Lesson 3: Determinants and Inverses
Algebra 2 B Unit 5: Matrices

Objective: Find the inverse of a matrix
Materials: *Course Materials are not available as of this time as this User has not been assigned to any Courses. Please check back once the User has been placed into a Course.*
Note: This lesson should take 2 days.

Making E-Commerce Safer

Have you ever bought anything over the Internet? Did you worry about sending a credit card number into cyberspace? One way to protect sensitive information is to encrypt, or encode, it. A matrix is used to encrypt the credit card number when it is sent to the vendor, and another matrix is used to decrypt the number for the vendor to use it. The two matrices have a special relationship: They are inverses of each other.

In this lesson, you will learn how to find the inverse of a matrix. You will also learn how to evaluate the determinant of a matrix, which is a value derived from the elements of the matrix. Finally, you will use inverses and determinants to solve problems.

Objective

- Find the inverse of a matrix

Key Words

- determinant
- multiplicative identity matrix
- multiplicative inverse matrix
- singular matrix
- square matrix

Tip: You will have two days to complete this lesson.

Inverses

If a matrix has an equal number of rows and columns it is a square matrix. For example, a 2×2 matrix or a 3×3 matrix is a square matrix.

For a square matrix, *A,* with *n* rows and *n* columns, there is a multiplicative identity matrix, *I*, that also has *n* rows and *n* columns. The elements of the multiplicative identity matrix along the main diagonal have a value of 1; all other elements have a value of 0. The product of A and the multiplicative identity matrix is A.

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$AI = A$$

A square matrix may have a multiplicative inverse matrix. The product of a square matrix and its multiplicative inverse is the multiplicative identity matrix. For square matrix, A, with inverse matrix A^{-1}, $A \bullet A^{-1} = I$.

Click on the link below to complete the Solve It! activity for Chapter 12, Lesson 3 from the PowerAlgebra website. You will see definitions of the multiplicative identify matrix and the multiplicative inverse matrix.

Solve It!

You can test whether matrices are inverses of each other by multiplying the matrices together. If the product of the matrices is the multiplicative identity matrix, then the matrices are inverses.

Fore example, given $A = \begin{bmatrix} -2 & 1 \\ 5 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & -1 \\ -5 & -2 \end{bmatrix}$, determine whether A and B are inverses.

$$\begin{aligned} AB &= \begin{bmatrix} -2 & 1 \\ 5 & -3 \end{bmatrix}\begin{bmatrix} -3 & -1 \\ -5 & -2 \end{bmatrix} \\ &= \begin{bmatrix} (-2)(-3)+(1)(-5) & (-2)(-1)+(1)(-2) \\ (5)(-3)+(-5)(-3) & (5)(-1)+(-3)(-2) \end{bmatrix} \\ &= \begin{bmatrix} 6-5 & 2-2 \\ -15+15 & -5+6 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{aligned}$$

Since $AB = I$, A and B are inverses of each other.

Determinants

A determinant is a number associated with a square matrix. For a 2 × 2 matrix, the determinant is equal to the difference between the products of the elements on each diagonal.

For example, for $A = \begin{bmatrix} -2 & 1 \\ 5 & -3 \end{bmatrix}$, find det A, the determinant of A:

$$\det\begin{bmatrix}-2 & 1\\ 5 & -3\end{bmatrix} = (-2)(-3)-(1)(5)$$

$$= 6 - 5$$

$$= 1$$

You can use the determinant of a matrix to determine whether the matrix has an inverse and to calculate the inverse.

For square matrix A, if det $A = 0$, then the matrix has no inverse. A square matrix with no inverse is called a singular matrix.

If det $A \neq 0$, then the matrix has an inverse.

Read and take notes on pp. 774–779 in *Algebra 2*. One of the problems you will solve involves encrypting a credit card number, as on the Getting Started page. While reading, complete Got It! for problems 1–5. Be sure to include in your notes the Key Concepts for the lesson:

- Identity and Multiplicative Inverse Matrices
- Determinants of 2 × 2 and 3 × 3 Matrices
- Area of a Triangle
- Inverse of a 2 × 2 Matrix

Click on the link below to access the online textbook.

Algebra 2

Complete the following activities.

