteaching**
Probability of Multiple Events

- If Event *A* can change the way Event *B* occurs, then the events are *dependent*.
- If Event *A* cannot change the way Event *B* occurs, then the events are *independent*.

If Event *A* and Event *B* are independent, the probability of Event *A* and Event *B* both occurring is the product of their individual probabilities.

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

Problem

A pet store has two cages of mice. The first cage has 10 white mice—4 females and 6 males. The second cage has 10 gray mice—6 females and 4 males. Suppose you randomly choose 1 mouse from each cage. What is the probability that you choose 2 female mice?

Cage 1		Cage 2	
F	F	F	F
F	F	F	F
M	M	F	F
M	M	M	M
M	M	M	M
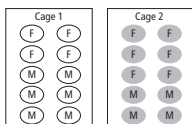
Step 1 Determine the events.
Event *A* is "you choose a white female mouse."
Event *B* is "you choose a gray female mouse."
You are looking for the probability of Event *A* and Event *B*.

Step 2 Decide if the events are independent.
Your choice of a white mouse does not affect your choice of a gray mouse. The events are independent.

Step 3 Use the formula.

$$P(\text{white female and gray female}) = P(\text{white female}) \cdot P(\text{gray female})$$
$$= \frac{4}{10} \cdot \frac{6}{10} = \frac{24}{100} = 0.24 = 24\%$$

The probability of choosing 2 female mice is 24%.

Exercises

Use the information from the problem above. You choose one mouse at random from each cage. Find each probability.

1. *P*(white male and gray male) **24%**

2. *P*(white mouse and gray male) **40%**

3. *P*(white male and gray female) **36%**

4. *P*(white male and white female) **0%**

5. *P*(white mouse and white mouse) **0%**

6. *P*(white female and gray male) **16%**

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11-3 **Reteaching** (continued)
Probability of Multiple Events

The probability of Event *A* or Event *B* occurring is the sum of their individual probabilities, minus the probability of both events occurring.

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

- If Event *A* cannot happen at the same time as Event *B*, then the events are said to be *mutually exclusive*.
- If Event *A* and Event *B* are mutually exclusive, *P*(*A* and *B*) = 0.

Problem

A pet store has one cage with 20 mice. Of the 20 mice, there are 4 white females, 6 white males, 6 gray females, and 4 gray males. Suppose you randomly choose 1 mouse from the cage. What is the probability that you choose a female or a gray mouse?

F	F	F	F
F	F	F	F
M	M	F	F
M	M	M	M
M	M	M	M
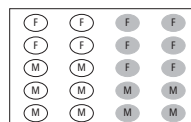
Step 1 Determine the events.
Event *A* is "you choose a female."
Event *B* is "you choose a gray mouse."
You are looking for the probability of Event *A* or Event *B*.

Step 2 Decide if the events are mutually exclusive.
You could choose one mouse that was both female and gray at the same time. The events are not mutually exclusive.

Step 3 Use the formula.

$$P(\text{female or gray}) = P(\text{female}) + P(\text{gray}) - P(\text{female and gray})$$
$$= \frac{10}{20} + \frac{10}{20} - \frac{6}{20} = \frac{14}{20} = \frac{7}{10} = 0.70 = 70\%$$

The probability of choosing a female or a gray mouse is 70%.

Exercises

Use the information from the problem above. You choose one mouse at random. Find each probability.

7. *P*(female or white) **80%**

8. *P*(male or white) **70%**

9. *P*(male or gray) **80%**

10. *P*(white male or gray female) **60%**

11. *P*(white female or gray male) **40%**

12. *P*(white male or gray male) **50%**

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11-4 **ELL Support**
Conditional Probability

Problem

Aaron asked 50 of his classmates whether they prefer soccer or basketball. What is the probability that a student prefers soccer given that the student is female? Explain your work.

Sports Survey

	Soccer	Basketball
Female	14	10
Male	7	19

$P(\text{female and soccer}) = \frac{14}{50}$ Determine the probability that a student is female and prefers soccer.

$P(\text{female}) = \frac{24}{50}$ Determine the probability that a student is female.

$\dfrac{P(\text{female and soccer})}{P(\text{female})} = \dfrac{\frac{14}{50}}{\frac{24}{50}}$ Apply the Conditional Probability formula.

$\dfrac{\frac{14}{50}}{\frac{24}{50}} = \frac{14}{24} = \frac{7}{12} \approx 0.58$ Simplify the fraction.

Exercise

An auto dealership sells two-wheel drive and four-wheel drive cars and trucks. What is the probability that an automobile is four-wheel drive given that it is a car? Explain your work.

Automobiles

	Car	Truck
2-wheel drive	24	12
4-wheel drive	6	38

$P(\text{four-wheel drive and car}) = \frac{6}{80}$ **Determine the probability that an automobile is a car and four-wheel drive**.

$P(\text{car}) = \frac{30}{80}$ **Determine the probability that an automobile is a car**.

$\dfrac{P(\text{female and soccer})}{P(\text{female})} = \dfrac{\frac{6}{80}}{\frac{30}{80}}$ **Apply the Conditional Probability formula**.

$\dfrac{\frac{6}{80}}{\frac{30}{80}} = \frac{6}{30} = 0.2$ **Simplify the fraction**.

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11-4 **Think About a Plan**
Conditional Probability

Transportation You can take Bus 65 or Bus 79. You take the first bus that arrives. The probability that Bus 65 arrives first is 75%. There is a 40% chance that Bus 65 picks up passengers along the way. There is a 60% chance that Bus 79 picks up passengers. Your bus picked up passengers. What is the probability that it was Bus 65?

Understanding the Problem

1. What is the probability that Bus 65 arrives first? **75%**

2. What is the probability that Bus 65 picks up passengers? **40%**

3. What is the probability that Bus 79 picks up passengers? **60%**

4. What is the problem asking you to determine?
   the probability that Bus 65 arrived first

Planning the Solution

5. Let B65 = Bus 65 arrived first, B79 = Bus 79 arrived first, P = passengers, NP = no passengers. What conditional probability are you looking for? **P(B65 | P)**

6. How can a tree diagram help you solve the problem?
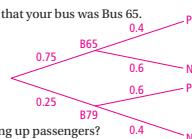   **Answers may vary. Sample: A tree diagram can help me organize the information in the problem and see all of the possible probabilities involved**.

7. Write an equation you can use to find the probability that your bus was Bus 65.
   $P(\text{B65} \mid \text{P}) = \dfrac{P(\text{B65 and P})}{P(\text{B65 and P}) + P(\text{B79} + \text{P})}$

Getting an Answer

8. Make a tree diagram for this problem.

0.75	B65	0.4 → P
		0.6 → NP
0.25	B79	0.6 → P
		0.4 → NP

9. Which two branches of the diagram show a bus picking up passengers?
   0.75 × 0.4 and 0.75 × 6

10. What is the probability your bus was Bus 65? $\frac{2}{3}$ **or 66.67%**

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11-4 Practice *Form G*
Conditional Probability

Use the table at the right to find each probability.

Education and Salary of Employees

	Under $20,000	$20,000 to $30,000	Over $30,000
Less than high school	69	36	2
High school	112	98	14
Some college	102	193	143
College degree	13	178	245

1. P(has less than high school education) **8.9%**

2. P(earns over $30,000 and has less than high school education) **0.2%**

3. P(earns over $30,000 | has only high school education) **6.25%**

4. P(has high school education or less | earns over $30,000) **4%**

Use the table below to find each probability. The table gives information about students at one school.

Favorite Leisure Activities

	Sports	Hiking	Reading	Phoning	Shopping	Other
Female	39	48	85	62	71	29
Male	67	58	76	54	68	39

5. P(sports | female) **11.7%**
6. P(female | sports) **36.8%**
7. P(reading | male) **21%**
8. P(male | reading) **47.2%**
9. P(hiking | female) **14.4%**
10. P(hiking | male) **16%**
11. P(male | shopping) **48.9%**
12. P(female | shopping) **51.1%**

13. The senior class is 55% female, and 32% of the class are females who play a competitive sport. What is the probability that a student plays a competitive sport, given that the student is female? **58.2%**

14. A softball game has an 80% chance of being cancelled if it rains and a 30% chance of being cancelled if there is fog when there is no rain. There is a 70% chance of fog with no rain and a 30% chance of rain.
   a. Make a tree diagram based on the information above.
   b. Find the probability that there will be fog and the game will be cancelled. **21%**
   c. Find the probability that there will be rain and the game will be played. **6%**
   d. Find the probability that the game will be cancelled. **45%**

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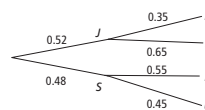**11-4 Practice** (continued) *Form G*
Conditional Probability

15. The population of a high school is 51% male. 45% of the males and 49% of the females attend concerts.
   a. Make a tree diagram based on the information above.
   b. Find the probability that a student is male and attends concerts. **about 23%**
   c. Find the probability that a student is female and does not attend concerts. **about 25%**
   d. Find the probability that a student attends concerts. **about 47%**

16. **Reasoning** A student says that if $P(A) = P(A | B)$, then A and B must be independent events. Is the student correct? Explain.
   Yes; if A and B are independent, then the probability of A will be the same whether or not B occurs.

17. A school's colors are blue and gold. At a pep rally, 65% of the students are wearing both blue and gold, and 90% of the students are wearing blue.
   a. What percent of students wearing blue are also wearing gold? **72%**
   b. **Writing** Describe how a tree diagram could help you solve this problem.
   Answers may vary. Sample: A tree diagram can help me determine P(B) and P(B and G) so that I can use the formula for conditional probability.

You survey a group of juniors and seniors. The tree diagram relates student's class and whether a student is employed after school. Find each probability. Let J, S, E, and U represent junior, senior, employed, and unemployed, respectively.

18. P(E) **44.6%**
19. P(J and U) **33.8%**
20. P(S | E) **59.2%**
21. P(J | U) **61%**
22. P(S | U) **39.0%**
23. P(J | E) **40.8%**

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11-4 Practice *Form K*
Conditional Probability

Use the table to find each probability.

Guitars

	Acoustic	Electric
Tan	78	42
Black	34	56
Blue	12	16

1. P(black | acoustic)

P(black | acoustic) $= \frac{34}{124}$

P(black | acoustic) $= $ **≈ 0.27**

2. P(tan | electric) **≈ 0.37**
3. P(blue | electric) **≈ 0.14**
4. P(acoustic | tan) **0.65**
5. P(tan | acoustic) **≈ 0.63**
6. P(electric | blue) **≈ 0.57**

The following table shows national employment statistics. Use the table to find each probability.

Number of Men and Women in Different Occupations (thousands)

	Professionals	Sales People	Laborers
Men	4190	2588	2951
Women	4747	3213	1432

Source: Equal Employment Opportunity Commission

7. P(male | professional) **≈ 0.47**
8. P(laborer | female) **≈ 0.15**
9. P(female | sales) **≈ 0.55**
10. P(professional | female) **≈ 0.51**
11. P(sales | male) **≈ 0.27**
12. P(male | laborer) **≈ 0.67**

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11-4 Practice (continued) *Form K*
Conditional Probability

Use the Conditional Probability Formula and the table below to answer the following questions.

Students' Reading Preferences

	Comic Books	Novels
Middle School	128	32
High School	86	98

13. What is the probability that a student prefers comic books, given that the student is in high school?

$P(B | A) = \frac{P(A \text{ and } B)}{P(A)}$

P(high school and comic books) $= \frac{86}{344}$ P(high school) $= \frac{184}{344}$

P(comic books | high school) $= \frac{\frac{86}{344}}{\frac{184}{344}} \approx$ **47%**

14. What is the probability that a student prefers novels, given that the student is in middle school? **20%**

15. **Writing** Tony wants to know the probability that his classmate is left-handed, given that she is female. Should he use conditional probability? Explain why or why not. **Answers may vary. Sample: Yes; in this case, the student's gender is this condition.**

Use a tree diagram to solve the following problem.

16. A car insurance company compiled the following information from a recent survey.
   • 75% of drivers carefully follow the speed limit.
   • Of the drivers who carefully follow the speed limit, 80% have never had an accident.
   • Of the drivers who do not carefully follow the speed limit, 65% have never had an accident.
   What is the probability that a driver does not carefully follow the speed limit and has never had an accident? **16.25%**

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11-4 Standardized Test Prep
Conditional Probability

Multiple Choice

For Exercises 1–2, choose the correct letter.

A local bookstore classifies its books by type of reader, type of book, and cost. Use the table at the right for Exercises 1–2.

		< $10	$10
Child	Fiction	120	255
	Nonfiction	35	60
Adult	Fiction	200	110
	Nonfiction	75	150

1. What is the probability that a book selected at random is a child's book, given that it costs $15? **C**

   Ⓐ $\frac{315}{1005}$ Ⓑ $\frac{470}{1005}$ Ⓒ $\frac{315}{575}$ Ⓓ $\frac{470}{575}$

2. What is the probability that a book selected at random is fiction, given that it costs $6? **I**

   Ⓕ $\frac{320}{1005}$ Ⓖ $\frac{430}{1005}$ Ⓗ $\frac{120}{430}$ Ⓘ $\frac{320}{430}$

Extended Response

3. Of the photographs produced in one day at a photo shop, 25% are black-and-white, and the rest are in color. Portraits make up 65% of the black-and-white photos and 45% of the color photos. Let B, C, P, and N represent black-and-white, color, portrait, and not a portrait, respectively. Draw a tree diagram to represent this situation. What is the probability that a photo chosen at random is not a portrait? Show your work.

   [4] Student used correct tree diagram and conditional probability formula to show $P(N) = 50\%$.
   [3] Student used diagram and formula, but misunderstood part of the problem or ignored important information.
   [2] Student attempted to use diagram and formula, but did so incorrectly, OR student used inappropriate strategy, but showed some understanding of the problem.
   [1] Student attempted to solve problem, but used inappropriate strategy and made little progress toward solution.
   [0] incorrect answers and no work shown OR no answers given
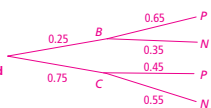
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11-4 Enrichment
Conditional Probability

The Probability of Receiving a Defective Item in a Shipment

Many manufactured items look interchangeable. Examples are ball bearings, light bulbs, and transistors. However, an individual ball bearing may be too large or too small, and a light bulb or a transistor that looks fine may prove to be defective. The following exercises require the computation of certain probabilities based on the number of defective items and the size of the sample. Answers can be given in terms of combination symbols.

1. A shipment contains 50 transistors, 3 of which are defective. What is the probability that a randomly chosen transistor from this shipment works? What is the probability that it is defective?
   $\frac{47}{50}, \frac{3}{50}$

2. A sample consisting of 2 transistors is chosen from this shipment. What is the probability that both transistors work? What is the probability that both transistors are defective? $\frac{_{47}C_2}{_{50}C_2}$ **or 88.2%;** $\frac{_3C_2}{_{50}C_2}$ **or 0.2%**

3. A shipment contains 80 ball bearings, 5 of which are defective. What is the probability that a randomly selected ball bearing is of an acceptable size? What is the probability that it is defective? $\frac{15}{16}, \frac{1}{16}$

4. A sample of three ball bearings is chosen from the above shipment.
   a. What is the probability that all three ball bearings are acceptable? $\frac{_{75}C_3}{_{80}C_3}$**; or 82.1%**
   b. What is the probability that two ball bearings are acceptable and one is defective? $\frac{_{75}C_2 \times {_5}C_1}{_{80}C_3}$ **or 16.9%**
   c. What is the probability that one is acceptable and two are defective? $\frac{_{75}C_1 \times {_5}C_2}{_{80}C_3}$ **or 0.9%**
   d. What is the probability that all three ball bearings are defective? $\frac{_5C_3}{_{80}C_3}$ **or 0.01%**
   e. What is the probability that an odd number of ball bearings are defective? $\frac{_{75}C_2 \times {_5}C_1 + {_5}C_3}{_{80}C_3}$ **or 16.9%**

5. A shipment of toy cars contains 40 red cars and 45 blue cars. Two cars of each color are defective. A sample of three toy cars is chosen from the shipment.
   a. What is the probability that all three toy cars are blue? $\frac{_{45}C_3}{_{85}C_3}$ **or 14.4%**
   b. What is the probability that two toy cars are blue and one is red? $\frac{_{45}C_2 \times {_{40}}C_1}{_{85}C_3}$ **or 40.1%**
   c. What is the probability that two toy cars are acceptable and one is defective? $\frac{_{81}C_2 \times {_4}C_1}{_{85}C_3}$ **or 13.1%**
   d. What is the probability that two toy cars are red and acceptable and one is blue and defective? $\frac{_{38}C_2 \times {_2}C_1}{_{85}C_3}$ **or 1.4%**
   e. What is the probability that one toy car is an acceptable blue car, another is a defective blue car, and the other is a defective red car? $\frac{_{43}C_1 \times {_2}C_1 \times {_2}C_1}{_{85}C_3}$ **or 0.2%**

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11-4 Reteaching
Conditional Probability

When events A and B are *dependent*, the probability of B occurring depends on whether A has already occurred. This kind of probability is called *conditional probability*. The probability of B given that A has occurred is written as $P(B|A)$.

