Lesson 2: Matrix Multiplication
Algebra 2 B Unit 5: Matrices

Objective: Multiply matrices using scalar and matrix multiplication
Materials: *Course Materials are not available as of this time as this User has not been assigned to any Courses. Please check back once the User has been placed into a Course.*

Calculating with Matrices

Maria is taking her twin nieces shopping for new school clothes. Hannah needs 3 pairs of jeans, 6 T-shirts, and 2 pairs of shoes. Isabelle needs 2 pairs of jeans, 9 T-shirts, and 1 pair of shoes. Jeans cost $15, T-shirts cost $5, and shoes cost $25. Maria has a coupon for 10% off the entire purchase. How can Maria use matrix multiplication to calculate the total cost of each twin's clothes?

In this lesson, you will learn how to multiply matrices by real-number factors and by other matrices.

Objective

- Multiply matrices using scalar and matrix multiplication

Key Words

- scalar
- scalar multiplication

Multiplying Matrices

There are two types of multiplication that can be performed with matrices.

1. Scalar multiplication is the multiplication of a matrix by a real number factor, or scalar. This type of multiplication gives you a simple way to resize, or scale, all of the elements in a matrix. The product of scalar c and matrix A is matrix cA, in which each element of matrix A is multiplied by scalar c.

$$4\begin{bmatrix} 2 & -3 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 4(2) & 4(-3) \\ 4(1) & 4(5) \end{bmatrix} = \begin{bmatrix} 8 & -12 \\ 4 & 20 \end{bmatrix}$$

2. Matrix multiplication is the multiplication of a matrix by another matrix. The product of two matrices is a matrix in which each element is the sum of the products of the elements in a row in the first matrix and a column in the second matrix.

$$\begin{bmatrix} 2 & 5 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} 4 & 6 \\ -2 & -3 \end{bmatrix} = \begin{bmatrix} (2)(4)+(5)(-2) & (2)(6)+(5)(-3) \\ (3)(4)+(-1)(-2) & (3)(6)+(-1)(-3) \end{bmatrix}$$
$$= \begin{bmatrix} 8+(-10) & 12+(-15) \\ 12+2 & 18+3 \end{bmatrix}$$
$$= \begin{bmatrix} -2 & -3 \\ 14 & 21 \end{bmatrix}$$

If you call the factor matrices *A* and *B*, respectively, and the product matrix *C*, then the element in position *c*12 is the product of the elements in row 1 of matrix *A* and the elements in column 2 of matrix *B*.

What is the position of the element that is the product of the elements in row 2 of matrix *A* and column 2 of matrix *B*? What is the value of that element?

Click on the Show Answer button below to check your answer.

Show Answer

Answer:
$c_{22} = 21$

Click on the link below to watch the "Matrix Multiplication" Discovery Education™ *streaming* movie. Look for a review of matrix addition and subtraction, as well as an explanation of matrix multiplication. Also look for an example of how matrix multiplication can be used to solve a cypher, or code. This concept will be covered in more detail in Lesson 3.

After viewing the movie, answer the following questions:

1. How does matrix multiplication differ from matrix addition or subtraction?
2. If the encryption key for a cypher were a 4 × 4 matrix, how many coded letters at a time would you need to decipher? Explain how you know.

Matrix Multiplication

Click on the Show Answer button below to check your answers.

Show Answer

Answers:

1. For matrix addition or subtraction, the two matrices must have the same dimensions because corresponding elements are added or subtracted. For matrix multiplication, the number of elements in each row of the first matrix must equal the number of elements in each column of the second

matrix.

2. I would need to decipher 4 letters at a time, because the number of elements in the 1-column matrix would have to be the same as the number of elements in each row of the 4 × 4 encryption matrix.

Click on the link below to complete the Solve It! activity for Chapter 12, Lesson 2 from the PowerAlgebra website. You will see examples of performing addition with scalar products of matrices and solving a matrix equation with scalars, both of which are extensions of the operations you performed in Lesson 1. Take notes as you are completing the instruction. Be sure to include in your notes the properties of scalar multiplication and matrix multiplication.

Solve It!

Click on the link below to watch the "Example 3: Using Matrices – Group Admission" Discovery Education™ *streaming* movie. Look for an example of using matrix multiplication to model and solve a problem.

After viewing the movie, explain the relationship in matrix multiplication between the elements in each row of the first matrix and the elements in each column of the second matrix in terms of coefficients and variables.

Example 3: Using Matrices – Group Admission

Click on the Show Answer button below to check your answers.

Show Answer

Answer:
Answers may vary but should be similar to:

The elements in each row of the first matrix are the coefficients of the variable elements in each column of the second matrix.

Complete the following activities.

1. Click on the link below to access and complete the 12-2 Think About a Plan worksheet. You will use matrix multiplication to determine the results of a track meet.

   12-2 Think About a Plan

2. Complete problems 11, 17, 27, 35, and 41 on pp. 769–770 in *Algebra 2*.
3. Think about the problem posed on the Getting Started page. Recall that Hannah needs 3 pairs of jeans, 6 T-shirts, and 2 pairs of shoes. Isabelle needs 2 pairs of jeans, 9 T-shirts, and 1 pair of shoes. Jeans cost $15, T-shirts cost $5, and shoes

cost $25. Maria has a coupon for 10% off the entire purchase. Use matrix multiplication to find the total cost of each twin's clothes.

