Lesson 4: Inverse Matrices and Systems
Algebra 2 B Unit 5: Matrices

Objective: Solve systems of equations using matrix inverses and multiplication
Materials: *Course Materials are not available as of this time as this User has not been assigned to any Courses. Please check back once the User has been placed into a Course.*

Finding Unknown Values
Salima is ordering pizza for a party. She orders a total of eight large pizzas. Cheese pizzas cost $10 each, and pepperoni costs $2 extra. If the total cost is $86 before tax, how many of each kind of pizza does she order?

There are several different strategies you could use to solve this problem, such as guess-and-check. You could also represent the problem as a linear system of equations and solve the system algebraically. In this lesson, you will learn another approach: using matrices and matrix equations to model and solve problems with linear systems of equations.

Objective

- Solve systems of equations using matrix inverses and multiplication

Key Words

- coefficient matrix
- constant matrix
- variable matrix

Using Matrices to Solve Systems of Equations
Click on the link below to complete the Solve It! activity for Chapter 12, Lesson 4 from the PowerAlgebra website. You will see an example of a problem that can be solved using a system of equations. You will also learn about solving multiplicative matrix equations.

Solve It!

You have previously solved systems of linear equations algebraically. You can also use matrices to solve systems of linear equations. First, you need to write a matrix equation to represent the system. Then, you can solve the matrix equation to find the values of the variables that satisfy the system.

A matrix equation that represents a system of linear equations consists of the following three parts:

1. The coefficient matrix is comprised of the coefficients of the variables in the equations. Each row of the matrix represents one equation from the system. Each column of the matrix represents the coefficients from each equation for the same variable in the system.
2. The variable matrix is a one-column matrix comprised of the variables in the equations.
3. The constant matrix is a one-column matrix comprised of the constant terms in the equations.

The product of the coefficient matrix and the variable matrix is the constant matrix.

Given the system of equations $\begin{cases} 2x+5y=3 \\ x-4y=9 \end{cases}$:

- The coefficient matrix is $\begin{bmatrix} 2 & 5 \\ 1 & -4 \end{bmatrix}$.
- The variable matrix is $\begin{bmatrix} x \\ y \end{bmatrix}$.
- The constant matrix is $\begin{bmatrix} 3 \\ 9 \end{bmatrix}$.

The matrix equation that represents the system is $\begin{bmatrix} 2 & 5 \\ 1 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 3 \\ 9 \end{bmatrix}$.

Using Matrices Continued

Click on the link below to watch the "Example 3: Inverse Matrix – Soccer" Discovery Education™ *streaming* movie. Look for an example of using a matrix equation to represent a system of linear equations, and using the inverse of the coefficient matrix to solve the matrix equation.

After viewing the movie, answer the following questions:

1. What does each row in the coefficient matrix represent about the team?
2. Why do you need to multiply both sides of the equation by the inverse of the

coefficient matrix?

Example 3: Inverse Matrix – Soccer

Click on the Show Answer button below to check your answers

Show Answer

Answers:
Answers may vary but should be similar to:

1. The first row represents the number of wins and ties that DC United had. The second row represents the number of points that DC United earned for each win, 3, and tie, 1.
2. You need to multiply both sides of the equation by the inverse of the coefficient matrix to isolate the coefficient matrix.
   Since $A^{-1} \bullet A = I$, $A^{-1} \bullet A \bullet X = X$. Multiplying the constant matrix by A^{-1} keeps the equation true.

Click on the link below to complete the "Systems of Linear Equations – Activity B" Gizmo. You will use a matrix to solve a system of linear equations. After completing the activity, answer the assessment questions. Then click on the Check Your Answers button.

Systems of Linear Equations - Activity B

Complete the following activities.

1. Read and take notes on pp. 784–789 in *Algebra 2*. Include in your notes an example of how to solve a matrix equation using an inverse matrix, and how to write a system of equations as a matrix equation.
2. Click on the link below to access and complete the 12-4 Puzzle: A Four-Word and Down-Word Puzzle worksheet. You will solve a puzzle by solving systems using matrices.

   12-4 Puzzle: A Four-Word and Down-Word Puzzle
3. Complete problems 26, 27, 28, 48, and 49 on pp. 790–791 in *Algebra 2*.
4. Continue working on the unit portfolio and participating in the unit discussion.

Tip: Please return to Unit 5, Lesson 1, page 5 to access the discussion link in order to add your comments.