Problem

A computer lab has 10 computers. Some have CD drives and some have DVD drives. Some are new and some are used. A student picks a computer at random. Use the table to find each probability.

	CD	DVD	Total
New	4	3	7
Used	2	1	3
Total	6	4	10

$P(\text{computer is new}) = \frac{7}{10}$ Out of 10 computers in the lab, 7 are new.

$P(\text{computer has a CD drive}) = \frac{6}{10} = \frac{3}{5}$ Out of 10 computers in the lab, 6 have CD drives.

$P(\text{computer is new and has a CD drive}) = \frac{4}{10} = \frac{2}{5}$ Out of 10 computers in the lab, 4 are new and have CD drives.

$P(\text{computer is new given it has a CD}) = \frac{4}{6} = \frac{2}{3}$ Out of 6 computers that have CD drives, 4 are new.

Note the difference between the last two probabilities. The conditional probability is based only on the number of computers that meet the condition, not on the total number of computers in the lab.

Exercises

A can holds 20 red balls with blue dots, 15 red balls without dots, 30 white balls with blue dots, and 25 white balls without dots. Find each probability.

1. $P(\text{red})$ $\frac{7}{18}$ 2. $P(\text{with dots})$ $\frac{5}{9}$ 3. $P(\text{red and with dots})$ $\frac{2}{9}$

4. $P(\text{red | with dots})$ $\frac{2}{5}$ 5. $P(\text{white | no dots})$ $\frac{5}{8}$ 6. $P(\text{no dots | white})$ $\frac{5}{11}$

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11-4 Reteaching (continued)
Conditional Probability

You can use a tree diagram to help you find the probabilities of dependent events.

Problem

Suppose a class is 55% male. Of the males, 34% are at least 68 in. tall. Of the females 12% are at least 68 in. tall. What is the probability that a randomly chosen student is a female at least 68 in. tall?

Step 1   Organize the information in a tree diagram.

M - male
F - female
G - at least 68 in. tall
L - less than 68 in. tall
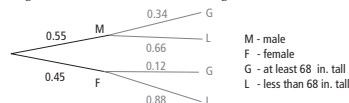
Each of the two black branches of the tree represents a simple probability: $P(M) = 0.55$ and $P(F) = 0.45$.

Each of the four gray branches represents a conditional probability: $P(G|M) = 0.34$, $P(L|M) = 0.66$, $P(G|F) = 0.12$, and $P(L|F) = 0.88$.

Step 2   Determine the probability you want to find.
$P(F \text{ and } G)$, the probability that a student is female and at least 68 in. tall.

Step 3   Rewrite the conditional probability formula to find $P(F \text{ and } G)$.
$P(F \text{ and } G) = P(F) \cdot P(G|F)$

Step 4   Substitute information from the tree diagram.
$P(F \text{ and } G) = P(F) \cdot P(G|F)$
$\qquad = 0.45 \cdot 0.12$
$\qquad = 0.054$

The probability that a randomly chosen student is a female at least 68 in. tall is 5.4%.

Exercises

Use the tree diagram above to find each probability.

7. $P(L \text{ and } M)$ **36.3%**   8. $P(G \text{ and } M)$ **18.7%**   9. $P(L \text{ and } F)$ **39.6%**

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11-5 ELL Support
Analyzing Data

Concept List

bimodal	box-and-whisker plot	interquartile range
mean	median	mode
outlier	quartiles	range of a data set

Choose the concept from the list above that best represents the item in each box.

1. (12), 36, 45, (12), 52, 27, (12)
mode

2. 29, 7, 35, 29, 56, 12, 75, 26, 39, 8

$Q_3 - Q_1 = 39 - 12 = 27$
interquartile range

3. 4, 7, 2, 9, 14, 8, (46)
outlier

4. 34, 72, 29, 25, 13, 81, 56

$81 - 13 = 68$
range of a data set

5. 15, 17, 17, (19), 21, 23, 24
median

6.
box-and-whisker plot

7. (54), 74, (35), 91, 23, (54), 21, (35)
bimodal

8. 44, 15, 76, 34, 91

$44 + 15 + 76 + 34 + 91 = 260$

$260 \div 5 = 52$
mean

9. 7, 9, 17, 26, 38, 40, 45, 53, 55, 62

$Q_1 = 17$

$Q_2 = 39$

$Q_3 = 53$

$Q_4 = 62$

quartiles

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11-5 Think About a Plan
Analyzing Data

Meteorology On May 3, 1999, 59 tornadoes hit Oklahoma in the largest tornado outbreak ever recorded in the state. Sixteen of these were classified as strong (F2 or F3) or violent (F4 or F5).

a. Make a box-and-whisker plot of the data for length of path.
b. Identify the outliers. Remove them from the data set and make a revised box-and-whisker plot.
c. **Writing** How does the removal of the outliers affect the box-and-whisker plot? How does it affect the median of the data set?

Major Tornadoes in Oklahoma, May 3, 1999

Length of path (miles)	Intensity
6	F3
9	F3
4	F2
37	F5
7	F2
12	F3
8	F2
7	F2
15	F4
39	F4
1	F2
22	F3
15	F3
8	F2
13	F3
2	F2

1. Arrange the data in increasing order.

1, 2, 4, 6, 7, 7, 8, 8, 9, 12, 13, 15, 15, 22, 37, 39

2. Minimum value = **1** $Q_1 = $ **6.5**

Maximum value = **39** $Q_2 = $ **8.5**

$Q_3 = $ **15**

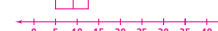**3.** Use your previous answers to make a box-and-whisker plot of the data for length of path.
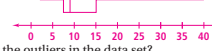
4. How can you identify the outliers in the data set?

Answers may vary. Sample: Look at the ends of the ordered data for values that are substantially different.

5. What are the outliers in the data set? **22, 37, 39**

6. Remove the outliers from the data set and make a revised box-and-whisker plot.

7. How does the removal of the outliers affect the box-and-whisker plot?

Answers may vary. Sample: The box is shifted left and the median moves to almost the center of the box. The whiskers become shorter, especially the right one.

8. How does the removal of the outliers affect the median of the data set?

The median shifts from 8.5 to 8.

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11-5 Practice
Analyzing Data
Form G

Find the mean, median, and mode of each set of values.

1. Customers per day: 98 87 79 82 101 99 97 97 102 91 93   **about 93.3; 97; 97**

2.
Weight (g)	2.3	2.4	2.5	2.6	2.8	2.9
Frequency	1	4	1	1	1	2

about 2.56; 2.45; 2.4

3.
Length (m)	12	13	14	15	16	17	18
Frequency	2	5	3	7	4	9	1

about 15.2; 15; 17

Identify the outlier of each set of values.

4. 32 35 3 36 37 35 38 40 42 34   **3**

5. 153 156 176 156 165 110 159 169 172   **110**

6. The table shows the average monthly rainfall for two cities. How can you compare the rainfall amounts?   **City A: mean = 3, mode = 3.1, min = 0.8, max = 5, range = 4.2, Q_1 = 2.25,**

	J	F	M	A	M	J	J	A	S	O	N	D
City A	3.2	3.1	4.5	5.0	4.1	2.9	1.8	0.8	2.2	2.3	3.1	3.0
City B	4.2	4.0	4.7	4.8	4.5	4.3	4.0	3.9	4.3	4.4	4.6	4.5

median = 3.05, Q_3 = 3.65, IQ range = 1.4; City B: mean = 4.35, modes = 4, 4.3, 4.5, min = 3.9, max = 4.8, range = 0.9, Q_1 = 4.1, median = 4.35, Q_3 = 4.55, IQ range = 0.45

7. The list gives the average temperatures in January for several cities in the mid-South. Make a box-and-whisker plot of the data.
49.1 50.8 42.9 44.0 44.2 51.4 45.7
39.9 50.8 46.7 52.4 50.4

Make a box-and-whisker plot for each set of values.

8. 2 8 3 7 3 6 4 9 10 15 21 29 32 30 5 7 32 4 11 13 11 14 10 12 13 15

9. 1054 1165 1287 1385 1456 1398 1298 1109 1067 1384 1499 1032 1222 1045

8.

9.

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11-5 Practice (continued)
Analyzing Data
Form G

Find the values at the 20th and 80th percentiles for each set of values.

10. 188 168 174 198 186 170 180 182 186 176   **174; 188**

11. 376 324 346 348 350 352 356 368 345 360   **346; 368**

Identify the outlier in each data set. Then find the mean, median, and mode of the data set when the outlier is included and when it is not.

12. 23 76 79 76 77 74 75   **23; about 68.6, 76, 76; about 76.2, 76, 76**

13. 43 46 49 50 52 54 78 47   **78; about 52.4, 49.5, no mode; about 48.7, 49, no mode**

14. The table shows the number of shaved-ice servings sold during the first week of July.   **See below**

Date	7/1	7/2	7/3	7/4	7/5	7/6	7/7
Number Sold	65	70	67	98	72	67	64

a. Make a box-and-whisker plot of the data for the number of shaved-ice servings sold.
b. Find any outliers. Remove them from the data set and make a revised box-and-whisker plot.   **98**
c. **Writing** How does removing the outliers affect the box-and-whisker plot? How does it affect the measures of central tendency?   **Answers may vary. Sample: Removing the outliers shortens the right-hand whisker and narrows the box; the mean is reduced from ≈ 71.9 to 67.5; the median and mode are unchanged.**

For Exercises 15–18, use the set of values below.

1 2 2 2 2 2 2 3 3 3 3 3 4 4 5 25 26 27

15. At what percentile is 1?   **0th**

16. At what percentile is 25?   **85th**

17. Find the mean, median, and mode of the data set.   **6.25, 3, 2**

18. **Writing** Suppose these values represent years of experience of the accountants at an accounting firm. Which measure(s) of central tendency best describe(s) the experience of the firm's accountants? Explain.   **Answers may vary. Sample: median and mode; 60% (12 out of 20) of the accountants have 2 or 3 years of experience, but only 15% (3 out of 20) of them have more than the mean years of experience.**

14a.

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11-5 **Practice** *Form K*
Analyzing Data

Find the mean, median, and mode of the following data set.

Points Per Game					
	Game 1	Game 2	Game 3	Game 4	Game 5
Points	24	17	15	30	24

1. mean

2. median **24**

3. mode **24**

$24 + 17 + 15 + 30 + 24 = 110$

$110 \div 5 =$ **22**

Identify the outlier in the following data set. Then find the mean, median, and mode.

Height of Students					
	Sue	Dalia	Ling	Roberto	Eleanore	Cayden
Height (in.)	58	60	74	58	62	64

4. outlier **74**

5. mean $\approx$ **62.7**

6. median **61**

7. mode **58**

8. Reasoning Which measure of central tendency would be most affected by removing the outlier from the above data set? Explain your reasoning.
The mean would be most affected. The median would decrease by 1 and the mode would stay the same. However, the mean would decrease by more than 2.

9. Compare the following sets of data.

Great Lakes Coastal Water Temperatures (°F)												
	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Buffalo, NY	34	28	33	46	56	65	72	70	61	50	44	38
Oswego, NY	49	48	48	49	52	57	62	65	64	62	58	54

Answers may vary. Sample: The mean for Oswego is about 6 degrees warmer than the mean for Buffalo, which indicates that the overall water temperature is warmer. The ranges and interquartile ranges indicate that the water temperature varies throughout the year much more in Buffalo than in Oswego.
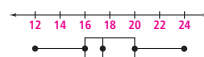
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11-5 **Practice** (continued) *Form K*
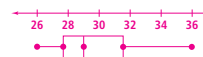Analyzing Data

Make a box-and-whisker plot for each set of values.

10. 15, 19, 24, 16, 12, 18, 20, 22, 16, 17

11. 26, 32, 27, 36, 28, 30, 31, 28

Find the following percentiles of the data set displayed below.

27, 28, 29, 29, 30, 31, 32, 33, 34, 35, 36, 36, 37, 38, 39, 40, 40, 41, 42, 43

12. 45th percentile **35**

13. 70th percentile **39**

14. 25th percentile **31**

15. 95th percentile **43**

16. 80th percentile **40**

17. 15th percentile **29**

18. Error Analysis Your friend calculated the tenth percentile of the data set shown above and got 35. What error did your friend make? What is the correct answer?
Your friend found the tenth number in the list, not the tenth percentile; the correct answer is 29.

19. Open-Ended Describe a situation in which the median would be a more useful measure of central tendency than the mean.
Answers may vary. Sample: If there are outliers in a set of data, then the median will give you a better sense of the data's central tendency.

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11-5 **Standardized Test Prep**
Analyzing Data

Multiple Choice

For Exercises 1–5, choose the correct letter. Use the data set below.

Day	9/1	9/2	9/3	9/4	9/5	9/6	9/7	9/8	9/9	9/10	9/11	9/12
Deliveries	14	15	19	15	15	16	19	20	21	29	16	17

1. What is the mean of the data set? **D**
Ⓐ 12 Ⓑ 15 Ⓒ 16.5 Ⓓ 18

2. How many modes does the data set have? **G**
Ⓕ 0 Ⓖ 1 Ⓗ 2 Ⓘ 3

3. What is the interquartile range of the data? **C**
Ⓐ 1.5 Ⓑ 3 Ⓒ 4.5 Ⓓ 15

4. What is the median value of the data set *without the outlier?* **F**
Ⓕ 16 Ⓖ 17 Ⓗ 19 Ⓘ 29

5. What value is at the 50th percentile? **B**
Ⓐ 16 Ⓑ 17 Ⓒ 19 Ⓓ 20

Short Response

6. Make a box-and-whisker plot of the data set. Label the median, minimum, maximum, first quartile, and third quartile.
[2]
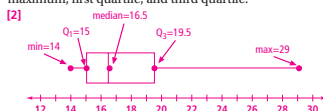
[1] incorrect or incomplete work shown
[0] no answer given

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11-5 **Enrichment**
Analyzing Data

Moments to Remember

You can convey statistical information in several different ways. If you obtain more than 20 or so scores, listing the scores becomes unwieldy. In such cases, you can present the information in a frequency distribution.

Suppose scores range from x_1 through x_n. The frequency of score x_i, written as f_i, is the number of times that score appears in the distribution.

1. Using this notation, express the total of the scores corresponding to x_i. $f_i x_i$

2. Using sigma notation, express the total of the data represented in the distribution. $\sum_{i=1}^{n} f_i x_i$

3. How would you express the mean of the distribution in terms of x_i and f_i? $\bar{x} = \dfrac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$

If x is a score in a data set and m is any number, the first moment of x about m is defined as the difference $x - m$. For example, if 73 is a score in a data set, the first moment of 73 about 80 is $73 - 80 = -7$.

4. Suppose a frequency distribution has scores x_1 through x_n and associated frequencies f_1 through f_n. Express the total of the first moments of x_i about m. $f_i(x_i - m)$

5. Using sigma notation, express the total of the first moments of the entire distribution about m. $\sum_{i=1}^{n} f_i(x_i - m)$

6. If y is used to represent this sum, express y as a function of m. $y = \sum_{i=1}^{n} f_i x_i - \left(\sum_{i=1}^{n} f_i \right) m$

7. What kind of function is y? **linear**

8. Let $y = T(m)$. Why is there only one number M such that $T(M) = 0$? **Any linear function with nonzero slope meets the x-axis at only one point.**

9. Solve the equation $T(M) = 0$ for M. $M = \dfrac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$

10. What term describes the value of M? **mean**

11. Complete the following sentence: The ____**mean**____ of a distribution is the unique number about which the ____**sum**____ of the first moments is ____**zero**____.

The table shows the frequency of five different scores on a challenging 15-point spelling test.

Score	15	14	13	12	11
Frequency	1	4	7	13	18

12. Find the mean for this set of scores. **12**

13. Show that the sum of the first moments about the mean is zero.
$\sum_{i=1}^{n} f_i(x_i - m) = 1(15 - 12) + 4(14 - 12) + 7(13 - 12) + 13(12 - 12) + 18(11 - 12) = 0$

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11-5 Reteaching
Analyzing Data

- The *mean* is the average of the values.
- The *median* is the middle value(s) when the values are listed in order.
- The *mode* is the most common value(s).