1. Complete problems 29, 37, 39, 43, 49, and 55 on pp. 780–781 in *Algebra 2*.
2. Click on the link below to access and complete the 12-3 Puzzle: That's Sum Matrix worksheet. You will use determinants and inverses to complete a grid.

   12-3 Puzzle: That's Sum Matrix

3. Check your understanding of determinants and inverses with the Lesson Check on p. 779 in *Algebra 2*.
4. Continue working on the unit portfolio and participating in the unit discussion.

Tip: Please return to Unit 5, Lesson 1, page 5 to access the discussion link in order to add your comments.

Click on the link below to access the online textbook.

Algebra 2

Complete the following review activities.

1. At the end of this lesson is a quiz on the lessons listed below.

- Lesson 1: Adding and Subtracting Matrices
- Lesson 2: Matrix Multiplication
- Lesson 3: Determinants and Inverses

Be sure you understand the concepts from these lessons and review vocabulary. The activities that follow will help you review for the quiz.

2. Practice for the quiz at the end of this lesson. Click on the link below to complete the Mid-Chapter Practice and Review for Chapter 12 from the PowerAlgebra website.

   Chapter 12 Mid-Chapter Practice and Review

3. To practice concepts you have learned throughout this course, complete the Mixed Review on p. 782 in *Algebra 2*. You may refer to the lessons indicated if you require additional guidance to solve the problems.

Tip: You may want to take the quiz on the next lesson day.

Click on the link below to access the online textbook.

Algebra 2

Tip: Now you will practice using WorkPad. You will use WorkPad to complete the assessment on this lesson. Select the link to access the WorkPad directions. Read the directions to understand how to use Workpad.

WorkPad

WorkPad Directions

Determinants and Inverses Quiz

Multiple Choice

1.

Use matrices *A* and *B*. Compute B – A, if you can.

$$A = \begin{bmatrix} -5 & 4 \\ -8 & 2 \end{bmatrix} \quad B = \begin{bmatrix} -2 & 7 & -3 \\ 1 & -6 & 0 \end{bmatrix}$$

(1 point)

○ $\begin{bmatrix} -7 & -4 & 2 \\ -4 & 6 & 6 \end{bmatrix}$

○ not possible

○ $\begin{bmatrix} -7 & 11 \\ -7 & -4 \end{bmatrix}$

○ $\begin{bmatrix} 3 & 10 & -4 \\ -2 & -6 & 6 \end{bmatrix}$

2.

Find the sum.

$$\begin{bmatrix} 1 & 4 \\ 0 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

(1 point)

○ $\begin{bmatrix} 1 & 4 \\ 0 & 3 \end{bmatrix}$

○ undefined

○ $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

○ $\begin{bmatrix} 1 & 0 \\ 4 & 3 \end{bmatrix}$

3.

Find –3*A*+ 6*B*.

$$A = \begin{bmatrix} -3 & 5 & -6 \\ 9 & -5 & 3 \end{bmatrix} \quad B = \begin{bmatrix} -2 & 6 & 7 \\ 2 & -1 & -6 \end{bmatrix}$$

(1 point)

○ $\begin{bmatrix} 21 & 12 & -24 \\ -15 & 9 & 36 \end{bmatrix}$

○ $\begin{bmatrix} -3 & 21 & 60 \\ 48 & -27 & 36 \end{bmatrix}$

○ $\begin{bmatrix} 21 & 12 & 60 \\ -15 & -27 & 36 \end{bmatrix}$

○ $\begin{bmatrix} -3 & 21 & 60 \\ -15 & 9 & -45 \end{bmatrix}$

4.

Solve the matrix equation.

$$X - \begin{bmatrix} 2 & -8 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} 4 & -6 \\ 2 & -8 \end{bmatrix}$$

(1 point)

○ $\begin{bmatrix} 6 & -14 \\ -2 & -6 \end{bmatrix}$

○ $\begin{bmatrix} 2 & -14 \\ -2 & -10 \end{bmatrix}$

○ $\begin{bmatrix} 6 & -14 \\ 6 & -10 \end{bmatrix}$

○ $\begin{bmatrix} 6 & 2 \\ 6 & -6 \end{bmatrix}$

5.

