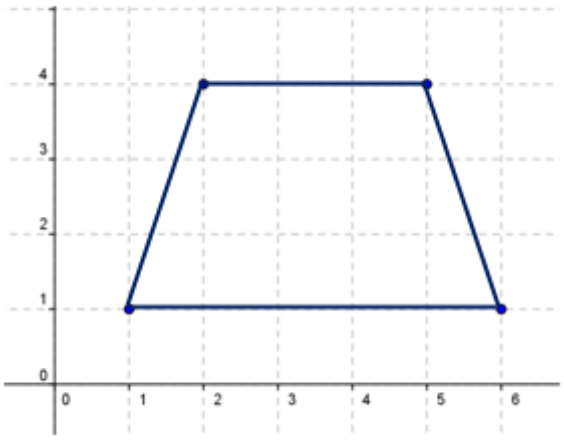
Lesson 5: Geometric Transformations
Algebra 2 B Unit 5: Matrices

Objective: Transform geometric figures using matrix operations
Materials: *Course Materials are not available as of this time as this User has not been assigned to any Courses. Please check back once the User has been placed into a Course.*
Note: This lesson should take 2 days.

Finding a Solution Without Graphing



Marissa drew a trapezoid on a coordinate grid with vertices at (1, 1), (2, 4), (5, 4), and (6, 1). She wants to draw another trapezoid with the same center and same proportions, but a different size and in a different position. Marissa could compute the side lengths of the original trapezoid and the new trapezoid, and then move it to its new location and read the coordinates of the vertices, but that would be very time consuming. How can she quickly determine the coordinates of the vertices of the second trapezoid?

In this lesson, you will learn how to use matrices to perform geometric transformations.

Objective

- Transform geometric figures using matrix operations

Key Words

- center of rotation
- dilation
- image
- preimage
- rotation

Tip: You will have two days to complete this lesson.

Transforming Figures Using Matrices

Before you begin to perform geometric transformations using matrices, review three types of transformations: rotations, reflections, and dilations. Watch the "Transformation" BrainPOP® movie to review these types, using the Geometric Transformations graphic organizer to take notes as you watch. You will add to the

graphic organizer throughout this lesson.

Click on the link below to access the Geometric Transformations graphic organizer.

Geometric Transformations

Click on the link below to watch the "Transformation" BrainPOP® movie.

Transformation

After viewing the movie, answer the following questions:

1. What is a translation? In terms of what two things can you define a translation?
2. What is a rotation? In terms of what two things can you define a rotation?
3. What is a reflection? In terms of what thing can you define a reflection?

Click on the Show Answer button below to check your answers.

Show Answer

Answers:

1. A translation is a movement from one place to another. You can define a translation in terms of distance and direction.
2. A rotation is a turn around a point. You can define a rotation in terms of the center point and the rotation angle.
3. A reflection is a flip over a line. You can define a rotation in terms of the mirror line, or axis.

Click on the link below to watch the "Dilations" Teachlet® tutorial. Take notes as you watch the tutorial. Pay close attention to the definitions of dilation and similar figures.

After you complete the tutorial, answer the following questions:

1. How does dilating a figure affect its dimensions?
2. How does dilating a figure affect its proportions?

Dilations

Click on the link below to access the Dilations Transcript.

Dilations Transcript

Click on the Show Answer button below to check your answers.

Show Answer

Answers:

1. Dilating a figure makes its dimensions larger or smaller.

2. Dilating a figure does not change its proportions.

Each of the transformations you have seen can be performed using matrices. Start by representing the coordinates of the preimage, or original figure, using a two-row matrix in which the *x*-coordinates of the vertices are written in the first row and the corresponding *y*-coordinates are written in the second row. Then, write a matrix equation to represent the transformation, in which the solution is a matrix that gives the coordinates of the image, or transformed figure.

- To translate a figure, add the preimage matrix to a transformation matrix in which each element in a single row has the same value.
- To dilate a figure, perform scalar multiplication on the preimage matrix.
- To rotate a figure, multiply the preimage matrix by a rotation matrix.
- To reflect a figure, multiply the preimage matrix by a reflection matrix.

Click on the links below to complete problems 1–4 from the PowerAlgebra website. Each problem below includes step-by-step instructions. You will see examples of geometric transformations using matrices. Use your graphic organizer to take notes as you complete the work. In the Transformation Matrix cell for dilations, note that dilations use a scale factor instead of a matrix. Be sure to include the four rotation matrices and four reflection matrices provided in the instruction.

Problem 1

Problem 2

Problem 3

Problem 4

Complete the following activities.