4. During this unit, you will complete a portfolio project. The links below will provide you with more information. The portfolio project is due at the end of Lesson 7.

   Click on the link below to access the Chapter 12 Project: Munching Microbes portfolio project document. You will complete all activities.

   Chapter 12 Project: Munching Microbes

   Click on the links below to access the Munching Microbes Portfolio Grading Rubric and the Munching Microbes Portfolio Checklist.

   Munching Microbes Portfolio Grading Rubric

   Munching Microbes Portfolio Checklist

Click on the link below to access the online textbook.

Algebra 2

Complete the following review activities.

In this lesson, you learned how to multiply a matrix by a scalar, or a matrix by another matrix.

1. Check your understanding of Matrix Multiplication with the Lesson Check on p. 769 in *Algebra 2*.
2. In your math writing journal, complete problem 44 on p. 770 in *Algebra 2*. Name the entry "Matrix Multiplication."

Click on the link below to access the online textbook.

Algebra 2

Lesson Answers

Click on the link below to check your answers to the 12-2 Think About a Plan worksheet.

12-2 Think About a Plan Answers

Click on the link below to check your answer to question 3 from the Activity page.

Activity: Question 3 Answer

Matrix Multiplication

Multiple Choice

1.

Find $-5A + 4B$

$$A = \begin{bmatrix} 6 & 1 \\ -4 & -6 \\ 7 & -7 \end{bmatrix} \qquad B = \begin{bmatrix} -5 & -1 \\ -3 & -8 \\ 6 & 8 \end{bmatrix}$$

(1 point)

- ○ $\begin{bmatrix} -10 & 9 \\ 8 & -2 \\ 16 & 67 \end{bmatrix}$
- ○ $\begin{bmatrix} -10 & 9 \\ 8 & -2 \\ -59 & -68 \end{bmatrix}$
- ○ $\begin{bmatrix} -50 & -9 \\ -1 & 16 \\ -11 & -68 \end{bmatrix}$
- ○ $\begin{bmatrix} -50 & -9 \\ 8 & -2 \\ -11 & 67 \end{bmatrix}$

Solve the matrix equation.

2.

$$2X + 2\begin{bmatrix} 2 & -8 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} 4 & -6 \\ 2 & -8 \end{bmatrix}$$

(1 point)

- ○ $\begin{bmatrix} 0 & 5 \\ -3 & -2 \end{bmatrix}$
- ○ $\begin{bmatrix} 0 & -11 \\ -3 & -6 \end{bmatrix}$
- ○ $\begin{bmatrix} 0 & 5 \\ 5 & -6 \end{bmatrix}$

○ $\begin{bmatrix} 4 & 5 \\ 5 & -2 \end{bmatrix}$

Find the product.

3.

$$\begin{bmatrix} -5 & -2 \\ -8 & -5 \end{bmatrix}\begin{bmatrix} -5 & 7 \\ -9 & -5 \end{bmatrix}$$

(1 point)

○ $\begin{bmatrix} 25 & 18 \\ -35 & 10 \end{bmatrix}$

○ $\begin{bmatrix} 43 & -25 \\ 85 & -31 \end{bmatrix}$

○ $\begin{bmatrix} 40 & 45 \\ -56 & 25 \end{bmatrix}$

○ $\begin{bmatrix} 43 & -25 \\ -31 & 85 \end{bmatrix}$

4.

The table summarizes the scoring of a football game between Team *A* and Team *B*. A touchdown (TD) is worth 6 points, a field goal (FG) is worth 3 points, a safety (S) is worth 2 points, and a point after touchdown (PAT) is worth 1 point. Use matrix multiplication to find the final score.

	TD	**FG**	**S**	**PAT**
Team *A*	3	2	1	0
Team *B*	4	4	1	2

(1 point)

○ Team *A*: 35
Team *B*: 36

○ Team *A*: 36
Team *B*: 39

○ Team *A*: 26
Team *B*: 40

○ Team *A*: 17
Team *B*: 38

5.

Determine whether the product is defined or undefined. If defined, give the dimensions of the product matrix.

$$\begin{bmatrix} 1 & 1 & -4 \\ 5 & 6 & 0 \end{bmatrix} \begin{bmatrix} 9 \\ 1 \\ -7 \end{bmatrix}$$

(1 point)

○ defined; 3 × 3
○ defined; 2 × 1
○ defined; 2 × 3
○ undefined

Activity: Question 3 Answer

Write the items and the cost per item in matrix form. The items form a 3 × 2 matrix in which each row represents one of the twins' purchases. The costs per item form a 1 × 3 matrix. Multiply the cost matrix by the scalar 0.9 before multiplying by the items matrix, or multiply the product matrix by the scalar 0.9.

$$0.9\begin{bmatrix} 3 & 6 & 2 \\ 2 & 9 & 1 \end{bmatrix}\begin{bmatrix} 15 \\ 5 \\ 25 \end{bmatrix} = 0.9\begin{bmatrix} (3)(15)+(6)(5)+(2)(25) \\ (2)(15)+(9)(5)+(1)(25) \end{bmatrix}$$

$$= 0.9\begin{bmatrix} 45+30+50 \\ 30+45+25 \end{bmatrix}$$

$$= 0.9\begin{bmatrix} 125 \\ 100 \end{bmatrix}$$

$$= \begin{bmatrix} 112.5 \\ 90 \end{bmatrix}$$

The total cost of Hannah's clothes is $112.50. The total cost of Isabelle's clothes is $90.