Find the product.

$$\begin{bmatrix} 5 & 5 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} -4 & 9 \\ 8 & 7 \end{bmatrix}$$

(1 point)

○ $\begin{bmatrix} 20 & 80 \\ 32 & 3 \end{bmatrix}$

○ $\begin{bmatrix} 20 & 80 \\ 3 & 32 \end{bmatrix}$

○ $\begin{bmatrix} -20 & 40 \\ 45 & 35 \end{bmatrix}$

○ $\begin{bmatrix} 8 & 24 \\ -18 & 21 \end{bmatrix}$

6.

Determine whether the product is defined or undefined. If defined, give the dimensions of the product matrix.

$$\begin{bmatrix} 4 & 5 \\ 9 & -2 \end{bmatrix}\begin{bmatrix} 1 & 7 \end{bmatrix}$$

(1 point)

○ defined; 2 × 2
○ defined; 2 × 1
○ defined; 1 × 2
○ undefined

7.

Are matrices A and B inverses?

$A = \begin{bmatrix} -5 & -18 \\ 2 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 7 & 18 \\ -2 & -5 \end{bmatrix}$

(1 point)

○ yes
○ no

8.

One factor in flood safety along a levee is the area that will absorb water should the levee break. The coordinates that make up the boundary area are (0, 0), (2.8, 2.1), and (4.3, 4.4). What is the area of the land that would absorb the water?

(1 point)

- ○ 1.65 mi^2
- ○ 9.03 mi^2
- ○ 1.1 mi^2
- ○ 10.68 mi^2

9.

Does the given matrix, *A*, have an inverse? If it does, what is A^{-1}?

$$A = \begin{bmatrix} -7 & -25 \\ 2 & 7 \end{bmatrix}$$

(1 point)

- ○ $\begin{bmatrix} -7 & -25 \\ -2 & -7 \end{bmatrix}$
- ○ $\begin{bmatrix} 7 & 25 \\ -2 & -7 \end{bmatrix}$
- ○ $\begin{bmatrix} 7 & 25 \\ -2 & 7 \end{bmatrix}$
- ○ does not exist

10.

The first table shows the teams with the four best records halfway through the season. The second table shows the full season records for the same four teams. Which team had the best record during the second half of the season?

Record for the First Half of the Season

Team	**Wins**	**Losses**
Team 1	26	14
Team 2	27	13
Team 3	25	15
Team 4	24	16

Record for the Season

Team	**Wins**	**Losses**
Team 1	59	21
Team 2	56	24
Team 3	56	24
Team 4	52	28

(1 point)

- ○ Team 2
- ○ Team 1
- ○ Team 3
- ○ Team 4

WorkPad

11.

Work Pad

Note: Remember to show all of the steps that you use to solve the problem. You can use the comments field to explain your work. Your teacher will review each step of your response to ensure you receive proper credit for your answer.

Find the values of the variables.

$$\begin{bmatrix} -12 & -w^2 \\ 2f & 3 \end{bmatrix} = \begin{bmatrix} 2k & -81 \\ -14 & 3 \end{bmatrix}$$

(2 points)

12.

Evaluate the determinant of the matrix.

$$\begin{bmatrix} -4 & 5 & 6 \\ 0 & 4 & 4 \\ -2 & -5 & 4 \end{bmatrix}$$

(2 points)

13.

For problems 13 and 14, assign each letter and a blank space to a number as shown by the alphabet table below.

0 = _	1 = A	2 = B	3 = C	4 = D	5 = E	6 = F	7 = G	8 = H
9 = I	10 = J	11 = K	12 = L	13 = M	14 = N	15 = O	16 = P	17 = Q
18 = R	19 = S	20 = T	21 = U	22 = V	23 = W	24 = X	25 = Y	26 = Z

Use $\begin{bmatrix} 1 & -2 \\ -3 & 7 \end{bmatrix}$ encode the phrase "ONE QUESTION TO GO."

(3 points)

14.

The matrix $C = \begin{bmatrix} 1 & -2 \\ -3 & 7 \end{bmatrix}$ was used to encode a phrase to

$\begin{bmatrix} 7 & -28 & -25 & -35 & -2 \\ -21 & 107 & 90 & 123 & 17 \end{bmatrix}$. Find C^{-1} and use it to decode the matrix.

(3 points)