1. Click on the link below to complete the Dynamic Activity for Chapter 12, Lesson 5 from the PowerAlgebra website. You will use sliders to translate or dilate points, line segments, and triangles that are represented by matrices. You will also explore the matrix equations that represent the transformations.

   Dynamic Activity

2. Click on the link below to access and complete the 12-5 Think About a Plan worksheet. You will identify the rotation matrices that can be used to perform specific rotations of a figure.

   12-5 Think About a Plan

3. Complete problems 9, 13, 15, 17, 21, 23, 27, 31, and 37 on pp. 798–799 in *Algebra 2*.
4. Continue working on the unit portfolio and participating in the unit discussion.

Tip: Please return to Unit 5, Lesson 1, page 5 to access the discussion link in order to add your comments.

Extension: Click on the link below to watch the "Computer Coordinates: Playing with Pixels" Discovery Education™ *streaming* movie. Look for how computers store graphics information, and how to use geometrical transformations to map information from a Cartesian grid to a computer graphic.

After viewing the movie, answer the following questions:

1. What is the position of the point (15, 20) on a computer coordinate grid, relative to the vertical and horizontal axes?
2. Write the matrix equation you would use to map a point on the Cartesian grid to a computer coordinate grid that is 1,024 pixels wide and 768 pixels high?

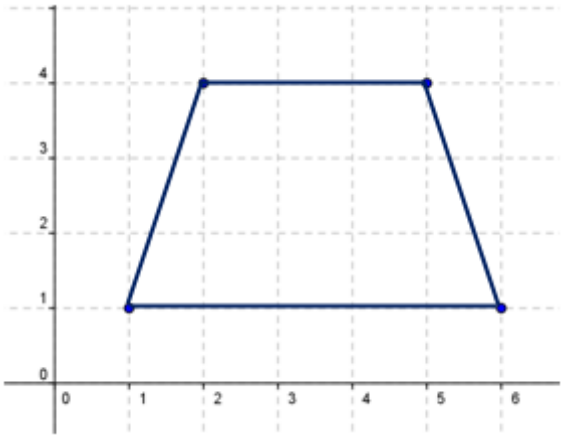
Computer Coordinates: Playing with Pixels

Click on the link below to access the online textbook.

Algebra 2

Complete the following review activities.

1. In this lesson, you learned how to use matrices to perform geometric transformations. Think back to the problem presented on the Getting Started page. Use matrices to find the vertices of Marissa's trapezoid after it has been dilated by a scale factor of 1.5 and translated 2 units up and 1 unit to the left.



2. Practice concepts from this lesson. Click on the link below to complete the Self-Assessment 12-5 activity from the PowerAlgebra website.

   Self-Assessment 12-5

3. In your math writing journal, complete problem 35 on p. 799 in *Algebra 2*. Name the entry "Geometric Transformations."

Click on the link below to access the online textbook.

Algebra 2

Lesson Answers

Click on the link below to check your answers to the 12-5 Think About a Plan worksheet.

12-5 Think About a Plan Answers

Click on the link below to check your answers to the "Computer Coordinates: Playing with Pixels" Discovery Education™ *streaming* movie.

Answers

Click on the link below to check your answer to question 1 on the Review page.

Review: Question 1 Answer

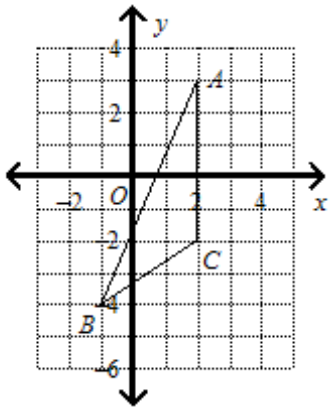
Geometric Transformations

Multiple Choice

1.

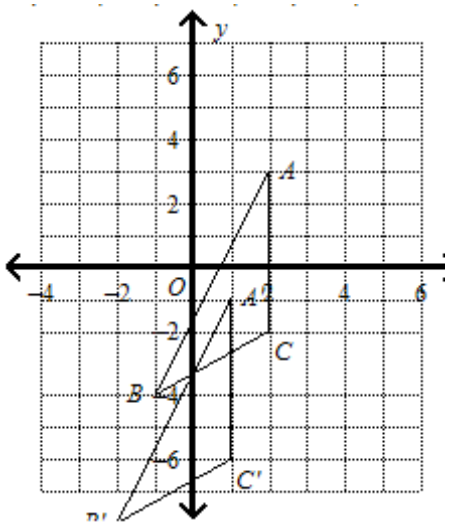
The points represent the vertices of a polygon. Use a matrix to find the coordinates of the image after the given transformation. Graph the preimage and the image.