Problem

What are the mean, median, and mode for the data set below?

2 2 5 5 1 3 6 6 3 5 3 4 3 2 4 4 5 2 4 1 3 5 5 3 5 3 4 3 5 3 3 1 5 6

Step 1 Find the mean. The mean is the average of the values. Add all the values, and then divide the sum by the number of values.
$\frac{124}{34} \approx 3.65$

Step 2 Find the median. Write the values in numerical order. For an odd number of values, the median is the middle value. For an even number of values, the median is the mean of the middle two values.

1 1 1 2 2 2 2 3 3 3 3 3 3 3 3 3 3 4 4 4 4 5 5 5 5 5 5 5 5 5 6 6 6

16 values　　2 values　　16 values

The mean of the middle two values is $\frac{3+4}{2} = \frac{7}{2} = 3.5$.

Step 3 Find the mode(s). If no value occurs more than once, then the data set has no mode. How many times does each value occur in the data set?

1: three times　　2: four times　　3: ten times
4: five times　　5: nine times　　6: three times

The most common value is 3.

The mean is about 3.65, the median is 3.5, and the mode is 3.

Exercises

Find the mean, median, and mode of each set of values.

1. 872 888 895 870 882 878 891 890 888　**about 883.8; 888; 888**

2. 2020 2040 2068 2120 2015 2301 2254　**about 2116.9; 2068; no mode**

3. 25 27 26 33 28 26 24 30 26 28 24 27　**27; 26.5; 26**

4. 4.4 5.6 1.5 2.1 3.8 1.9 4.7 2.5 4.7 2.8　**3.4; 3.3; 4.7**

5. 194 502 413 768 986 616 259 351 825　**546; 502; no mode**

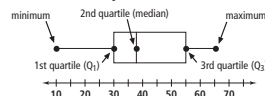6. 36 37 38 38 38 37 26 36 39 40 40 40 35　**about 36.9; 38; 38 and 40**

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11-5 Reteaching (continued)
Analyzing Data

A *box-and-whisker plot* is a visual representation of a data set. The plot shows the minimum and maximum data values, and organizes the data values into four groups separated by three *quartiles*. The second quartile is the median of the values.

Problem

What is a box-and-whisker plot for the data set?　5 6 8 12 11 9 4 3 7 10

Step 1 Write the values in numerical order. Find the minimum and maximum.
3 4 5 6 7 8 9 10 11 12
minimum: 3　　maximum: 12

Step 2 Find the second quartile. Q_2 is the median of all the data values.
3 4 5 6 7 8 9 10 11 12　　$\frac{7+8}{2} = 7.5$

Step 3 Find the first quartile. Q_1 is the median of the lower half of the values.
3 4 ⑤ 6 7 8 9 10 11 12

Step 4 Find the third quartile. Q_3 is the median of the upper half of the values.
3 4 5 6 7 8 9 ⑩ 11 12

Step 5 Draw a number line for the base of the plot. Above the number line, plot the three quartiles, the minimum value, and the maximum value.

Step 6 Draw a box through Q_1 and Q_3, a vertical line through the median, and line segments from the box outward to the minimum and maximum values.

Exercises

Make a box-and-whisker plot for each data set.

7. 43.4 46.5 47.9 51.0 50.2 49.5 42.5 41.6 46.8 50.0

8. 19 20 21 22 23 25 27 12 19 31 53 52 48 41 29 33 48 46 44 42

9. 721 242 567 541 589 234 965 845 566 487 486 515 577 875 698 564 654

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11-6 ELL Support
Standard Deviation

Rita was studying for a quiz on standard deviation. She wrote the steps to find standard deviation on a set of note cards, but the cards got mixed up.

Calculate the variance by finding the mean of these squares.
$$\sigma^2 = \frac{\sum (x - \bar{x})^2}{n}$$

Square each difference, $(x_i - \bar{x})^2$.

Find the mean, $\bar{x}$, of the n values in the data set.

Take the square root of the variance.
$$\sigma = \sqrt{\frac{\sum (x - \bar{x})^2}{n}}$$

Find the difference, $x_1 - \bar{x}$, between each value x_1 and the mean.

Use the note cards to write the steps in order.

1. First, **find the mean, $\bar{x}$, of the n values in the data set**

2. Second, **find the difference, $x_1 - \bar{x}$, between each value x_1 and the mean**

3. Next, **square each difference, $(x_i - \bar{x})^2$**

4. Then, **calculate the variance by finding the mean of these squares.** $\sigma^2 = \frac{\sum (x - \bar{x})^2}{n}$

5. Finally, **take the square root of the variance.** $\sigma = \sqrt{\frac{\sum (x - \bar{x})^2}{n}}$

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11-6 Think About a Plan
Standard Deviation

Energy The data for daily energy usage of a small town during ten days in January is shown.

83.8 MWh　　87.1 MWh　　92.5 MWh　　80.6 MWh　　82.4 MWh
77.6 MWh　　78.9 MWh　　78.2 MWh　　81.8 MWh　　80.1 MWh

a. Find the mean and the standard deviation of the data.
b. How many values in the data set fall within one standard deviation of the mean? Within two standard deviations? Within three standard deviations?

Know

1. The data values are: **83.8, 87.1, 92.5, 80.6, 82.4, 77.6, 78.9, 78.2, 81.8, 80.1**

2. The mean of a set of data is **sum of the data values / number of data values**.

3. The standard deviation of a set of n data values is $\sigma = \sqrt{\frac{\sum (x - \bar{x})^2}{n}}$

Need

4. To solve the problem I need to find:
the mean of the data values, the standard deviation of the data values, and how many of the data values fall within 1, 2, and 3 standard deviations of the mean.

Plan

5. The mean of the data is **82.3**. The standard deviation of the data is **about 4.33**.

6. Plot the data values on a number line. Mark off intervals of the standard deviation on either side of the mean.

69.31	73.64	77.97	82.3	86.63	90.96	95.29
	77.6	80.6	83.8	87.1		92.5
		78.9	82.4			
		78.2				
		81.8				
		80.1				

70　　75　　80　　85　　90　　95

7. How many values in the data set fall within one standard deviation of the mean? Within two standard deviations? Within three standard deviations? **7; 9; 10**

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11-6 Practice   Form G
Standard Deviation

Find the mean, variance, and standard deviation for each data set.

1. 232 254 264 274 287 298 312 342 398 **about 295.7; about 2246.8; about 47.4**

2. 26 27 28 28 28 29 30 30 32 35 35 36 **about 30.3; about 10.2; about 3.2**

3. 2.2 2.2 2.3 2.4 2.4 2.4 2.5 2.5 2.5 2.6 **about 2.4; about 0.02; about 0.1**

4. 75 73 77 79 79 74 81 74 70 68 70 72 **about 74.3; about 15.1; about 3.9**

Graphing Calculator Find the mean and the standard deviation.

5. price of XYZ Company stock for the first 12 weeks of 2006 **about 5.54; about 0.12**

| 5.34 | 5.40 | 5.41 | 5.42 | 5.50 | 5.55 |
| 5.55 | 5.57 | 5.70 | 5.65 | 5.66 | 5.68 |

6. price of XYZ Company stock for the first 12 weeks of 2009 **about 5.69; about 0.26**

| 6.00 | 5.95 | 5.92 | 5.80 | 5.81 | 5.75 |
| 5.75 | 5.75 | 5.64 | 5.52 | 5.40 | 5.03 |

Determine the whole number of standard deviations that includes all data values.

7. The hours students in your study group study is 66.1 min; the standard deviation is 2.9 min. **3**

62 63 65 64 64 68 68 69 72 66

8. The mean weight of your pets is 18.25 lb; the standard deviation is 30.1 lb. **3**

0.25 0.25 6 8 10 85

9. Use the data for average daily water usage of a family during the past 10 months. Find the mean and the standard deviation of the data. How many items in the data set fall within one standard deviation of the mean? Within two standard deviations? **249.4; about 162.5; 7; 10**

| 124 gal | 113 gal | 152 gal | 545 gal | 150 gal |
| 490 gal | 442 gal | 207 gal | 124 gal | 147 gal |

10. **Reasoning** In Lesson 11-5 an outlier is defined as a value "substantially different from the rest of the data in a set." How could you use the concept of standard deviation to rewrite this definition? **Answers may vary. Sample: An outlier is any data value more than 2 standard deviations from the mean.**

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11-6 Practice (continued)   Form G
Standard Deviation

Find the standard deviation for each data set. Use the standard deviations to compare each pair of data sets.

11. prices of the first 10 cars sold at Joe's Used Car Lot in 1998: **$\sigma \approx 176.18$**
$900 $1300 $1200 $850 $800 $1250 $795 $950 $1020 $975

prices of the first 10 cars sold at Joe's Used Car Lot in 2008: **$\sigma \approx 2240.70$**
$2500 $2700 $3600 $5000 $1900 $6175 $4000 $7200 $9250 $3000
The 1998 prices are clustered more closely around the mean price than are the 2008 prices.

12. times of boys in 100-m dash state high-school finals in 1998: **$\sigma \approx 0.15$**
10.43 10.48 10.49 10.51 10.61 10.63 10.66 10.92

times of boys in 100-m dash state high-school finals in 2008: **$\sigma \approx 0.21$**
10.32 10.38 10.39 10.48 10.70 10.74 10.83 10.90
The 2008 times are slightly more spread out from the mean than the 1998 times.

Use the chart at the right for Exercises 13–17.

13. Find the mean amount of money raised for each year. **about $771.88; $1115**

14. Find the standard deviation for each year. **about $697.70; about $1374.03**

15. **Writing** Use the standard deviation for each year to describe how school fundraising varied from 2006–2007 to 2007–2008. **Fundraising amounts varied more widely from the mean in 2007–08 than they did in 2006–07.**

16. For 2007–2008, the amounts raised by which clubs are not within one standard deviation of the mean? **Service Club**

Fundraising at Smithburg High School

Club	2006–2007	2007–2008
Adventure	$500	$600
Car	$250	$250
Chess	$100	$120
Drama	$1500	$1400
Ecology	$475	$300
Film	$150	$250
Service	$2200	$4500
Spirit	$1000	$1500

17. **Error Analysis** A student says that the amounts raised in 2006–2007 by the Drama Club, Service Club, and Spirit Club are not within one standard deviation of the mean. Do you agree? Explain. **No; the Spirit Club's $1000 is within one standard deviation of the mean.**

18. **a.** Make a table showing the heights of ten books in your home. **Check students' work.**
b. Find the mean and standard deviation of the data. **Check students' work.**

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11-6 Practice   Form K
Standard Deviation

Find the mean, variance, and standard deviation for each data set.

1. 6, 13, 12, 9, 10

Mean

$\bar{x} = 50 \div 5 = 10$

Variance

x	$\bar{x}$	$x - \bar{x}$	$(x - \bar{x})^2$
6	10	−4	16
13	10	3	9
12	10	2	4
9	10	−1	1
10	10	0	0

Standard Deviation

$\sigma = \sqrt{\sigma^2} = \sqrt{6} \approx 2.4$

$$\sigma^2 = \frac{\sum(x - \bar{x})^2}{n} = \frac{30}{5} = 6$$

2. 8, 16, 12, 15, 4
mean: 11
variance: 20
standard deviation: ≈ 4.5

3. 25, 18, 20, 19, 22, 16
mean: 20
variance: ≈ 8.3
standard deviation: ≈ 2.9

4. 27, 34, 45, 30, 26, 42
mean: 34
variance: ≈ 52.3
standard deviation: ≈ 7.2

Use a graphing calculator to solve the following problems.

5. The most recent test scores for a math class are displayed in the table below. What are the mean and the standard deviation for this data set?

Student	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Score	77	86	79	94	65	82	76	97	65	77	89	78	84	79	88

mean ≈ 81; standard deviation ≈ 8.8

6. Your sister's bowling scores for the last 12 games are displayed in the table below. What are the mean and standard deviation for this data?

Game	1	2	3	4	5	6	7	8	9	10	11	12
Score	212	187	176	205	193	229	201	175	203	216	227	235

mean ≈ 205; standard deviation ≈ 19

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11-6 Practice (continued)   Form K
Standard Deviation

Determine the whole number of standard deviations that include all of the following data values.

7. You brother is buying his textbooks for his first semester of college. The price of each of his books is shown in the table below. The mean of the data set is $65.85, and the standard deviation is about 36. Within how many standard deviations of the mean do all of the prices fall? .

Book	1	2	3	4	5	6
Price	$25.60	$57.00	$38.25	$126.40	$84.00	$63.85

All of the prices fall within two standard deviations of the mean.

8. The table below shows the weights of the five starting players on a basketball team. Within how many standard deviations of the mean do all of the weights fall?

Player	1	2	3	4	5
Weight (lb)	146	189	246	178	203

All of the weights fall within two standard deviations of the mean.

9. **Open-Ended** Describe an example of how it can be useful to know the standard deviation of a data set.
Answers may vary. Sample: A teacher can calculate the standard deviation of a class's test scores to see whether the whole class is achieving a similar level of academic performance. If the standard deviation is high, that indicates that there is a great deal of variation between students' academic performance.

10. **Writing** How is standard deviation similar to range and interquartile range?
All three measures give a sense of how much variation exists in a set of data.

11. **Error Analysis** Your classmate calculated the standard deviation of the data set shown below and got 46.53. What error did she make? What is the correct standard deviation?

Day	Mon	Tue	Wed	Thu	Fri	Sat	Sun
High Temperature (°F)	76°	82°	63°	69°	79°	84°	75°

She calculated the variance rather than the standard deviation. She needs to find the square root of the variance. The correct standard deviation is about 6.82.

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11-6 Standardized Test Prep
Standard Deviation

Multiple Choice

For Exercises 1–4, choose the correct letter.

1. Of the 25 students who take a standardized test, the minimum score is 98 and the maximum score is 472. The mean score is 216, and the standard deviation is 52. What is the number of standard deviations that includes all the data values? **B**

Ⓐ 3 　　Ⓑ 5 　　Ⓒ 8 　　Ⓓ 9

2. What is the standard deviation of the data set below? **F**
87 21 90 43 54 23 123 110 90 44 50

Ⓕ 33.1 　　Ⓖ 47.0 　　Ⓗ 66.8 　　Ⓘ 89.0

3. A data set has a mean of 255 and a standard deviation of 12. All the data values are within two standard deviations of the mean. Which could be the maximum value of the data? **C**

Ⓐ 232 　　Ⓑ 244 　　Ⓒ 268 　　Ⓓ 280

4. The scores on a math test are:
67 69 71 75 78 78 83 85 85 85 86 87 89 92 95 98 98 98 100.
Within how many standard deviations of the mean is a score of 100? **F**

Ⓕ 2 　　Ⓖ 3 　　Ⓗ 10 　　Ⓘ 15

Short Response

5. The ages of students in a club are:
13 17 18 15 16 14 15 18 17 16 15 16 13.
Calculate the mean and standard deviation. What is the number of standard deviations that includes all the data values? Show your work.

[2] $\bar{x} = \dfrac{13 + 17 + 18 + 15 + 16 + 14 + 15 + 18 + 17 + 16 + 15 + 16 + 13}{13} \approx 15.6$

$\sigma = \sqrt{\dfrac{\sum_{i=1}^{13}(x_i - 15.6)^2}{13}} = \sqrt{\dfrac{33.08}{13}} \approx 1.6$

All values are within two standard deviations of the mean.
[1] incorrect or incomplete work shown
[0] incorrect answers and no work shown OR no answers given

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11-6 Enrichment
Standard Deviation

Speedier Standard Deviations

Standard deviation and variance are both defined in terms of the mean of the distribution. Variance is the square of standard deviation. Using their formulas requires first computing the mean, then using the mean to compute the variance and standard deviation.

Is it possible to compute the variance and standard deviation without first computing the mean? Consider the following reasoning.

1. Write the definition of the mean of the scores x_1 through x_n, using sigma notation. $\bar{x} = \dfrac{\sum_{i=1}^{n} x_i}{n}$

2. Write the formula for the variance in terms of the mean. $\sigma^2 = \dfrac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$

3. Expand the right-hand side of the formula. $\sigma^2 = \dfrac{1}{n}\sum_{i=1}^{n}(x_i^2 - 2x_i\bar{x} + \bar{x}^2)$

Notice that the middle term has two constants, 2 and $\bar{x}$, so they may be factored outside the sigma symbol. The last term is equivalent to adding the square of the mean n times.

