Lesson 6: Completing the Square
Algebra 2 A Unit 4: Quadratic Functions and Equations

Objectives: Solve equations by completing the square; Rewrite functions by completing the square
Materials: *Course Materials are not available as of this time as this User has not been assigned to any Courses. Please check back once the User has been placed into a Course.*
Note: This lesson should take 2 days.

Maximum Area

Suppose the local youth group is hosting a dance and you are on the decoration committee. You want to enclose a rectangular area for dancing with a string of lights. If you have 140 feet of lights, what is the maximum area that can be enclosed? In this lesson, you will learn how to solve this problem and others using another method for solving quadratic equations called completing the square.

Objectives

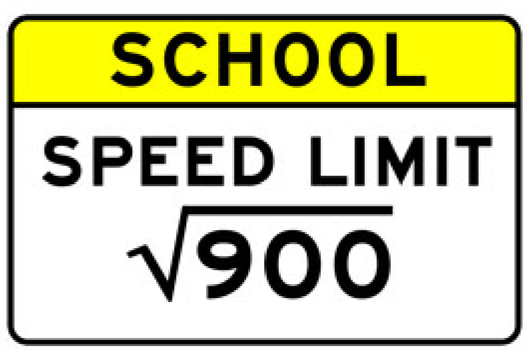
- Solve equations by completing the square
- Rewrite functions by completing the square

Key Word

- completing the square

Solving Quadratic Equations

In this lesson, you will continue to learn methods for solving quadratic equations algebraically. Some of these methods involve simplifying square roots. Below is a list of things to keep in mind when simplifying square roots.



- An expression is not in simplest form if any of the following is true:
  - There is a fraction under a radical.
  - There is a radical in the denominator.
  - There is a perfect square or perfect square factor under the radical.

- There is an exponent under the radical.

- When working with real numbers, it is not possible to take the square root of a negative number.
- When taking the square root of a variable, the result is positive and thus indicated with absolute value symbols.

To practice simplifying radicals, complete the even-numbered problems of the Square Roots and Radicals activity on p. 225 in *Algebra 2*. Be sure to read the information provided at the top of the page before starting the exercises. When you are finished, click on the Show Answer button below to check your work.

Show Answer

Answers:

2. $\sqrt{75} = \sqrt{25 \bullet 3} = \sqrt{25} \bullet \sqrt{3} = 5\sqrt{3}$

4. Taking the square root of a negative number will not produce a real solution.

6. $\sqrt{\frac{3}{15}} = \frac{\sqrt{3}}{\sqrt{15}} = \frac{\sqrt{3}}{\sqrt{15}} \bullet \frac{\sqrt{15}}{\sqrt{15}} = \frac{\sqrt{45}}{15} = \frac{\sqrt{9 \bullet 5}}{15} = \frac{3\sqrt{5}}{15} = \frac{\sqrt{5}}{5}$

8. $5\sqrt{320} = 5\sqrt{64 \bullet 5} = 5 \bullet 8\sqrt{5} = 40\sqrt{5}$

10. $-\sqrt{10^4} = -10^2 = -100$

12. $\sqrt{\frac{8}{x^2}} = \frac{\sqrt{4 \bullet 2}}{\sqrt{x^2}} = \frac{2\sqrt{2}}{|x|}$

14. $\sqrt{\frac{(-3)^4}{12}} = \frac{\sqrt{(-3)^4}}{\sqrt{12}} = \frac{(-3)^2}{\sqrt{4 \bullet 3}} = \frac{9}{2\sqrt{3}} = \frac{9}{2\sqrt{3}} \bullet \frac{\sqrt{3}}{\sqrt{3}} = \frac{9\sqrt{3}}{2 \bullet 3} = \frac{9\sqrt{3}}{6} = \frac{3\sqrt{3}}{2}$

16. $\sqrt{120x} = \sqrt{4 \bullet 30x} = 2\sqrt{30x}$

Equations involving perfect squares can be solved using radicals. You can study how this method works. Click on the links below to complete problems 1–3 for Chapter 4, Lesson 6 from the PowerAlgebra website. Each problem below includes step-by-step instructions.

Problem 1

Problem 2

Problem 3

A quadratic equation may not contain a perfect square, but it can be rewritten to include one. This process is called completing the square. Read more about this process on pp. 235–236 in *Algebra 2*. Click on the link below to access the

Completing the Square chart to organize your notes. Be sure to include the completed chart in your math binder.