$A(2, 3)$, $B(-1, -4)$, and $C(2, -2)$; a translation 1 unit left and 4 units down.

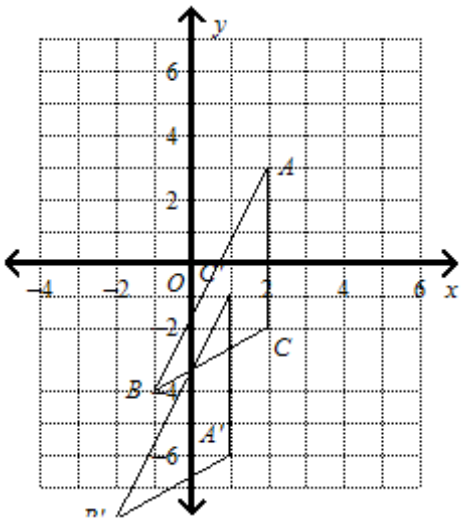


(1 point)

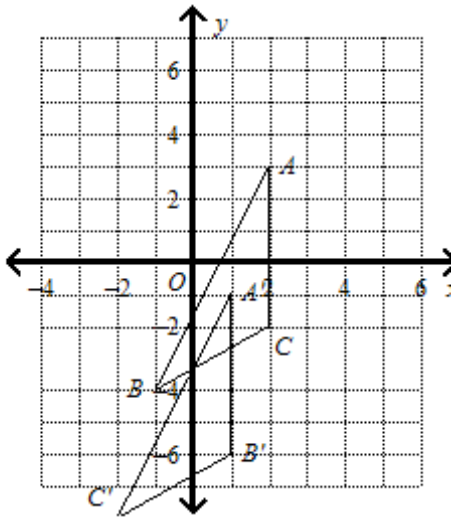
○ $A'(1, -1)$, $B'(-2, -8)$, $C'(1, -6)$



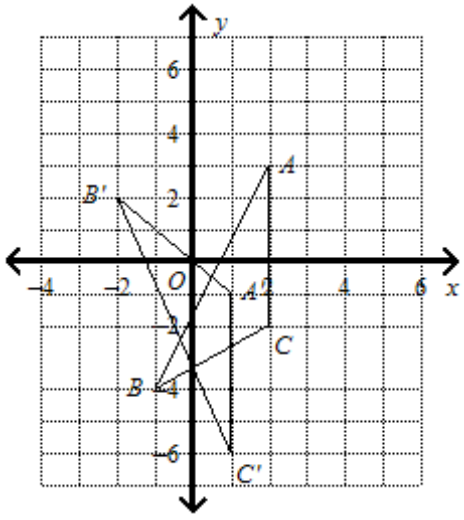
○ $A'(1, -6)$, $B'(-2, -8)$, $C'(1, -1)$



○ $A'(1, -1)$, $B'(1, -6)$, $C'(-2, -8)$



○ $A'(1, -1)$, $B'(-2, 2)$, $C'(1, -6)$



2.

A graphics designer has a star-shaped figure with vertices whose coordinates are represented by the matrix below. Her customer wants the figure increased in size by a factor of 1.5. Find the coordinates of the vertices of the enlargement.

$$\begin{bmatrix} 0 & 5 & -5 & -3 & 3 \\ 4 & 0 & 0 & -4 & -4 \end{bmatrix}$$

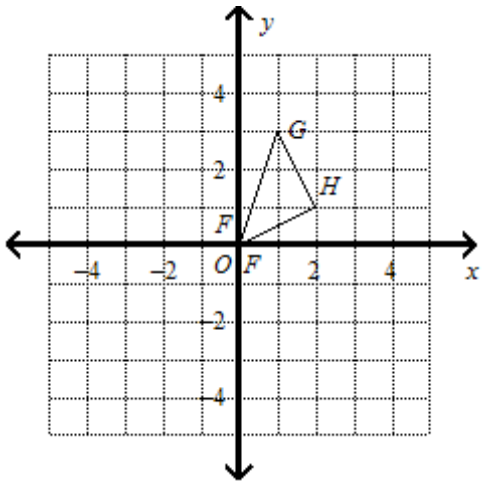
(1 point)

- ○ $\begin{bmatrix} 0 & 7.5 & -7.5 & -4.5 & 4.5 \\ 6 & 0 & 0 & -6 & -6 \end{bmatrix}$
- ○ $\begin{bmatrix} 0 & 7.5 & -7.5 & -4.5 & 4.5 \\ -6 & 0 & 0 & 6 & 6 \end{bmatrix}$
- ○ $\begin{bmatrix} 0 & 7.5 & -7.5 & 4.5 & -4.5 \\ 6 & 0 & 0 & -6 & -6 \end{bmatrix}$
- ○ $\begin{bmatrix} 0 & 6.5 & -3.5 & 4.5 & -1.5 \\ 5.5 & 0 & 0 & -2.5 & -2.5 \end{bmatrix}$

3.