4. What formula results when these substitutions are made? $\sigma^2 = \dfrac{1}{n}\left(\sum_{i=1}^{n}x_i^2 - 2\bar{x}\sum_{i=1}^{n}x_i + n\bar{x}^2\right)$

5. Now replace the mean, $\bar{x}$, with its sigma notation. What formula results? $\sigma^2 = \dfrac{1}{n}\left[\sum_{i=1}^{n}x_i^2 - \dfrac{1}{n}\left(\sum_{i=1}^{n}x_i\right)^2\right]$

6. What is the corresponding formula for the standard deviation? $\sigma = \sqrt{\dfrac{1}{n}\sum_{i=1}^{n}x_i^2 - \dfrac{1}{n^2}\left(\sum_{i=1}^{n}x_i\right)^2}$

Notice that only the number of scores, the sum of the scores, and the sum of the squares of the scores are needed to find the standard deviation and variance with these formulas.

Suppose the data is presented in terms of the scores x_1 through x_n and their associated frequencies f_1 through f_n.

7. Express the sum of the scores in terms of x_i and f_i. $\sum_{i=1}^{n} f_i x_i$

8. Express the sum of the squares of the scores. $\sum_{i=1}^{n} f_i(x_i)^2$

9. During league play last Friday evening the scores of the last 12 games bowled were 186, 165, 193, 216, 174, 184, 187, 209, 198, 143, 217, and 192. Find the standard deviation of this set of scores without first finding the mean. **about 20.4**

10. The last 12 games bowled Saturday evening were very close. The score of four games was 186, the score of three games was 184, the score of three games was 187, and the score of two games was 178. Find the standard deviation of this set of scores without first finding the mean. **about 3.1**

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11-6 Reteaching
Standard Deviation

The mean tells you what the center of a set of data values looks like. But two very different data sets can have the same mean. For example, each of these data sets has a mean of 25.

Set A: {23 24 25 26 27} 　　Set B: {1 5 25 45 49}
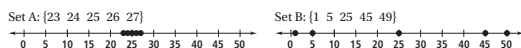
Notice that the data values on the number line for set B are much more spread out from the mean than the data values for set A. *Variance* and *standard deviation* are measures of how widely data values differ from the mean.

The lowercase Greek letter sigma, σ, is the symbol for standard deviation. Variance is the square of the standard deviation, and is written as σ^2. For a set of n data values:

$$\sigma^2 = \dfrac{\sum(x - \bar{x})^2}{n} \qquad \sigma = \sqrt{\dfrac{\sum(x - \bar{x})^2}{n}}$$

Problem

What are the variance and standard deviation for the data set {100 158 170 192}?

Step 1 Find the mean of the values.

$\bar{x} = \dfrac{100 + 158 + 170 + 192}{4} = 155.$

Step 2 Subtract the mean from each value in the data set. Then square each difference.

$(100 - 155)^2 = 3025$ 　　$(158 - 155)^2 = 9$
$(170 - 155)^2 = 225$ 　　$(192 - 155)^2 = 1369$

Step 3 Find the mean of the squared differences. This is the variance.

$\sigma^2 = \dfrac{3025 + 9 + 225 + 1369}{4} = 1157$

Step 4 Find the square root of the variance. This is the standard deviation.

$\sigma = \sqrt{1157} \approx 34$

The variance for the data set is 1157 and the standard deviation is about 34.

Exercises

Find the variance and standard deviation for each data set.

1. 6.5 7.0 9.0 8.0 7.5 **0.74; about 0.86** 　　**2.** 5.6 5.8 5.9 6.1 **0.0325; about 0.18**

3. 201 203 208 210 211 **15.44; about 3.93** 　　**4.** 12 14 15 17 19 **5.84; about 2.42**

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11-6 Reteaching (continued)
Standard Deviation

You can describe the spread of a set of data values by counting the number of standard deviations from the mean that it takes to include some or all of the values.
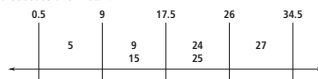
2 standard deviations $\bar{x} - 2\sigma$	1 standard deviations $\bar{x} - \sigma$	mean $\bar{x}$	1 standard deviations $\bar{x} + \sigma$	2 standard deviations $\bar{x} + 2\sigma$
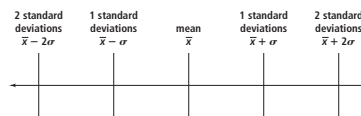
Problem

The data set {5 9 15 24 25 27} has a mean of 17.5 and a standard deviation of about 8.5. Within how many standard deviations of the mean do all the values fall?

Step 1 Draw five lines to represent the mean and two standard deviations on either side of the mean.

Step 2 Substitute the values for $\bar{x}$ and σ. Simplify each expression.

$\bar{x} - 2\sigma$	$\bar{x} - \sigma$	$\bar{x} = 17.5$	$\bar{x} + \sigma$	$\bar{x} + 2\sigma$
$= 17.5 - 2(8.5)$	$= 17.5 - 8.5$		$= 17.5 + 8.5$	$= 17.5 + 2(8.5)$
$= 0.5$	$= 9$		$= 26$	$= 34.5$

Step 3 Label each line with the appropriate value. Write each data value in the appropriate section. If a value falls on a line between two sections, write it in the section closest to the mean.

0.5	9	17.5	26	34.5
	5	9 15	24 25	27

The drawing shows that all the values fall within two standard deviations of the mean.

Exercises

A family buys groceries weekly. Over four weeks their grocery costs are $72.42, $91.50, $58.99, and $69.02.

5. Within how many standard deviations of the mean do all the costs fall? **2**

6. How many costs fall within one standard deviation of the mean? **2**

7. Within how many standard deviations of the mean would a cost of $102.00 fall? **3**

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11-7 ELL Support
Samples and Surveys

Choose the word from the list that best matches each sentence.

> bias   controlled experiment   observational study
> population   self-selected sample

1. The members of a set.   **population**

2. A study method that involves observing members of a sample without affecting them.   **observational study**

3. Systematic error caused by the sampling method.   **bias**

4. A sample that includes only volunteers.   **self-selected sample**

5. A study method that involves a control group and a treated group.   **controlled experiment**

Choose the word from the list that best completes each sentence.

> convenience sample   random sample   sample
> survey   systematic sample

6. When conducting a ___**survey**___, you ask members of a sample a set of questions.

7. All members of the population are equally likely to be chosen in a ___**random sample**___.

8. A ___**convenience sample**___ includes members of the population who are readily available.

9. A ___**sample**___ is a part of the population.

10. To create a ___**systematic sample**___, you must order the population and then select from it at regular intervals.

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11-7 Think About a Plan
Samples and Surveys

Entertainment A magazine publisher mails a survey to every tenth person on a subscriber list that is alphabetized by last name. The survey asks for three favorite leisure-time activities. What sampling method is the survey using? Identify any bias in the sampling method.

Know

1. The company sending out the survey is a **magazine publisher**.

2. The surveys are mailed to **every tenth person on a subscriber list, alphabetized by last name**.

3. The survey asks for **three favorite leisure-time activities**.

Need

4. To solve the problem I need to find:
   the sampling method used by the survey and any bias in the sampling method

Plan

5. What sampling method is the survey using? **systematic sampling**

6. Do the people who receive the survey represent the general population? Explain.
   Answers may vary. Sample: No; only people who already subscribe to the publisher's magazines receive the survey.

7. Do the people who return the survey represent the general population? Explain.
   Answers may vary. Sample: No; only people who choose to return the survey are represented.

8. Is there any bias in the sampling method? Explain.
   Answers may vary. Sample: Yes; the people who receive the survey subscribe to magazines, so they are likely to list reading magazines as a favorite leisure-time activity. The sample is also self-selected, depending on who returns the survey. These people may overrepresent or underrepresent some choices of activities.

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11-7 Practice   Form G
Samples and Surveys

Identify the sampling method. Then identify any bias in each method.

1. A teacher committee wants to find how much time students spend reading each week. They ask students as they enter the library. **Convenience; the data is likely to include a disproportionate number of students who spend more time reading.**

2. The students planning the junior class party want to know what kinds of pizza to buy. They ask the pizza restaurant what kinds sell the most. **Convenience; the restaurant sells to people other than students, and students may prefer something different than the population as a whole.**

3. The county road department wants to know which roads cause the most concern among the residents of the county. They ask the local restaurants to hand out survey forms for customers to return by mail. **Self-selected; the restaurant patrons are probably not representative of all the different kinds of people who live in the county.**

Identify any bias in each survey question.

4. Do you believe that kids should go to school year-round because they are responsible for the rise in petty crime during the summer months? **Two issues in the same question and "crime" makes the question loaded**

5. Isn't our local government not aware of our current traffic problems? **Double negative**

6. Shouldn't our school do its part to end global warming by starting a recycling program? **Leading question**

   Answers may vary. Sample: Call randomly selected people from the city telephone book.

7. a. What sampling method could you use to find the percent of people in your community who support tougher penalties for running red lights?
   b. What is an example of a survey question that is likely to yield unbiased information? **Answers may vary. Sample: Do you think the city should add bicycle lanes to our city's streets?**

A committee surveys public response to a plan to add bicycle lanes to downtown city streets. Describe a sampling method that can be used for each population.

8. bicyclists **Answers may vary. Sample: Interview random people at several city bicycle shops.**

9. car drivers **Answers may vary. Sample: Call random people from a list of all driver's license holders in the city.**

10. downtown business owners **Answers may vary. Sample: Send surveys to all downtown businesses.**

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11-7 Practice (continued)   Form G
Samples and Surveys

11. a. Write a survey question to find out the number of students in your class who plan to travel out of state after graduation. **Check student's work.**
    b. Describe the sampling method you would use. **Check student's work.**
    c. Conduct your survey. **Check student's work.**

12. A television show's website asks every 20th person who visits the site to name their favorite TV star.
    a. What sampling method is the survey using? **convenience**
    b. Describe any bias in the sampling method. **People who visit the show's website are more likely than the general television viewer to pick the show's star as their favorite.**

When you take a random sample of size n from a large population, the sample has a *margin of error* of approximately $\pm\frac{1}{\sqrt{n}}$. Approximate the margin of error for each sample.

13. In a traffic survey, 42% of the 1287 drivers passing through the checkpoint were traveling more than 100 mi from home. **±3%**

14. In one lake, 30% of the last 323 fish caught have a certain chemical present in their body. **±6%**

You can use the margin of error ME to find an interval that is likely to contain the result you would get if you asked the entire population. If the percentage found from a survey is p, the percentage from the total population is likely to be between $p - ME$ and $p + ME$. For each margin of error, find a small interval that is likely to contain the result from the total population given that the result from the survey is $p = 63\%$.

15. $ME = \pm2\%$ **61% – 65%**

16. $ME = \pm7$ **56% – 70%**

17. $ME = \pm1.4\%$ **61.6% – 64.6%**

18. $ME = \pm3.7$ **59.3% – 66.7%**

19. **Reasoning** A certain survey has a margin of error of $\pm3\%$. About how many people participated in the survey? **≈ 1111 people**

20. **Writing** Describe the relationship between a change in the sample size and the change in the margin of error. **Answers may vary. Sample: They vary inversely. As one increases, the other decreases. If sample size increases by a factor of x, margin of error decreases by a factor of $x^{-0.5}$.**

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11-7 Practice   Form K
Samples and Surveys

Identify the sampling methods used in each of the following situations. Then state whether the sampling method has any bias.

1. A television station invites viewers to call in and name their favorite game show.
This is a self-selected sample. All of the sample members will be viewers, so the entire population will not be accurately represented.

2. A school principal gathers an alphabetical list of all the students at her school. Then she selects every 15th student to take a survey about the cafeteria's lunch menu.
This is a systematic sample. There is most likely no bias in this sample.

3. A reporter asks people leaving a movie theater to take a survey about their television viewing habits.
This is a convenience sample. Because the members of the sample are leaving a movie theater, they may have different television viewing habits than most other members of the population.

4. A psychologist uses a computer program to randomly select names from a list of students at a university. The members of the sample will take a survey about student housing at the university.
This is a random sample. There is most likely no bias in this sample.

5. **Writing** A group of television producers plans to survey 10-year-olds to determine their opinions about a new cartoon. Describe a sampling method that could be used to gather a biased sample in this situation. Then describe a method to gather an unbiased sample. **Answers may vary. Sample: In order to gather a biased sample, the producers could stand outside of a toy store and give the survey to the children entering and exiting. To gather an unbiased sample, the producers could randomly select children from elementary schools across the country.**

6. **Multiple Choice** A school psychologist sits in a school cafeteria and takes notes on students' behavior while they eat lunch. Which of the following types of studies is the researcher conducting? **B**

   Ⓐ controlled experiment   Ⓑ observational study   Ⓒ survey

7. **Open-Ended** Your classmate is randomly selecting a sample of students at his high school to take a survey. You say that your classmate's sample is biased because it only contains high-school students. In what case might you be wrong?
Answers may vary. Sample: Your classmate's sample is not biased if the survey deals only with issues pertaining to the high school.

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11-7 Practice (continued)   Form K
Samples and Surveys

Identify and describe the bias in the following survey questions.

8. Isn't summer a much more pleasant season than winter?
This is a leading question. The question suggests that summer is more pleasant than winter.

9. Are college students better off studying useful subjects such as math or impractical subjects such as art history?
This is a loaded question. Positive terms are used to describe one option and negative terms are used to describe the other option.

10. Do you believe that this year's class field trip was fun and educational?
This question combines two issues. Whether or not the field trip was fun and whether or not it was educational are two separate issues.

11. Do you agree that Mrs. Regis's class is more interesting than Mr. Wright's class?
This is a leading question. The question suggests that Mrs. Regis's class is more interesting than Mr. Wright's class.

Rewrite the following survey questions so that they are no longer biased.

12. Do you prefer the excitement of rock and roll or the tediousness of classical music?
Answers may vary. Sample: Do you prefer rock and roll or classical music?

13. Would you agree that dogs make better pets than cats?
Answers may vary. Sample: Which pet would you prefer, a dog or a cat?

14. Do you believe that Mayor Johnson is friendly and effective?
Answers may vary. Sample: Do you believe that Mayor Johnson is effective?

15. **Writing** A supervisor wants to determine what percent of people in his office building believe it is important to have an Internet connection at home. What sampling method can he use to gather an unbiased sample? What is an example of a survey question that is likely to yield unbiased information?
He could ask every eighth person leaving the building at the end of the day whether or not he or she believes it is important to have an Internet connection at home; "Do you believe that it is important to have an Internet connection at home?"

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11-7 Standardized Test Prep
Samples and Surveys

Multiple Choice

For Exercises 1–4, choose the correct letter.

1. The School Dance Committee conducts a survey to find what type of music students would like to hear at the next dance. Which is an example of a random sample? **B**

   Ⓐ Call 20% of the people in the senior class directory.
   Ⓑ Interview every 10th student as they enter the school.
   Ⓒ Ask every 5th person leaving a school orchestra concert.
   Ⓓ Set up a jazz website where students can list their 3 favorite songs.

2. Which is a characteristic of a biased survey question? **H**

   Ⓕ It is about a controversial issue.   Ⓗ It produces inaccurate results.
   Ⓖ It is about a well-known person.   Ⓘ It is about a very unpopular person.

3. In a survey, 36% of 1600 students said they spent at least 5 h online during the past week. What is the approximate margin of error for this sample? **B**

   Ⓐ ± 0.6%   Ⓑ ±2.5%   Ⓒ ± 6%   Ⓓ ± 25%

4. A newspaper surveys a sample of 2500 people and finds that 64% agree with a certain political position. What interval is most likely to contain the percentage of the total population who agree with the position? **F**

   Ⓕ 62−66%   Ⓖ 62−64%   Ⓗ 63−65%   Ⓘ 64−66%

Short Response

5. A city council surveys a sample of citizens about a new law. The survey finds that 38% of citizens think the law should be repealed. The survey has a margin of error of about 8%. About how many people did the council survey? Show your work.

   [2] $ME = \frac{1}{\sqrt{n}}$
   $\sqrt{n} = \frac{1}{ME}$
   $n = \left(\frac{1}{ME}\right)^2 = \left(\frac{1}{0.08}\right)^2 \approx 156$
   The council surveyed about 156 people.
   [1] incorrect or incomplete work shown
   [0] incorrect answer and no work shown OR no answer given

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11-7 Enrichment
Samples and Surveys

By choosing a random sample and avoiding bias in survey questions, you can get results that accurately reflect a larger population. However, it is important that the survey be reliable. Reliability is the extent to which a survey will produce the same results on repeated trials. There are three key types of reliability:

Test-retest reliability	the extent to which two items measure the same concept at the same level of difficulty
Internal consistency	how well items measure the same characteristics
Interrater reliability	the extent to which two people conducting a survey get the same results

State which type of reliability is illustrated in each situation.