[Completing the Square](#)

Click on the link below to access the online textbook.

[Algebra 2](#)

Click on the Show Answer button to check your answers.

Show Answer

Answers:

$x^2 - 10 = -6x$

Step 1: $x^2 + 6x = 10$ ↔ Rewrite the equation to isolate all x-terms on one side.

Step 2: $\left(\frac{6}{2}\right)^2 = (3)^2 = 9$ ↔ Divide the *x*-coefficent by 2 and square the result.

Step 3: $x^2 + 6x + 9 = 10 + 9$ ↔ Add the result from Step 2 to both sides of the equation.

Step 4: $(x + 3)^2 = 19$ ↔ Factor the quadratic polynomial and simplify the right side.

Step 5: $\sqrt{(x+3)^2} = \sqrt{19}$ ↔ Take the square root of both sides to eliminate the exponent.

Step 6: $(x + 3) = \pm\sqrt{19}$ ↔ Simplify, and remember to use the plus/minus sign in front of the radical.

Step 7: $x = -3 \pm \sqrt{19}$ ↔ Solve for x .

Now click on the links below to complete problems 4–6 of Chapter 4, Lesson 6 from the PowerAlgebra website. Each problem below includes step-by-step instructions.

[Problem 4](#)

[Problem 5](#)

Problem 6

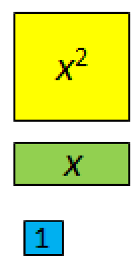
Click on the link below to watch the "Form 4 and Completing the Square Algebraically" Discovery Education™ *streaming* movie. Focus on the process used to complete the square for a quadratic equation.

Form 4 and Completing the Square Algebraically

Tip: The square of a number will always be positive, even if the number is negative. In the example below, you can see that both 5 squared and –5 squared equal 25. Since taking the square root is the reverse of the process below, the ± symbol is used to indicate that the square root can be positive or negative number.

$$5^2 = 5 \cdot 5 = 25$$
$$(-5)^2 = 5 \cdot -5 = 25$$

Complete the following activities.



1. Now explore completing the square using algebra tiles. Click on the link below to complete the Dynamic Activity for Chapter 4, Lesson 6 from the PowerAlgebra website.

   Dynamic Activity

2. To practice solving equations by completing the square and writing equations in vertex form, complete problems 29, 33, 35, 37, 41, 45, 47, 48, and 49 from pp. 237–238 in *Algebra 2*.
3. To practice solving quadratic equations algebraically, click on the link below to complete the 4–6 Puzzle worksheet.

   4-6 Puzzle Worksheet

4. Continue working on the unit portfolio and participating in the unit discussion, both of which are due in Lesson 10. Recall that the information regarding the portfolio is located in Lesson 2.

Tip: Please return to Unit 4, Lesson 1, page 4 to access the discussion link in order to add your comments.

Click on the link below to access the online textbook.

Algebra 2

Tip: When solving an equation using square roots, you must take the square root of both sides of the equation.

Complete the following review activities.

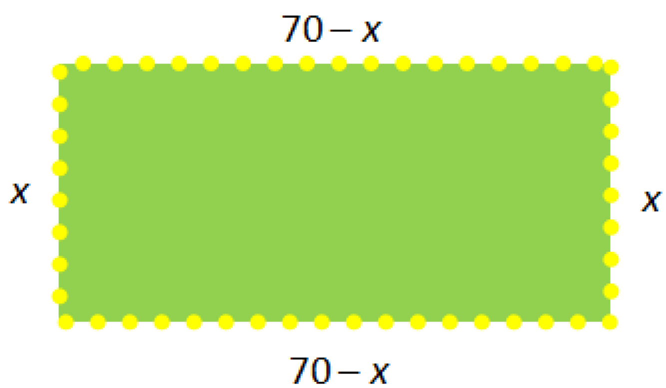
In this lesson, you learned how to solve quadratic equations algebraically by using square roots. You explored the process of completing the square, which can be used to write a quadratic equation as a perfect square trinomial. The activities that follow will help you review these concepts.

1. Recall the problem from the beginning of this lesson. Suppose the local youth group is hosting a dance and you are on the decoration committee. You want to enclose a rectangular area for dancing with a string of lights. If you have 140 feet of lights, what is the maximum area that can be enclosed? First, you will need to write an equation based on the perimeter of the rectangle, which you know will be 140 feet. Then use the process of completing the square to write the equation in vertex form.