Rotate the triangle with the given vertices by the indicated amount. What are the vertices of the image? Graph the preimage and the image in the same coordinate plane.

$F(0, 0)$, $G(1, 3)$, and $H(2, 1)$; a rotation of 180º



(1 point)

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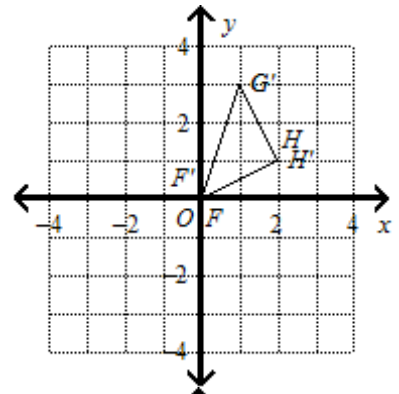
y
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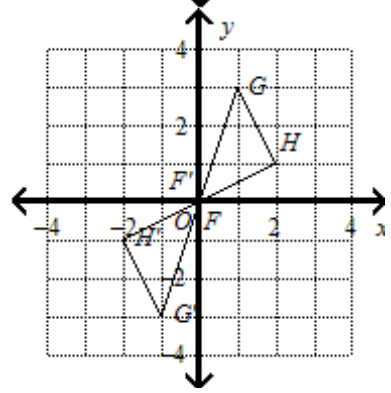
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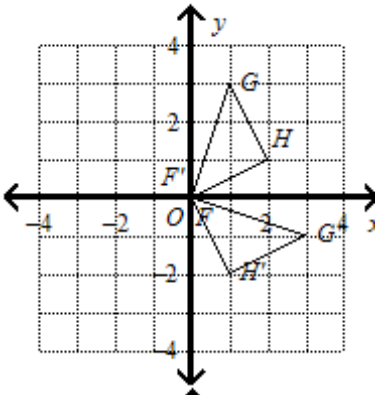
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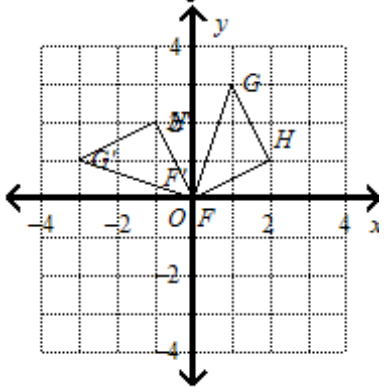
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F'

4.

Find the coordinates of the image after a reflection in the given line.

$\begin{bmatrix} -8 & 1 & -7 \\ -7 & -5 & 1 \end{bmatrix}$; y-axis

(1 point)

- ○ $\begin{bmatrix} -7 & -5 & 1 \\ -8 & 1 & -7 \end{bmatrix}$
- ○ $\begin{bmatrix} -8 & 1 & -7 \\ 1 & -5 & -7 \end{bmatrix}$
- ○ $\begin{bmatrix} 8 & -1 & 7 \\ -7 & -5 & 1 \end{bmatrix}$
- ○ $\begin{bmatrix} 7 & -1 & 8 \\ -7 & -5 & 1 \end{bmatrix}$

Answers

1. the 16th pixel to the right of the vertical axis and the 21st point below the horizontal axis
2. $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x-1 \\ y+768 \end{bmatrix}=\begin{bmatrix} x^1 \\ y^1 \end{bmatrix}$

Answer:
Answers may vary, but should be similar to:

Multiply the preimage matrix by the scale factor, and add the translation matrix.

$$1.5\begin{bmatrix} 1 & 2 & 5 & 6 \\ 1 & 4 & 4 & 1 \end{bmatrix}+\begin{bmatrix} -1 & -1 & -1 & -1 \\ 2 & 2 & 2 & 2 \end{bmatrix}=\begin{bmatrix} 1.5 & 3 & 7.5 & 9 \\ 1.5 & 6 & 6 & 1.5 \end{bmatrix}+\begin{bmatrix} -1 & -1 & -1 & -1 \\ 2 & 2 & 2 & 2 \end{bmatrix}=\begin{bmatrix} 0.5 & \\ 3.5 & \end{bmatrix}$$