1. A researcher wants to determine how prepared high school students are for a mathematics class. Several questions in the survey measure the same mathematical concept. **internal consistency reliability**

2. A group of students were given an IQ test. Each student was given the test twice two weeks apart. **test-retest reliability**

3. A certain level of communication skills is needed for a telemarketing position. When hiring, an employer gives a communication skills test to each applicant. The interviewer rates the candidate on a scale of 1 to 10. The test is given during the first round of interviews and then again by a second interviewer to all candidates invited back for a second interview. **interrater reliability**

4. A researcher is designing a survey to find out how satisfied readers are with a particular newspaper. Certain questions are analyzed to make sure that they indicate that the person is satisfied with the newspaper. **internal consistency reliability**

5. Two researchers are observing an English classroom. The class is discussing a movie the class recently watched as a group. The researchers separately rate each student's level of discussion on a scale of 1 to 5. **interrater reliability**

6. Before each Olympic wrestling match, each wrestler is weighed twice during the sign-in process. **test-retest reliability**

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11-7 Reteaching
Samples and Surveys

When doing a survey, it usually is not practical to get the opinion of every member of a population. You can get a fairly accurate picture of the opinion of a population by surveying a *sample* of the population. A sample is a smaller group that represents the whole population. There are several ways to choose a sample:

Convenience	choosing any people easily available
Self-selection	having people volunteer to participate in the survey
Systematic	ordering the population and choosing participants at regular intervals (such as choosing every fifth person from the telephone book)
Random	all members of the population have an equal chance of being asked to participate

The way you choose the sample can introduce *bias*, or systematic error, into the survey. When a survey is biased, the results are inaccurate.

Problem

An athletic shoe company wants to learn which brand of athletic shoes is worn most often by local high-school students. The company sets up a booth in a local mall and offers a coupon for a free pair of their athletic shoes to anyone who answers the question, "What is your favorite brand of athletic shoes?"

a. What is the sampling method used? There may be more than one.
b. Is there any bias in the company's sampling method?

a. People in the mall are readily available to the booth. Also, people must volunteer to participate. The sample is a convenience sample and is self-selected.
b. The survey is biased in several ways:
- People who do not shop at the mall are excluded.
- Only people who choose to walk up to the booth participate in the survey.
- People who are not high-school students may participate in the survey.
- People may be more likely to say this company makes their favorite shoes when they are offered a free pair.

Exercises

A politician wants to know what issues are most important to the voters in his district. Identify the sampling method and any bias in the method.

1. The politician spends 9:00 A.M. to 4:00 P.M. on Tuesday talking to people as they enter a grocery store. **convenience; excludes people who don't shop during that day**

2. The politician sets up a questionnaire on his website. **self-selected; excludes people without Internet access**

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11-7 Reteaching (continued)
Samples and Surveys

You can introduce bias into a survey by using poorly written questions. Survey questions should NOT be:

Confusing	by asking about more than one issue or by using double negatives
Ambiguous	by offering answer choices that overlap
Loaded	by using words that might provoke strong reactions in all or some people
Leading	by suggesting that one particular answer is correct

Usually simple questions make the best survey questions.

Problem

An athletic shoe company wants to learn which shoe features are important to local high-school students. Is there any bias in any of these survey questions? Explain.

a. Choose the feature that is most important to you in a shoe.
   (A) fit   (B) style   (C) color   (D) appearance
b. Isn't fit the most important feature in any shoe?
c. How important is it to you that shoe materials have not been tested on laboratory animals?

a. Style and color both contribute to the appearance of a shoe. The question is ambiguous by offering overlapping answer choices and leaves out other choices.
b. This question implies that fit is the most important feature. It is a leading question.
c. The question introduces a second issue, laboratory-animal testing. This is a loaded question.

Exercises

A politician wants to know what issues are most important to the voters in his district. Identify the type of bias in each survey question.

3. How do you feel about the toxic pollution being released into the air by the local manufacturing plants? **Loaded question; using the words *toxic pollution* rather than *emissions* can cause strong reactions.**

4. Isn't a school bond not the right way to raise money for local education? **Confusing question and leading question; it has a double negative and suggests an answer.**

5. Which city service is most important to you?
   (A) road maintenance   (B) fire   (C) public works   (D) police
Ambiguous question; choices of road maintenance and public works overlap.

6. Do you think your local library should offer videos and Internet access? **Confusing question; it asks about more than one issue.**

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11-8 ELL Support
Binomial Distributions

In the town of Rainesville, 15% of the 10,000 houses are made of brick. If 7 houses are randomly selected, what is the probability that 4 of them will be made of brick?

There are two sets of cards below that show how to solve the above problem. The set on the left explains the thinking. The set on the right shows the steps. Write the thinking and the steps in the correct order.

Think Cards

- Simplify the exponents.
- Evaluate $_7C_4$.
- Multiply.
- Write the Binomial Probability formula.
- Substitute values from the problem into the formula.

Write Cards

- $P(4) \approx 35(0.0005)(0.6141)$
- $P(x) = {_n}C_x\, p^x q^{n-x}$
- $P(4) \approx 0.01$
- $P(4) = {_7}C_4(0.15)^4(0.85)^3$
- $P(4) = 35(0.15)^4(0.85)^3$

Think	Write
First, write the Binomial Probability formula.	Step 1 $P(x) = {_n}C_x\, p^x q^{n-x}$
Second, substitute values from the problem into the formula.	Step 2 $P(4) = {_7}C_4(0.15)^4(0.85)^3$
Next, evaluate $_7C_4$.	Step 3 $P(4) = 35(0.15)^4(0.85)^3$
Then, simplify the exponents.	Step 4 $P(4) \approx 35(0.0005)(0.6141)$
Finally, multiply.	Step 4 $P(4) \approx 0.01$

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11-8 Think About a Plan
Binomial Distributions

Weather A scientist hopes to launch a weather balloon on one of the next three mornings. For each morning, there is a 40% chance of suitable weather. What is the probability that there will be at least one morning with suitable weather?

Understanding the Problem

1. What is the probability that a morning will have suitable weather? **0.4**

2. What is the probability that a morning will have unsuitable weather? **0.6**

3. How many chances does the scientist have to launch the balloon? **3**

4. What is the problem asking you to determine?
Answers may vary. Sample: the probability that there will be one, two, or three mornings with suitable weather

Planning the Solution

5. What binomial can help you find the binomial distribution for this problem? **$(p + q)^3$**

6. Expand your binomial. **$p^3 + 3p^2q + 3pq^2 + q^3$**

7. What should you substitute for the variables in your binomial expansion?
$p = 0.4$; $q = 0.6$

8. Which terms of your binomial expansion do you need to solve the problem?
Explain. **Answers may vary. Sample: I need the first three terms because they represent having 3, 2, or 1 successes. A success is a morning with suitable weather**

Getting an Answer

9. Use your binomial expansion to find the probability that there will be at least one morning with suitable weather. **0.784 or 78.4%**

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11-8 Practice — Form G
Binomial Distributions

Find the probability of x successes in n trials for the given probability of success p on each trial.

1. $x = 5, n = 5, p = 0.4$ **about 1%**

2. $x = 2, n = 8, p = 0.9$ **about 0.002%**

3. $x = 3, n = 10, p = 0.25$ **about 25%**

4. $x = 1, n = 3, p = 0.2$ **about 38%**

5. A light fixture contains 6 light bulbs. With normal use, each bulb has an 85% chance of lasting for 4 months. What is the probability that all 6 bulbs will last for 4 months? **about 38%**

Expand each binomial.

6. $(2a + 4b)^3$
$8a^3 + 48a^2b + 96ab^2 + 64b^3$

7. $(m + 3n)^4$
$m^4 + 12m^3n + 54m^2n^2 + 108mn^3 + 81n^4$

8. $(2c - d)^5$
$32c^5 - 80c^4d + 80c^3d^2 - 40c^2d^3 + 10cd^4 - d^5$

9. $(5s + t)^4$
$625s^4 + 500s^3t + 150s^2t^2 + 20st^3 + t^4$

Find the indicated term of each binomial expansion.

10. third term of $(2a - b)^8$ **$1792a^6b^2$**

11. fifth term of $(r + 3s)^5$ **$405rs^4$**

12. fourth term of $(-2x + 3y)^6$ **$-4320x^3y^3$**

13. first term of $(8g + 6h)^3$ **$512g^3$**

Use the binomial expansion of $(p + q)^n$ to calculate each binomial distribution.

14. $n = 5, p = 0.6$
$P(5) = 0.08, P(4) = 0.25, P(3) = 0.35, P(2) = 0.23, P(1) = 0.08, P(0) = 0.01$

15. $n = 3, p = 0.7$
$P(3) = 0.34, P(2) = 0.44, P(1) = 0.19, P(0) = 0.03$

16. The probability that the weather will be acceptable for a launch of a satellite over the next 3 days is 70% each day. What is the probability that the weather will be acceptable at least 1 of the next 3 days? **97.3%**

17. A poll shows that 60% of a school district's home owners favor an increase in property tax to fund a new high school. What is the probability that exactly 4 of 5 people chosen at random favor a tax increase? **about 26%**

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11-8 Practice (continued) — Form G
Binomial Distributions

There is a 60% probability of rain each of the next 5 days. Find each probability. Round to the nearest whole percent.

18. It will rain on at least 3 of the next 5 days. **68%**

19. It will rain on at least 1 of the next 5 days. **99%**

20. It will rain on at least 1 of the next 4 days. **97%**

21. It will rain on at least 1 of the next 2 days. **84%**

[graph: bars at 0.45, 0.35, 0.25, 0.15, 0.05 labeled P(4) P(3) P(2) P(1) P(0)]

22. Open-Ended Describe a situation with a 20% probability of success in each of 4 trials. Graph the binomial distribution. **Check students' work. See sample graph above.**

23. The probability that an egg from one farm is small is 10%. What is the probability that exactly 1 egg in a sample of 4 eggs is too small? **about 29%**

In one neighborhood the probability of a power outage during a rainstorm is 4%. Find each probability.

24. P(at least 1 power outage in the next 5 rainstorms) **about 18%**

25. P(at least 2 power outages in the next 10 rainstorms) **about 6%**

**26.** P(at least 1 power outage in the next 20 rainstorms) **about 56%**

27. Writing Explain the relationship between the expansion of $(x + y)^{12}$ and the 12th row of Pascal's triangle. **The numbers in the 12th row of Pascal's triangle are the coefficients of the expansion of $(x + y)^{12}$.**

28. A newspaper carrier can throw the paper and have it land on a customer's porch 85% of the time. Use the Binomial Theorem to calculate each probability for the deliverer's first 3 throws of the morning.
 a. The carrier does not land any papers on a porch. **about 0.34%**
 b. The carrier lands only 1 paper on a porch. **about 5.7%**
 c. The carrier lands exactly 2 papers on a porch. **about 32.5%**
 d. The carrier lands all 3 papers on a porch. **about 61.4%**

29. Reasoning The probability that a baby born in Scotland has red hair is 13%. A certain Scottish hospital has an average of 20 babies born per week. At the beginning of the week, the hospital has 3 "It's a Redhead!" stickers available to put on the babies' cribs. Does this seem to be an adequate amount? Justify your answer. **Yes; the probability that the hospital will have 3 or fewer redheads out of the 20 babies born is about 74%.**
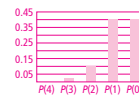
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11-8 Practice — Form K
Binomial Distributions

Find the probability of x successes in n trials for the given probability of success p on each trial.

1. $x = 5, n = 8, p = 0.6$
$P(x) = {}_nC_x\, p^x q^{n-x}$
$P(5) = {}_8C_5(0.6)^5(0.4)^3$
$P(5) = 56(0.6)^5(0.4)^3$
$P(5) \approx$ **0.28**

2. $x = 3, n = 9, p = 0.5$ **≈ 0.16**

3. $x = 6, n = 12, p = 0.3$ **≈ 0.08**

4. $x = 2, n = 7, p = 0.25$ **≈ 0.31**

5. $x = 4, n = 10, p = 0.45$ **≈ 0.24**

6. $x = 5, n = 14, p = 0.2$ **≈ 0.09**

7. At a pet shop, 30% of the cats have short hair. The owner of the pet shop will randomly choose 6 cats to take to an animal show. What is the probability that 3 of the cats will have short hair? **≈ 0.19**

8. Your brother baked a large batch of cookies. He put chocolate chips in 45% of the cookies. He randomly selects 10 cookies to give to a friend. What is the probability that 6 of the cookies contain chocolate chips? **≈ 0.16**

9. Reasoning Does rolling a number cube 10 times fit all of the conditions for a binomial experiment? Explain why or why not. **Answers may vary. Sample: No, it does not fit all of the conditions for a binomial experiment. Each trial has 6 possible outcomes, not two.**

10. Multiple Choice Which of the following is not one of the conditions for a binomial experiment? **C**
 Ⓐ There is a fixed number of trials.
 Ⓑ Each trial has two possible outcomes.
 Ⓒ The trials are dependent.
 Ⓓ The probability of each outcome is constant throughout the trials.

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11-8 Practice (continued) — Form K
Binomial Distributions

Expand each binomial.

11. $(2x + y)^4$
$= {}_4C_0(2x)^4 + {}_4C_1(2x)^3y + {}_4C_2(2x)^2y^2$
$\quad + {}_4C_3 2xy^3 + {}_4C_4 y^4$
$= 16x^4 + 32x^3y + 24x^2y^2 + 8xy^3 + y^4$

12. $(x + y)^3$
$x^3 + 3x^2y + 3xy^2 + y^3$

13. $(c + 3d)^5$
$c^5 + 15c^4d + 90c^3d^2 + 270c^2d^3 + 405cd^4 + 243d^5$

14. $(m + 2n)^4$
$m^4 + 8m^3n + 24m^2n^2 + 32mn^3 + 16n^4$

Find the indicated term of each binomial expansion.

15. fourth term of $(2x + 2y)^5$
$= {}_5C_3(2x)^2(2y)^3$
$= 10(4x^2)(8y^3)$
$320x^2y^3$

16. second term of $(m + n)^6$
$6m^5n$

17. third term of $(3x + y)^5$
$270x^3y^2$

Use the binomial expansion of $(p + q)^n$ to solve each of the following problems.

18. Your friend is shopping at a used CD store. Of all the used CDs in the store, 10% of them are badly scratched. She buys 5 used CDs.
 a. What is the probability that at least 2 of your friend's CDs are badly scratched? **≈ 0.08**
 b. What is the probability that at least 4 of the CDs are badly scratched? **≈ 0.0005**
 c. What is the probability that none of the CDs are badly scratched? **≈ 0.59**

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11-8 Standardized Test Prep
Binomial Distributions

Multiple Choice
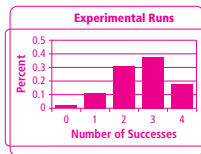
For Exercises 1–5, choose the correct letter.

1. The probability that a newborn baby at a certain hospital is male is 50%. What is the probability that exactly 2 of 3 babies born in the hospital on any day are male? **A**
   - (A) 37.5% (B) 50% (C) 66.7% (D) 75%

2. The probability that a newborn baby at the hospital is female is 50%. What is the probability that at least 2 babies of 3 children born on a certain day are female? **H**
   - (F) 33.3% (G) 37.5% (H) 50% (I) 66.7%

3. What is the fifth term of the expansion of $(2x - y)^8$? **D**
   - (A) $-1792x^5y^3$ (B) $-448x^3y^5$ (C) $256x^4y^4$ (D) $1120x^4y^4$

4. A poll shows that 30% of voters favor an earlier curfew. Find the probability that all of five voters chosen at random favor an earlier curfew. **F**
   - (F) 0.24% (G) 1.5% (H) 4.1% (I) 16.7%

5. The probability that a machine part is defective is 10%. Find the probability that no more than 2 out of 12 parts tested are defective. **C**
   - (A) 28% (B) 66% (C) 89% (D) 98%

Short Response

6. A scientist runs an experiment 4 times. Each run has a 65% chance of success. Calculate and graph the distribution of binomial probabilities for the experiment.

   [2] $(p + q)^4 = p^4 + 4p^3q + 6p^2q^2 + 4pq^3 + q^4$
   $= (0.65)^4 + 4(.65)^3(.35) + 6(.65)^2(.35)^2$
   $+ 4(.65)(.35)^3 + (.35)^4$
   $\approx 0.18 + 0.388 + 0.318 + 0.118 + 0.02$
   [2] correct graph with all work shown
   [1] incorrect or incomplete work shown
   [0] incorrect answer and no work shown
   OR no answers given

   Experimental Runs

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11-8 Enrichment
Binomial Distributions

You can use mean and standard deviation to describe a set of data. However, mean and standard deviation each depend on the number of trials in the experiment, which makes comparisons between experiments difficult. Comparisons are possible, however, if you standardize the results and describe them by a *z-score*. A *z*-score converts the data to a distribution with a mean of 0 and a standard deviation of 1. Once standardized, you can compare the distributions.