   Click on the Show Answer button below to check your answer.

   Show Answer

   Answer:
   Let x be the width of the rectangular dance floor. Subtract the two known side lengths, each equal to x, from the total length of the string of lights to get the expression $140–2x$. This expression represents the sum of the two unknown side lengths. Because the two unknown side lengths are also equal to each other, each length is half of $140 – 2x$, which is $70 – x$.



   Use these dimensions to write the area of the rectangle.

   $A = x(70 – x)$
   $= 70x – x^2$

   Now complete the square to write the area in vertex form.

   $-x^2 + 70x$

   $-\left(x^2 - 70x\right) \leftrightarrow$ Factor out a -1 in order to make the leading coefficient positive

   $-\left(x^2 - 70x + 1,225\right) + 1,225 \leftrightarrow$ Complete the square

   $-\left(x - 35\right)^2 + 1,225 \leftrightarrow$ Simplify

   The vertex of the parabola is (35, 1,225). Therefore, it turns out the maximum area that you can enclose with a string of lights 140 feet long is a square with side lengths of 35 feet. The total area of the enclosure will be 1,225 square feet.

2. To review concepts from this lesson, complete the Lesson Check on p. 237 of *Algebra 2*.
3. To review previous concepts, complete the Mixed Review on p. 239.

Click on the link below to access the online textbook.

Algebra 2

Completing the Square

Multiple Choice

1.

What is the solution of the equation?
$36x^2 = 100$

(1 point)

- ○ $-\frac{9}{25}, \frac{9}{25}$
- ○ $-\frac{3}{5}, \frac{3}{5}$
- ○ $-\frac{25}{9}, \frac{25}{9}$
- ○ $-\frac{5}{3}, \frac{5}{3}$

2.

A landscaper is designing a flower garden in the shape of a trapezoid. She wants the shorter base to be 3 yards greater than the height, and the longer base to be 7 yards greater than the height. She wants the area to be 225 square yards. The situation is modeled by the equation, $\frac{1}{2}h(b_1+b_2)$, where h is the height, in yards, and b1 and b2 are the length of the two bases, in yards. Complete the square to find the height that will give the desired area. Round to the nearest hundredth of a yard.

(1 point)

- ○ 25.41 yards
- ○ 460 yards

○ 15.21 yards
○ 12.71 yards

3.

Solve the equation.
$x^2 + 12x + 36 = 25$

(1 point)

○ 11, 1
○ –1, 1
○ –1, –11
○ 11, –11

4.

Rewrite the equation in vertex form. Name the vertex and y-intercept.
$y = x^2 - 10x + 15$

(1 point)

○ $y = (x - 10)^2 - 10$
vertex: (–10, –10)
y-intercept: (0, –10)
○ $y = (x - 5)^2 - 10$
vertex: (5, –10)
y-intercept: (0, 15)
○ $y = (x - 5)^2 + 40$
vertex: (5, –10)
y-intercept: (0, 15)
○ $y = (x - 10)^2 + 20$
vertex: (–10, –10)
y-intercept: (0, –10)

5.

Solve the quadratic equation by completing the square.
$-3x^2 + 9x = 1$

(1 point)

○ $\frac{3}{2} \pm \frac{\sqrt{6}}{6}$
○ $-3 \pm \frac{\sqrt{69}}{3}$

○ $3 \pm \frac{\sqrt{-3}}{3}$

○ $\frac{3}{2} \pm \frac{\sqrt{69}}{6}$

6.

Rewrite the equation in vertex form. Name the vertex and *y*-intercept.

$y = \frac{3}{5}x^2 + 30x + 382$

(1 point)

○ $y = \frac{3}{5}(x + 25)^2 + 7$
vertex: (—25, 7)
y-intercept: (0, 382)

○ $y = (x + 30)^2 + 518$
vertex: (30, 518)
y-intercept: (0, 7)

○ $y = (\frac{3x}{5} + 25)^2 + 7$
vertex: (25, —7)
y-intercept: (0, 7)

○ $y= \frac{3}{5}(x + 5)2 + \frac{21}{5}$
vertex: (—5, $\frac{21}{5}$)
y-intercept: (0, 382)