Extension: Click on the link below to watch the "Chinese Method for Solving a 3 by 3 System of Equations" Discovery Education™ *streaming* movie. Look for an introduction to the Chinese Method for solving a matrix that represents a system of equations.

After viewing the movie, answer the following questions:

1. How is this method similar to solving systems algebraically?

2. Why is the first step of the method to multiply each element in the second row by the first element in the third row?
3. Why do you need to get zeros in the first and second rows of at least one column?

Chinese Method for Solving a 3 by 3 System of Equations

Click on the link below to access the online textbook.

Algebra 2

Complete the following activities.

1. Check your understanding of Inverse Matrices and Systems with the Lesson Check on p. 789 in *Algebra 2*.
2. In your math writing journal, list the methods for solving a system of linear equations. Explain which method you prefer and why. Name the entry “Solving Systems using Matrices.”

Click on the link below to access the online textbook.

Algebra 2

Lesson Answers

Click on the link below to check your answers to the “Chinese Method for Solving a 3 by 3 System of Equations” Discovery Education™ *streaming* movie.

Answers

Inverse Matrices and Systems

Multiple Choice

1.

What is the solution of the matrix equation?

$$\begin{bmatrix} 9 & 4 \\ 2 & 1 \end{bmatrix} X = \begin{bmatrix} -9 & -6 \\ -1 & -8 \end{bmatrix}$$

(1 point)

- ○ $\begin{bmatrix} 5 & -26 \\ 9 & -60 \end{bmatrix}$
- ○ $\begin{bmatrix} -5 & 26 \\ -1 & -8 \end{bmatrix}$

- $\begin{bmatrix} -5 & 26 \\ 9 & -60 \end{bmatrix}$
- $\begin{bmatrix} 9 & 26 \\ 9 & 1 \end{bmatrix}$

2.

Write the system $\begin{cases} 2y + 7z = -3 \\ -3x - 3y = 3 \\ 7x - 8z = 3 \end{cases}$ as a matrix equation.

(1 point)

- $\begin{bmatrix} 2 & 7 \\ -3 & -3 \\ 0 & -8 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 \\ 3 \\ 3 \end{bmatrix}$
- $\begin{bmatrix} 0 & -3 & 7 \\ 2 & -3 & 0 \\ 7 & 0 & -8 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 \\ 3 \\ 3 \end{bmatrix}$
- $\begin{bmatrix} 0 & 2 & 7 \\ -3 & -3 & 0 \\ 7 & 0 & -8 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 \\ 3 \\ 3 \end{bmatrix}$
- $\begin{bmatrix} -3 & 7 \\ 2 & -3 \\ 7 & -8 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 \\ 3 \\ 3 \end{bmatrix}$

3.

What is the solution of the system? Solve using matrices.

$$\begin{cases} -3x + 2y = 10 \\ -4x + 3y = 2 \end{cases}$$

(1 point)

○ $\begin{bmatrix} -26 \\ -34 \end{bmatrix}$

○ $\begin{bmatrix} -34 \\ -26 \end{bmatrix}$

○ $\begin{bmatrix} 26 \\ 34 \end{bmatrix}$

○ no solution

4.

$$\begin{cases} x + 2y = -1 \\ -x - 3y = -2 \end{cases}$$

(1 point)

○ $\begin{bmatrix} -7 \\ 3 \end{bmatrix}$

○ $\begin{bmatrix} 3 \\ -7 \end{bmatrix}$

○ $\begin{bmatrix} 7 \\ -3 \end{bmatrix}$

○ no solution

5.

Solve the system.

$$\begin{cases} 3x + y - 4z = -30 \\ 3x + 2y + 2z = -8 \\ 5x + 5y + z = -26 \end{cases}$$

(1 point)

○ (–30, –8, –26)
○ (4, –2, 4)

○ (–4, –2, 4)
○ (–4, 2, –4)

Answers

Answers may vary, but should be similar to:

1. To solve a system algebraically, equations are multiplied by factors and subtracted from each other in order to eliminate variables. In this method, rows are multiplied by factors and subtracted from each other in order to eliminate elements.
2. The first step is to multiply each element in the second row by the first element in the third row because that will ensure that the repreated subtraction of the first element in teh third row from teh modified first element in teh second row will result in 0.
3. You need to get zeros in the first and second rows of at least one column so that you can solve for z, whose coefficient is the third term in each row.