Different binomial distributions can be compared with *z*-scores. The *z*-score is found using the formula $z = \frac{X - \overline{X}}{\sigma}$ where X is the data point in the distribution, $\overline{X}$ is the mean, and σ is the standard deviation.

1. Suppose you conduct n independent trials of an event. If the probability of a success on any one trial is p, what is the probability of a failure on any one trial? **$1 - p$**

2. If there are r successes, how many failures will there be? **$n - r$**

3. What is the probability that in a sequence of n trials, r are successes? **$p^r(1 - p)^{n-r}$**

4. Use combinations to express the number of sequences of n trials in which r are successes. **$_nC_r$**

5. What is the probability of exactly r successes in n trials? **$_nC_r p^r(1 - p)^{n-r}$**

You might recognize this formula as the binomial distribution with a mean of np and a standard deviation of $\sqrt{np(1 - p)}$.

6. A single number cube is rolled 180 times. What is the probability of rolling a four 28 times? Find the *z*-score associated with rolling 28 fours. **about 0.0754; –0.4**

7. A coin is flipped 100 times. What is the probability of getting 55 heads? Find the *z*-score for 55 heads. **about 0.0485; 1**

8. The probability of catching a fish on a single cast of a fly rod at your favorite stream is calculated to be 2%. If you make 210 casts, what is the probability of catching 8 fish? Find the number of fish caught that would be associated with a *z*-score of 0.4. **about 0.0354; about 3 fish**

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11-8 Reteaching
Binomial Distributions

Suppose you repeat an experiment n times, and each time you run the experiment it has a probability of success p and a probability of failure q. Then, the probability of x successes in n trials is:

$$_nC_x\, p^x q^{n-x}, \text{ where } q = 1 - p$$

Problem

What is the probability of two successes in five trials, where the probability of success for each trial is 0.2?

$_nC_x = {_5C_2}$ Find $_nC_x$.

$= \dfrac{5!}{2!(5 - 2)!}$

$= 10$

$q = 1 - p$ Find q.

$= 1 - .2$

$= 0.8$

$P(2 \text{ successes}) = {_5C_2}\,(0.2)^2\,(0.8)^{5-2}$ Substitute for n, x, p, and q in the formula.

$= 10(0.2)^2(0.8)^3$ Simplify.

$= 10(0.04)(0.512)$

$= 0.2048$

The probability of two successes in five trials is about 20%.

Exercises

Find the probability of x successes in n trials for the given probability of success p on each trial. Round to the nearest tenth of a percent.

1. $x = 3$, $n = 4$, $p = 0.3$ **7.6%**
2. $x = 4$, $n = 6$, $p = 0.1$ **0.1%**
3. $x = 7$, $n = 9$, $p = 0.4$ **2.1%**
4. $x = 5$, $n = 6$, $p = 0.3$ **1.0%**

5. A light fixture contains six light bulbs. With normal use, each bulb has a 95% chance of lasting for 2 yr. What is the probability that all six bulbs last for 2 yr? **about 73.5%**
6. Use the information from Exercise 5. What is the probability that five of the six bulbs will last for 2 yr? **about 23.2%**
7. Suppose the bulbs have an 80% chance of lasting for 2 yr. Find the probability that three of the six bulbs will last for 2 yr. **0.8%**

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11-8 Reteaching (continued)
Binomial Distributions

Suppose you repeat an experiment n times, and each time you run the experiment it has a probability of success p and a probability of failure q. You can expand the binomial $(p + q)^n$ to find the distribution of binomial probabilities.

- The first term of the expansion is the probability of n successes out of n trials.
- The second term of the expansion is the probability of $n - 1$ successes out of n trials.
- The third term of the expansion is the probability of $n - 2$ successes out of n trials.
- The pattern continues to the last term, the probability of zero successes out of n trials.

Problem

Find the distribution of binomial probabilities for four trials with a probability of success of 0.3 for each trial. What is the probability of at least three successes?

Step 1 Use the Binomial Theorem to expand $(p + q)^n$.
$_4C_4(0.3)^4(0.7)^{4-4} + {_4C_3}(0.3)^3(0.7)^{4-3} + {_4C_2}(0.3)^2(0.7)^{4-2}$
$+ {_4C_1}(0.3)^1(0.7)^{4-1} + {_4C_0}(0.3)^0(0.7)^{4-0}$

Step 2 Simplify.
$= (0.3)^4 + 4(0.3)^3(0.7)^1 + 6(0.3)^2(0.7)^2 + 4(0.3)^1(0.7)^3 + (0.7)^4$
$= 0.0081 + 0.0756 + 0.2646 + 0.4116 + 0.2401$

Step 3 Determine the distribution of binomial probabilities.
$P(4 \text{ successes}) = 0.0081$ $P(3 \text{ successes}) = 0.0756$ $P(2 \text{ successes}) = 0.2646$
$P(1 \text{ success}) = 0.4116$ $P(0 \text{ successes}) = 0.2401$

Step 4 Find the probability of at least 3 successes.
$P(\text{at least 3 successes}) = P(4 \text{ successes}) + P(3 \text{ successes})$
$= 0.0081 + 0.0756 = 0.0837$

The probability of at least three successes in four trials is 8.37%.

Exercises

8. In a population of laboratory mice, the probability that a mouse has black spots is 0.85. What is the probability of randomly choosing 7 mice and getting at least 5 mice with black spots? **about 92.6%**

9. A construction site has a 97% rate of accident-free workdays. What is the probability of no more than 1 accident in the next 5 days? **about 99.2%**

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11-9 ELL Support
Normal Distributions

Choose the word from the list that best completes each sentence.

| continuous probability distribution | discrete probability distribution |
| normal distribution | normal curve | scatter plot |

1. The data vary randomly from the mean in a ___**normal distribution**___ .

2. The graph of a discrete probability distribution is a ___**scatter plot**___ .

3. In a ___**continuous probability distribution**___ , the events can be any value in an interval of real numbers.

4. The graph of a normal distribution is a ___**normal curve**___ .

5. There are a finite number of possible values in a ___**discrete probability distribution**___ .

Identify each of the following graphs as *positively skewed*, *normally distributed*, or *negatively skewed*.

6. ___**normally distributed**___   7. ___**positively skewed**___   8. ___**negatively skewed**___

Multiple Choice

9. In a normal distribution, about what percent of the data are within one standard deviation of the mean? **C**

 Ⓐ 16%   Ⓑ 34%   Ⓒ 68%   Ⓓ 95%

10. A normal curve is shaped like a symmetric bell centered around the _____. **G**

 Ⓕ mode   Ⓖ mean   Ⓗ median   Ⓘ range

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11-9 Think About a Plan
Normal Distributions

Agriculture To win a prize, a tomato must be greater than 4 in. in diameter. The diameters of a crop of tomatoes grown in a special soil are normally distributed, with a mean of 3.2 in. and a standard deviation of 0.4 in. What is the probability that a tomato grown in the special soil will be a winner?

Know

1. A tomato must have a diameter greater than **4 in.** to win a prize.

2. The mean diameter of the crop of tomatoes is **3.2 in.** .

3. The standard deviation of the diameters of the crop of tomatoes is **0.4 in.** .

Need

4. To solve the problem I need to find:
   the probability that the diameter of a tomato grown in the special soil is greater than 4 in. .

Plan

5. Draw a normal curve. Label the mean and intervals that are multiples of the standard deviation from the mean.

   2.35% 13.5% 34% 34% 13.5% 2.35%
   0 2.0 2.4 2.8 3.2 3.6 4.0 4.4

6. What is the percent of the crop with diameters that are greater than the mean?
   50%

7. What is the percent of the crop with diameters that are greater than the mean and less than 4 in.? How do you know? **47.5%; I summed the percent of the crop that are between 3.2 in. and 3.6 in. and between 3.6 in. and 4.0 in. in diameter** .

8. How can you find the percent of the crop with diameters greater than 4 in.?
   I can subtract the percent of the crop with diameters between 3.2 in. and 4.0 in. from the percent of the crop with diameters that are greater than the mean

9. What is the probability that a tomato grown in the special soil will be a winner? **2.5%**
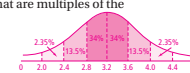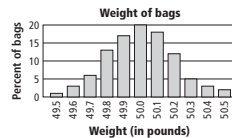
page 83

11-9 Practice   *Form G*
Normal Distributions

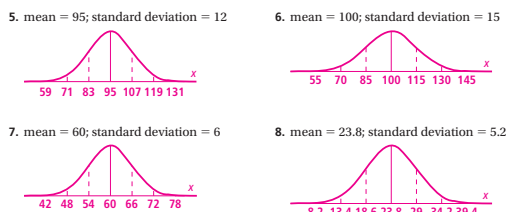**The actual weights of bags of pet food are normally distributed about the mean. Use the graph at the right for Exercises 1–4.**

Weight of bags

1. About what percent of bags of pet food weigh 49.9 lb–50.1 lb? **about 55%**

2. About what percent of bags weigh less than 49.8 lb? **about 10%**

3. In a group of 250 bags, how many would you expect to weigh more than 50.4 lb? **about 5 bags**

4. The mean of the data is 50, and the standard deviation is 0.2. Approximately what percent of bags are within one standard deviation of the mean weight? **about 80%**

Sketch a normal curve for each distribution. Label the *x*-axis values at one, two, and three standard deviations from the mean.

5. mean = 95; standard deviation = 12
   59 71 83 95 107 119 131

6. mean = 100; standard deviation = 15
   55 70 85 100 115 130 145

7. mean = 60; standard deviation = 6
   42 48 54 60 66 72 78

8. mean = 23.8; standard deviation = 5.2
   8.2 13.4 18.6 23.8 29 34.2 39.4

A set of data has a normal distribution with a mean of 5.1 and a standard deviation of 0.9. Find the percent of data within each interval.

9. from 4.2 to 5.1 **≈34%**   10. from 6.0 to 6.9 **≈13.5%**   11. greater than 6.9 **≈2.5%**

12. The number of miles on a car when a certain part fails is normally distributed, with a mean of 60,000 and a standard deviation of 5000.
   a. Sketch the normal curve for the distribution. Label the *x*-axis values at one, two, and three standard deviations from the mean.
   b. What is the probability that the part will NOT fail between 55,000 and 65,000 miles? **32%**
   45 50 55 60 65 70 75

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11-9 Practice (continued)   *Form G*
Normal Distributions

13. **Writing** The list shows the number of siblings for each person in a class:
   2, 4, 2, 0, 2, 5, 2, 2, 1, 0, 2
   Does the number of siblings appear close to being distributed normally? Explain.
   Yes; the mean value is 2 and the standard deviation is about 1.4. About 67% of the values are within 1 standard deviation of the mean, and 25% are between 1 and 2 standard deviations of the mean. This closely matches the normal distribution, where 68% is within 1 standard deviation and 27% is between 1 and 2 standard deviations.

14. **Open-Ended** On a math test the mean score is 82 with a standard deviation of 3. A passing score is 70 or greater. Choose a passing score that you would consider to be an outlier. Justify your choice. **Check students' work.**

15. A college only accepts students who score in the top 16% on the entrance exam. The exam scores are normally distributed, with a mean of 25 and a standard deviation of 3.8. To the nearest whole number, what is the least score you could earn and still be accepted to the college? **29**

A normal distribution has a mean of 50 and a standard deviation of 6. Find the probability that a value selected at random is in the given interval.

16. from 44 to 50 **34%**   17. from 38 to 56 **81.5%**   18. from 50 to 62 **47.5%**

19. at least 50 **50%**   20. at most 56 **84%**   21. at least 38 **97. 5%**

22. The table at the right shows the heights of sunflowers planted at the same time in a garden.
   a. Draw a histogram to represent the data. **See below.**
   b. Does the histogram approximate a normal curve? Explain. **No; the data do not fit a normal curve.**

Height (in.)	Frequency
56	3
57	2
58	2
59	6
60	3
61	4
62	5
63	5
64	3

23. **Reasoning** In a set of data, the value 591 is 2 standard deviations from the mean and the value 462 is 1 standard deviation from the mean. Name two possible values for the mean. Justify your answers. **333 or 505; check student's work.**
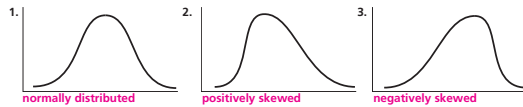
22b.
Sunflower Heights
56 57 58 59 60 61 62 63 64
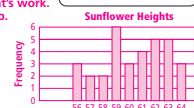
page 85

11-9 Practice *Form K*
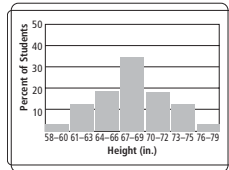Normal Distributions

Identify each of the following distributions as *positively skewed, negatively skewed,* or *normally distributed.*

1. normally distributed
2. positively skewed
3. negatively skewed

The bar graph below displays the heights of the students at a high school. Use the graph to answer the following questions.

4. Approximately what percent of students are between 61 in. and 67 in. tall?
approximately 30%

5. Approximately what percent of students are between 64 in. and 75 in. tall?
approximately 81%

6. Approximately what percent of students are between 70 in. and 79 in. tall?
approximately 32%

7. Reasoning Your mother has a rose garden. Every day, she sprays fertilizer on the roses in one section of the garden. Do you expect that the heights of the rose bushes in her garden are normally distributed? Explain why or why not.
Answers may vary. Sample: No; the fact that she sprays fertilizer on some of the roses will most likely skew the data.

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11-9 Practice (continued) *Form K*
Normal Distributions

Sketch a normal curve to represent each of the following normal distributions.

8. The average weight of a tomato in a tomato garden is 10 oz. The standard deviation is 1.6 oz. Sketch a normal curve showing the tomato weights at one, two, and three standard deviations from the mean.
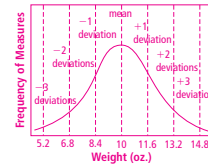
9. The average score on a math test is 76. The standard deviation is 6.2. Sketch a normal curve showing the test scores at one, two, and three standard deviations from the mean.
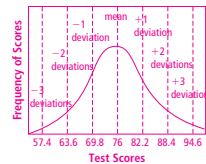
Draw a normal curve to solve the following problems.

10. A local bakery makes chocolate chip cookies. The number of chocolate chips in the cookies is approximately normally distributed, with mean 11.4 and standard deviation 1.3. What percent of the cookies have between 8.8 and 14 chocolate chips? 95%

11. The bakery described in Exercise 10 sold 200 chocolate chip cookies. How many of the cookies had less than 8.8 chocolate chips? about 5 of the cookies

12. Reasoning One of the cookies sold by the bakery had 18 chocolate chips. Would this be considered an outlier? Explain why or why not.
Answers may vary. Sample: Yes; 18 chocolate chips is more than 5 standard deviations above the mean.

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11-9 Standardized Test Prep
Normal Distributions

Multiple Choice

For Exercises 1–5, choose the correct letter.

1. The mean number of pairs of shoes sold daily by a shoe store is 36, with a standard deviation of 3. On what percent of days would you expect the store to sell from 33 to 42 pairs of shoes? D
Ⓐ 13.5% Ⓑ 50% Ⓒ 68% Ⓓ 81.5%

2. What is the standard deviation for the normal distribution shown at the right? F
496 556 616 676 736 796 856
Ⓕ 60 Ⓗ 120 Ⓖ 360 Ⓘ 676

3. A normal distribution has a mean of 700 and a standard deviation of 35. What is the probability that a value selected at random is at most 630? B
Ⓐ 0.0235 Ⓑ 0.025 Ⓒ 0.700 Ⓓ 0.975

4. Scores on an exam are distributed normally with a mean of 76 and a standard deviation of 10. Out of 230 tests, about how many students score above 96? H
Ⓕ 2 Ⓖ 3 Ⓗ 6 Ⓘ 8

5. A hardware store sells bags of mixed nails. The number of nails of a given length is distributed normally with a mean length of 5 in. and a standard deviation of 0.03 in. About how many nails in a bag of 120 are between 4.97 in. and 5.03 in. long? D
Ⓐ 34 Ⓑ 41 Ⓒ 68 Ⓓ 82
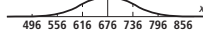
Short Response

6. The heights of the girls in a school choir are distributed normally, with a mean of 64 and a standard deviation of 1.75. If 38 girls are between 60.5 in. and 67.5 in. tall, how many girls are in the choir? Show your work.
[2] 64 − 1.75 − 1.75 = 60.5; 64 + 1.75 + 1.75 = 67.5
The range 60.5 − 67.5 is within two standard deviations of the mean.
13.5 + 34 + 34 + 13.5 = 95%; 95% of the data is within the range 60.5 − 67.5.
38 = 0.95%
x = 40; there are 40 girls in the choir.
[1] incorrect or incomplete work shown
[0] incorrect answer and no work shown OR no answer given

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11-9 Enrichment
Normal Distributions

Tchebycheff's Theorem

Tchebycheff's Theorem states that given a number k greater than or equal to 1 and a set of n measurements, at least $\left(1 - \frac{1}{k^2}\right)$ of the measurements will lie within k standard deviations of the mean. Note that the theorem is true for any number you wish to choose for k as long as it is greater than or equal to 1.

The mean and standard deviation of a sample of n = 25 measurements are 75 and 10, respectively.

1. Using Tchebycheff's Theorem and k = 2, what can you assume?
that $\frac{3}{4}$ of data lie within 55 and 95

2. What is the least number of measurements in the sample that will lie in the interval for k = 2? 19

3. Using Tchebycheff's Theorem and k = 3, what can you assume?
that $\frac{8}{9}$ of data lie within 45 and 105

4. What is the least number of measurements in the sample that will lie in the interval for k = 3? 23

5. If k = 1, do you learn anything about the data? No; it says that at least 0 measurements lie within 65–85.

Consider the following data:
23 45 12 56 34 37 85 26 77 74
15 80 65 47 37 55 26 44 73 86
85 16 37 85 74 57 43 63 37 34
72 65 37 75 77 34

6. Calculate the mean and standard deviation. Round to the nearest whole number. 52; 22

7. Using Tchebycheff's Theorem and k = 2, what is the least number of measurements in the sample that will lie within 2 standard deviations of the mean? What interval corresponds to within 2 standard deviations of the mean? 27; between 8 and 96

8. How many measurements in the sample actually lie in that interval? 36

9. Using Tchebycheff's Theorem and k = 3, what is the least number of measurements in the sample that will lie within 3 standard deviations of the mean? What interval corresponds to within 3 standard deviations of the mean? 32; between 0 and 118

10. How many measurements in the sample actually lie in that interval? 36

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11-9 Reteaching
Normal Distributions

If a data set has a *normal distribution*:
- 2.35% of the values will be between 2 and 3 standard deviations below the mean.
- 13.5% of the values will be between 1 and 2 standard deviations below the mean.
- 34% of the values will be within 1 standard deviation below the mean.
- 34% of the values will be within 1 standard deviation above the mean.
- 13.5% of the values will be between 1 and 2 standard deviations above the mean.
- 2.35% of the values will be between 2 and 3 standard deviations above the mean.

The graph of a normal distribution is a *normal curve.*
- A normal curve is shaped like a bell, with the highest point at the mean and tapering down evenly on either side of the bell.

Problem

The weight in pounds of newborn calves on a farm is distributed normally, with a mean of 85 and a standard deviation of 4. What percent of newborn calves on the farm weigh between 77 lb and 89 lb?

Step 1 Draw a normal curve. Label the mean.

Step 2 Divide the graph into 6 equal sections. Each section should be one standard deviation wide, which is 4 lb in this problem. Label each section with the appropriate percent for a normal distribution.
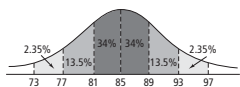
Step 3 Add the percents for the sections with weights 77 lb–81 lb, 81 lb–85 lb, and 85 lb–89 lb.
$$13.5 + 34 + 34 = 81.5$$

About 82% of newborn calves will weigh 77 lb–89 lb.

Exercises

Use the graph above to find the percent of calf weights within each interval.

1. from 73 lb to 81 lb **about 16%**

2. greater than 81 lb **about 84%**

3. from 77 lb to 97 lb **about 97%**

4. less than 85 lb **about 50%**

5. at most 89 lb **about 84%**

6. at least 93 lb **about 2.5%**

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11-9 Reteaching (continued)
Normal Distributions

You can use the percents associated with a normal distribution to make predictions.

Problem

The number of hours a certain type of battery will last is distributed normally with a mean of 500. The standard deviation is 50. Out of 250 batteries tested, how many batteries would you expect to be still working after 550 h?

Step 1 Represent the normal distribution on a number line.

Step 2 Find the percent of batteries that last 550 h or longer. Which sections of the distribution contain values of 550 or greater? $13.5 + 2.35 = 15.85 \approx 16\%$

Step 3 Find 16% of 250 batteries.
$$x = 16(0.01)(250)$$
$$x = 40$$

You can expect about 40 of the 250 batteries to be still working after 550 h.

Exercises

Sketch the normal distribution for the following data. Make a prediction based on your sketch.

7. A certain type of light bulb lasts an average of 219 h. Out of 1000 bulbs, how many would you expect to last less than 79 h if the standard deviation is 70 h?
about 25 bulbs

8. The 26 students in a math class can finish 100 problems in a mean time of 4 min. The standard deviation is 1 min. How many students in the class will still be working after 5 min?
about 4 students

9. A group of 71 frogs hops a mean distance of 66 in. with a standard deviation of 3 in. How many frogs would you expect to hop more than 72 in.? **about 2 frogs**

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Chapter 11 Quiz 1
Lessons 11-1 through 11-4
Form G

Do you know HOW?

1. In how many different orders can 6 numbered blocks be chosen from a set of 23 blocks? **72,681,840**

Find the experimental probability of each event.

2. a goal by a soccer player who made 2 of 5 shots on goal **0.40**

3. a glass broken by a dishwasher who broke 3 of 25 glasses **0.12**

Find the theoretical probability of each event when rolling a fair number cube.

4. $P(1, 3, \text{or } 4)$ **0.50**

5. $P(2 \text{ or odd})$ **0.67**

6. $P(6)$ **0.17**

7. C and D are independent events, $P(C) = \frac{2}{7}$, and $P(D) = \frac{1}{3}$. What is $P(C \text{ and } D)$? $\mathbf{\frac{2}{21}}$

8. X and Y are mutually exclusive events, $P(X) = \frac{5}{6}$, and $P(Y) = \frac{1}{9}$. What is $P(X \text{ or } Y)$? $\mathbf{\frac{17}{18}}$

9. A certain county court can assign any one of its 50 lawyers to a case. Of the lawyers, 5 are female and over 40 and 25 are male. What is the probability that the lawyer is under age 40, given that the lawyer is female? **0.80**

Do you UNDERSTAND?

10. **Writing** Explain the difference between $_4P_2$ and $_4C_2$. **Answers may vary. Sample: $_4P_2$ is the number of permutations of 4 things taken 2 at a time, and $_4C_2$ is the number of combinations of 4 things taken 2 at a time.**

11. **Reasoning** A jar holds red, green, and blue marbles. The probability of randomly selecting marbles from the jar is red = $\frac{1}{3}$, green = $\frac{1}{4}$, and blue = $\frac{5}{12}$. What is the minimum number of marbles in the jar? **12 marbles**

12. **Writing** How do $P(B \mid A)$ and $P(A \text{ and } B)$ differ? **Answers may vary. Sample: $P(B \mid A)$ is the probability of B occurring given that A occurred, and $P(A$ and $B)$ is the probability of both A and B occurring.**

13. **Open-Ended** Describe two independent events. **Check student's work.**

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Chapter 11 Quiz 2
Lessons 11-5 through 11-9
Form G

Do you know HOW?

Find the mean, median, and mode of each data set.

1. 5 7 10 10 7 9 5 6 7 7 4 **7, 7, 7**

2. 12 13 14 9 7 8 15 10 11 6 **10.5, 10.5, none**

3. What are the mean and standard deviation for the data set below? **11.25; 4.2**
   5 8 12 15 17 12 14 7

4. A school newspaper reporter is investigating students' study habits. She asks students as they are leaving the library, "How many hours each week do you spend studying?" Is this survey biased? Explain. **Yes; answers will vary. Sample: The survey doesn't include students who don't use the library. These students may study more or fewer hours than students who use the library.**

Each trial of an experiment has a probability of success p. Find the probability of x successes in n trials.

5. $p = 0.3, x = 4, n = 6$ **≈ 5.95%**

6. $p = 0.9, x = 5, n = 7$ **≈ 12.4%**

7. The amount of money spent per person at the state fair is distributed normally, with a mean of $32 and a standard deviation of $3.75. If 1200 people attend the fair, how many people would you expect to spend between $24.50 and $35.75? **978 people**

Do you UNDERSTAND?

8. **Error Analysis** Using the data set {4, 2, 9, 8, 5, 5, 3, 9, 1, 1, 9, 3}, a student says the median is 4. Explain the student's error. What is the correct median? **The student did not put the values in ascending order; 4.5**

9. **Reasoning** For a given data set, all of the values fall within 1 standard deviation of the mean. What does this tell you about the data? **Answers will vary. Sample: The data are not normally distributed.**

10. **Writing** Why would a large sample give a better estimate of the response of a total population than a small sample? **Answers will vary. Sample: Sample size and margin of error vary inversely.**

11. **Reasoning** The ratings of athletes attending a training camp are distributed normally. If 170 athletes' ratings fall within 1 standard deviation of the mean, how many athletes were rated at the camp? **250 athletes**

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Chapter 11 Chapter Test
Form G

Do you know HOW?

1. You must complete the following chores: take out the trash, wash the dishes, vacuum the carpet, clean your room, make your bed, and feed the fish. Does this situation involve a permutation or combination? In how many different ways can you do the chores? **permutation; 720**

You randomly select a number from the sample space {5, 7, 9, 11, 13, 15, 17}. Find each theoretical probability.

2. P(less than 13) $\frac{4}{7}$

3. P(odd) $\frac{7}{7} = 1$

4. P(multiple of 5) $\frac{2}{7}$

5. A basketball player made 27 free throws in her last 45 tries. What is the experimental probability that she will make her next free throw? **0.6**

6. You have a CD with 8 rock songs, 3 blues songs, and 2 jazz songs. Today you hit the shuffle button on your CD player, which plays the songs in a random order. Tomorrow you do the same thing. What is the probability that the CD player plays a blues song first each day? $\frac{9}{169}$

Use the results of the survey below to find each conditional probability.

How many pets do you have in your home?

	0 Pets	1 Pet	2 Pets	3 + Pets
Male respondents	5	10	5	2
Female respondents	10	13	6	2

7. P(male | 2 pets) $\frac{5}{11}$

8. P(2 pets | male) $\frac{5}{22}$

9. P(0 pets | female) $\frac{10}{31}$

10. P(0 or 1 pet | male) $\frac{15}{22}$

Use the following set of values for Exercises 11–16. Round answers to the nearest hundredth, if necessary.

0.3 0.6 0.9 1.3 0.4 0.6 1.2 1.4 1.1 0.2 0.2

11. Find the mean, median, and mode(s). **0.75; 0.6; 0.2, 0.6**

12. Find the range. **1.2**

13. Draw a box-and-whisker plot.

0.4 0.8 1.2

14. Find the interquartile range. **0.9**

15. Find the standard deviation. **0.43**

16. Find the 25th and 75th percentiles. **0.3; 1.1**

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Chapter 11 Chapter Test (continued)
Form G

Find the margin of error for each sample, rounded to the nearest whole percent. Then find an interval that is likely to contain the true population proportion.

17. 6% of 525 artists **±4%; 2–10%**

18. 97% of 4000 readers **± 2%; 95–99%**

19. A researcher is interested in an upcoming election. Choose the poll that is the least biased. **D**
    Ⓐ a poll taken at a political convention
    Ⓑ a poll on a candidate's website
    Ⓒ a poll taken at a high school
    Ⓓ a phone poll of registered voters

20. Find the probability of $x = 14$ successes in $n = 22$ trials with the probability of success $p = 0.6$ on each trial. **about 16%**

21. A bicycle-rental shop has ten bicycles for rent. On an average day, 60% of the bicycles are rented out. To the nearest whole percent, what is the probability that on one day, only four of ten bicycles are rented out? **11%**

22. The exam scores of 200 students are distributed normally with a mean of 72 and a standard deviation of 10. Find the number of students with scores between 72% and 82%. **about 68**

Do you UNDERSTAND?

23. **Open-Ended** Write and solve a problem using combinations. **Check students' work.**

24. **Reasoning** Which measure of central tendency best describes the values at the right? Justify your answer. **Answers may vary. Sample: mean; all but 2 values are within 1 standard deviation of the mean.**

12	1	20	1	15	2	1
10	12	2	3	12	2	

25. **Open-Ended** Give an example of a binomial experiment. **Check students' work.**

26. **Writing** What happens to the standard deviation of a data set if you remove an outlier? Explain. **Answers may vary. Sample: It will decrease. The standard deviation is the "mean" of the distances between each data value and the mean. If you remove an outlier, you remove the largest difference, decreasing the standard deviation.**

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Chapter 11 Part A Test
Form K

Lessons 11–1 through 11–4

Do you know HOW?

Use the Fundamental Counting Principle to solve the following problem.

1. Your grandfather goes to dinner at a restaurant that is having a special on a three-course meal. He may choose one appetizer, one entrée, and one dessert. There are 4 appetizer choices, 8 entrée choices, and 6 dessert choices. How many different three-course meals could he choose? **192 three-course meals**

Evaluate each expression.

2. 8! **40,320**

3. $\frac{7!}{3!}$ **840**

4. $_4P_2$ **12**

5. $_{14}P_5$ **240,240**

6. $_7C_4$ **35**

7. $6(_6C_3)$ **120**

8. Your brother is ordering 5 pizzas for the family. There are 18 different kinds of pizza. How many different ways could he order 5 pizzas? **8568 ways**

Find the experimental probability.

9. You got an A on 6 of your last 18 quizzes. What is the probability that you will get an A on your next quiz? **0.33 or 33%**

10. Your friend ran more than 10 mi on 3 of her last 15 runs. What is the probability she will run more than 10 mi on her next run? **0.2 or 20%**

Find the theoretical probability. Use combinatorics if needed.

11. A jar holds 6 blue blocks, 8 red blocks, 10 green blocks, 2 yellow blocks, and 4 black blocks.
    a. P(blue) **0.2 or 20%**
    b. P(red or green) **0.6 or 60%**

12. Your classmate rolls a pair of fair number cubes. What is the theoretical probability that he will roll doubles (two of the same number)? **0.17 or 17%**

13. Five of 20 wigs in a shop have red hair. Your aunt chooses 6 wigs. What is the probability that exactly 2 of the wigs she chooses will have red hair? **≈ 0.35 or 35%**

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Chapter 11 Part A Test (continued)
Form K

Lessons 11–1 through 11–4

Tell whether the events are *independent, dependent,* or *mutually exclusive.*

14. You roll two fair number cubes and the sum of the outcome is prime or equals 6. **mutually exclusive**

15. Randomly choose a card from a deck. Then choose another card from the deck. **dependent**

16. Roll a fair number cube. Then randomly choose a card from a deck. **independent**

Find the following probabilities.

17. A number from 1 to 25 is randomly chosen.
    a. What is the probability that the number chosen is a multiple of 2 or 4? **0.48 or 48%**
    b. What is the probability that the number chosen is a multiple of 2 and 4? **0.24 or 24%**
    c. What is the probability that the number chosen is a multiple of 2 or 5? **0.6 or 60%**

Do you UNDERSTAND?

18. **Reasoning** When using a simulation, are you finding the experimental probability or the theoretical probability of an event? Why? **Answers may vary. Sample: You are finding the experimental probability. When you simulate an event, you use the data from that simulation (experiment) to calculate probability.**

19. **Writing** Explain why the probability of two mutually exclusive events occurring at the same time is zero. **Answers may vary. Sample: Mutually exclusive events are events that cannot happen at the same time. If they cannot happen, then the probability is zero.**

20. An alumni association compiled the following information about its recent graduates.
    • 20% graduated with a B average or better
    • 95% of those students who graduated with a B average or better were employed within 6 months of graduation
    • 50% of those that graduated with less than a B average were employed within 6 months of graduation
    a. What is the probability that someone is employed within 6 months of graduation, given that he had less than a B average? **0.50 or 50%**
    b. What is the probability that someone is not employed within 6 months of graduation, given that she had a B average or better? **0.05 or 5%**

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Chapter 11 Part B Test *Form K*

Lessons 11–5 through 11–9

Do you know HOW?

Find the mean, median, and mode of the following data set.

Top Wind Speeds					
	Mon	Tue	Wed	Thu	Fri
Speed (mph)	19	23	14	16	23

1. mean **19**   **2.** median **19**   **3.** mode **23**

Find the following percentiles of the data set displayed below.

22, 22, 25, 26, 29, 29, 31, 34, 34, 34,
38, 41, 45, 45, 49, 53, 55, 57, 58, 61

4. 20th percentile **29**   **5.** 65th percentile **45**   **6.** 85th percentile **57**

Calculate the mean, variance, and standard deviation of the following data set.

Pounds of Trash Produced Each Week					
	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6
Trash (lb)	16	12	22	13	15	18

7. mean **16**   **8.** variance **11**   **9.** standard deviation **≈ 3.32**

Identify the sampling methods used in each of the following situations. Then state whether the sampling method has any bias.

10. A local newspaper places an ad in the paper asking readers to phone in and name their favorite section of the newspaper.
Answers may vary. Sample: This is a self-selected sample. The sample consists only of people who volunteer to participate, so the opinions of the population may not be completely represented.

11. A rock and roll radio station surveys listeners to determine what types of music people are most commonly listening to.
Answers may vary. Sample: This is a convenience sample. The survey results will most likely be biased, because all of the members of the sample are listening to a rock and roll radio station.

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Chapter 11 Part B Test (continued) *Form K*

Lessons 11–5 through 11–9

Find the probability of x successes in n trials for the given probability of success p on each trial.

12. $x = 4, n = 9, p = 0.3$
≈ 0.17

13. $x = 7, n = 12, p = 0.6$
≈ 0.23

14. $x = 2, n = 7, p = 0.5$
≈ 0.16

Expand each binomial.

15. $(x + y)^4$
$x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

16. $(2m + n)^3$
$8m^3 + 12m^2n + 6mn^2 + n^3$

17. $(x + 2y)^5$
$x^5 + 10x^4y + 40x^3y^2 + 80x^2y^3 + 80xy^4 + 32y^5$

Find the indicated term of each binomial expansion.

18. third term of $(x + 3y)^5$
$90x^3y^2$

19. fifth term of $(2x + 3y)^6$
$4860x^2y^4$

20. second term of $(3x + 3y)^4$
$324x^3y$

Do you UNDERSTAND?

21. Writing The following question appeared on a survey: "Do you agree that the town would be better off with a new mayor?" Classify and describe the bias in this question. Then rewrite the question so that it is no longer biased.
Answers may vary. Sample: This is a leading question. The question suggests an answer. "Do you approve of the job that the mayor is doing?"

22. In a cooler, 40% of the drinks are diet. Someone grabs 4 drinks without looking. What is the probability that at least 2 of the drinks are diet? **≈ 0.52**

23. A baseball team is reviewing their batting statistics. The mean number of hits for the batters on the team is 124, and the standard deviation is 14.5. What percent of the batters have between 95 and 124 hits? **47.5%**

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Chapter 11 Performance Tasks

Task 1
a. Cassie has seven skirts, five blouses, and ten pairs of shoes. How many possible outfits can she wear? **350 outfits**
b. Cassie decides that four of her skirts should not be worn to school. How many possible outfits can she wear to school today? **150 outfits**
c. Two of Cassie's friends come over and share her clothes. In how many different ways can the three girls wear the seven skirts? **35 ways**
d. Cassie has six bracelets. In how many different ways can she wear three bracelets at a time? **20 ways**

[4] Student uses appropriate methods to calculate combinations and permutations. The only errors are minor computational or copying errors.
[3] Student uses appropriate methods to calculate combinations and permutations, with several errors.
[2] Student uses appropriate methods to calculate at least one of the combinations or permutations correctly OR student gives correct answers, without work shown.
[1] Student attempts a solution, but shows little understanding of problem.
[0] Student makes no attempt or no response is given.

Task 2
You evaluate six overhead lights to find the intensity of light at work stations that are about 2 m from the light. Use the data below. Round to the nearest whole percent, if necessary.

Trial Number	1	2	3	4	5	6
Intensity of Light (lux)	102	99	105	97	100	98
Distance from the Light (m)	2.1	2.0	1.7	2.2	2.1	1.9

a. What is the probability that a random light is more than 2 m from a work station? Has intensity of less than 99 lux? **50%; 33%**
b. What is the probability that a random light is more than 2 m from a work station and has an intensity of more than 100 lux? **17%**
c. What is the probability that a random light is more than 100 lux or less than 99 lux? **67%**

[4] Student uses appropriate methods to find probabilities. The only errors are minor computational or copying errors.
[3] Student uses appropriate methods to find probabilities, with several errors.
[2] Student uses appropriate methods to find at least one of the probabilities correctly OR student gives correct answers, without work shown.
[1] Student attempts a solution, but shows little understanding of problem.
[0] Student makes no attempt or no response is given.

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Chapter 11 Performance Tasks (continued)

Task 3
The heights of seven students in a class are 5'2", 5'6" 5'7", 5'2", 5'9", 6'0", and 5'7".
a. Find the mean, median, and mode of the data set. **about 66.4"; 67"; 62", 67"**
b. Find the standard deviation of the data set. **about 3.33"**
c. Make a bar graph of the data set.
d. Is the data for this class distributed normally? Explain your answer.
No; there is no central grouping of data that falls off as you move away from the mean.

[4] Student uses appropriate methods to find statistics, graph the data, and identify the distribution. The only errors are minor computational or copying errors.
[3] Student applies appropriate methods, but misunderstands part of the problem or ignores a condition in the problem.
[2] Student chooses appropriate methods, but applies them incorrectly or incompletely. Student shows some understanding of the problem OR student gives correct answers, without work shown.
[1] Student attempts a solution, but shows little understanding of the problem.
[0] Student makes no attempt or no response is given.

Task 4
Determine the number of students in your high school who plan to attend college.
a. Survey a sample proportion that is least likely to be biased. Explain your choice.
b. Find the margin of error for your sample.
c. Use the percent of your sample who plan to attend college to find each probability:
P(exactly 3 of 10 students plan to attend college)
P(exactly 7 of 10 students plan to attend college)
P(at least 4 of 10 students plan to attend college)   **a.–c. Check students' work.**

**[4] Student selects an unbiased sample, gives a sufficient explanation, and uses appropriate methods to find margin of error and probabilities. The only errors are minor computational or copying errors.**
[3] Student selects an unbiased sample, gives a sufficient explanation, and uses appropriate methods to find margin of error and probabilities, with several errors.
[2] Student's sample may show some bias. Student chooses appropriate methods to find margin of error and probabilities, but applies them incorrectly or incompletely. Student shows some understanding of the problem OR student gives correct answers, without work shown.
[1] Student attempts a solution, but shows little understanding of problem.
[0] Student makes no attempt or no response is given.

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Chapter 11 Cumulative Review

Multiple Choice

For Exercises 1–12, choose the correct letter.

1. Which is equivalent to $_6C_3$? **A**

   Ⓐ $\frac{6!}{3!(6-3)!}$ Ⓑ $\frac{6!}{(6-3)!}$ Ⓒ $\frac{6!}{9!}$ Ⓓ $\frac{3!}{6!(6-3)!}$

2. Solve $(x+7)(x-5)=0$. **G**

   Ⓕ $x=7$ or $x=5$ Ⓗ $x=7$ or $x=-5$
   Ⓖ $x=-7$ or $x=5$ Ⓘ $x=-7$ or $x=-5$

3. Which graph best models exponential decay? **A**

   Ⓐ
   Ⓒ
   Ⓑ
   Ⓓ

4. Which of these is the equation of a hyberbola with foci at $(5,0)$ and $(-5,0)$? **F**

   Ⓕ $\frac{x^2}{9}-\frac{y^2}{16}=1$ Ⓖ $\frac{x^2}{5}-\frac{y^2}{5}=1$ Ⓗ $\frac{y^2}{5}-\frac{x^2}{5}=1$ Ⓘ $\frac{y^2}{9}-\frac{x^2}{16}=1$

5. Divide $2x^2-14x+24$ by $(x-4)$. **C**

   Ⓐ $x-6$ Ⓑ $-2x+6$ Ⓒ $2x-6$ Ⓓ $2x+6$

6. Which of these parabolas opens to the left? **J**

   Ⓕ $y=4x^2$ Ⓖ $x=3y^2$ Ⓗ $y=-6x^2$ Ⓘ $x=-2y^2$

7. Simplify $\frac{x-3}{x^2+3x-18}$. **B**

   Ⓐ $\frac{1}{x-6}$ Ⓑ $\frac{1}{x+6}$ Ⓒ $\frac{1}{6-x}$ Ⓓ 0

8. A and B are two independent events. $P(A)=\frac{1}{5}$ and $P(B)=\frac{3}{10}$. What is $P(A \text{ and } B)$? **H**

   Ⓕ $\frac{1}{10}$ Ⓖ $\frac{1}{2}$ Ⓗ $\frac{3}{50}$ Ⓘ $\frac{4}{15}$

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Chapter 11 Cumulative Review (continued)

9. Which of the following is the margin of error for a sample of 5000? **A**

   Ⓐ $\pm1.4\%$ Ⓑ $\pm0.14\%$ Ⓒ $\pm14\%$ Ⓓ $\pm0.014\%$

10. Which value is the smallest? **G**

    Ⓕ $\log_{12}12$ Ⓖ $\log_{15}1$ Ⓗ $\log_3 9$ Ⓘ $\log_2 64$

11. Which of the following is the sum of series $\sum_{n=1}^{4}(-2)^{n-1}$? **A**

    Ⓐ -5 Ⓑ -1 Ⓒ -3 Ⓓ 15

12. Which is the solution to $\sqrt{3x-5}+4=0$? **G**

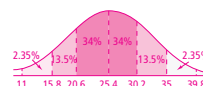   Ⓕ 0 Ⓖ No solution   Ⓗ $\frac{7}{3}$ Ⓘ 7

Short Response

13. Find the slope of a line perpendicular to $y=3x+2$. Show your work.
    The slope of $y=3x+2$ is $m=3$. The slope of a line perpendicular to it is equal to the opposite reciprocal of the slope of the original line, or $m=\frac{1}{m}=-\frac{1}{3}$.

14. Solve $5x^2-7=18$.
    $5x^2-7=18$, $5x^2-25=0$, $5(x^2-5)=0$, $x^2-5=0$, $x^2=5$, $x=\pm\sqrt{5}$

Extended Response

15. The heights of dogs at the City Animal Shelter are distributed normally, with a mean of 25.4 in. and a standard deviation of 4.8 in.
    a. Sketch a normal curve and divide the area under the curve into sections that are one, two, and three standard deviations from the mean.
    b. Of the 73 dogs at the shelter, what percent of dogs would you expect to be less than 20.6 in. tall?
    c. The shelter has one dog that is 40 in. tall. Would you consider this height to be an outlier? Explain.

    [4] Student sketches curve and labels standard deviations as shown. Student finds $50-34\%\approx16\%$ would be less than 20.6 in., and a height of 40 in. is likely to be an outlier because it is more than three standard deviations from the mean.

    2.35% 13.5% 34% 34% 13.5% 2.35%
    11 15.8 20.6 25.4 30.2 35 39.8

    [3] Student uses appropriate strategies, but misunderstands part of the problem or ignores a condition in the problem.
    [2] Student attempts to use appropriate strategies, but applies them incorrectly or incompletely.
    [1] Student work contains significant errors and little evidence of correct strategies used.
    [0] incorrect answers and no work shown OR no answers given

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Chapter 11 Project Teacher Notes: On the Move

About the Project

The Chapter Project gives students an opportunity to conduct research to identify transportation problems. They develop products or processes to solve these problems. They conduct marketing surveys to see how their inventions meet the market's needs and they make marketing decisions based on the results of the surveys. Then they develop presentations that introduce their new products.

Introducing the Project

- Ask students: *Have you ever been included in a market survey? How can market surveys be used to help develop something new to sell? How might they be used to help make a product sell better?*
- Instruct students to begin to make lists of questions they can use in their initial surveys.

Activity 1: Interviewing

Students conduct surveys to identify transportation problems.

Activity 2: Analyzing

Students make graphs and calculate summary statistics to identify problems and issues.

Activity 3: Designing

Students propose and develop details for products or services to solve the problems they identified.

Activity 4: Interviewing

Students conduct market surveys for their inventions, analyze the results, and make marketing decisions based on the results.

Finishing the Project

You may wish to plan a project day on which students share their completed projects. Encourage students to explain their processes as well as their results.

- Have students review their methods for conducting market research, making graphs, and calculating summary statistics for the project.
- Ask groups to share their insights that resulted from completing the project, such as any shortcuts they found for calculating summary statistics or making graphs.

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Chapter 11 Project: On the Move

Beginning the Chapter Project

Surveys show that many people list traffic as one of their top problems. Creative people in the transportation industry are designing faster, safer, less expensive, and environmentally cleaner ways to get around. In Curitiba, Brazil, a highly efficient bus system uses design features of a modern subway to move people faster and more smoothly.

In this project, you will identify a transportation problem. Then, you will design a new product or service to solve this problem. Finally, you will conduct a survey to decide whether your product or service is practical and marketable.

List of Materials

- Calculator
- Graph paper

Activities

Activity 1: Interviewing

Conduct a survey to identify a transportation problem in your community. **Check students' work.**

- Choose the group of people you want to survey.
- Design the survey. Before you write questions, decide what data you want to collect. You can collect data about types of transportation people use, how far or how often they use each type, and how satisfied they are.
- Test the survey on a few people to make sure the questions are clear. Revise it if necessary.
- Collect the data.

Activity 2: Analyzing

Organize the data you gathered in the survey in Activity 1. **Check students' work.**

- Make graphs of the data.
- Calculate summary statistics.
- Use your graphs and summary statistics to analyze the data.
- List problems or issues revealed by your data.

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Chapter 11 Project: On the Move (continued)

Activity 3: Designing
Suppose the members of your survey population are potential customers for your business. **Check students' work.**
- What problem seems the most important to the people you surveyed?
- Propose a product or service that could solve this problem. Be sure your idea is practical. Make a drawing, scale model, or written description of your new transportation product or service. Include a price or charge that you think is appropriate.

Activity 4: Interviewing
Conduct a market survey for the transportation product or service you proposed. **Check students' work.**
- In a series of interviews, identify your potential customers, what they want the product to do, and what changes they would like for the product.
- Graph the data. Analyze your results. Should your business market this new product or service? If so, what changes should you make first, if any? Be sure you can defend your marketing decisions on the basis of the data you collected.

Finishing the Project
The answers to the four activities should help you complete your project. Prepare a presentation that unveils the new product or service you invented and describes the results of your surveys. Present it to your classmates. Then discuss with them the marketing decision you made on the basis of your survey. Do they agree with your decision?

Reflect and Revise
Before giving a presentation, review your analysis of the market survey. Are your graphs clear and correct? Have your summary statistics been calculated correctly? Are your decisions and conclusions supported by the data? Practice your presentation in front of at least two people before presenting it to the class. Ask for suggestions for improvement.

Extending the Project
Market research plays an important role in many business decisions. Find out about some of the survey techniques used by market researchers.

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Chapter 11 Project Manager: On the Move

Getting Started
Read the project. As you work on the project, you will need a calculator, materials on which you can record your calculations, and materials to make accurate and attractive graphs. Keep all of your work for the project in a folder.

Checklist
☐ Activity 1: designing and conducting a survey
☐ Activity 2: graphing and analyzing data
☐ Activity 3: designing a product or service
☐ Activity 4: analyzing a market survey
☐ marketing decision

Suggestions
☐ Test and revise the survey.
☐ Try different types of graphs to see which best displays the data.
☐ Make your product or service practical and feasible.
☐ Identify customers and their needs. What changes do they suggest for your product?
☐ Have you considered the cost of manufacturing your product or of implementing your service? How will you make a profit on your investment? What are the potential consequences of over-estimating or under-estimating consumer demand for your product or service?

Scoring Rubric
4   Surveys are well-designed. Graphs and summary statistics are accurate, and clearly show the results of your surveys. Explanations, decisions, and presentation are based on your research. All work is presented in a clear and organized manner.

3   Graphs and summary statistics are mostly correct, with some minor errors. Decisions are mostly based on the research. Some work could be presented more clearly.

2   Graphs and summary statistics contain major errors. Decisions are not based on sound research.

1   Major concepts are misunderstood. Project satisfies few of the requirements and shows poor organization and effort.

0   Major elements of the project are incomplete or missing.

Your Evaluation of Project Evaluate your work, based on the *Scoring Rubric.*

Teacher's Evaluation of Project