Lesson 6: Vectors
Algebra 2 B Unit 5: Matrices

Objective: Use basic vector operations and the dot product

Materials: *Course Materials are not available as of this time as this User has not been assigned to any Courses. Please check back once the User has been placed into a Course.*

Note: This lesson should take 2 days.

Resultant Forces

While on a hike, Zoey and Darius came across a large boulder blocking their path. They tried to move the boulder by pushing on it. Because of their positions, they pushed the rock in slightly different directions. They each pushed as hard as they could, but the forces they applied were not equal. In what direction will the boulder move?

Since the forces that Zoey and Darius apply have both magnitude and direction, they can be represented by vectors. The direction in which the boulder ultimately moves, and the amount of force with which it moves, will be the sum of the forces that Zoey and Darius apply. In this lesson, you will learn how to represent vectors, as well as perform operations with vectors, including scalar multiplication, addition, and the dot product.

Objective

- Use basic vector operations and the dot product

Key Words

- dot product
- initial point
- magnitude
- normal vectors
- terminal point
- vector

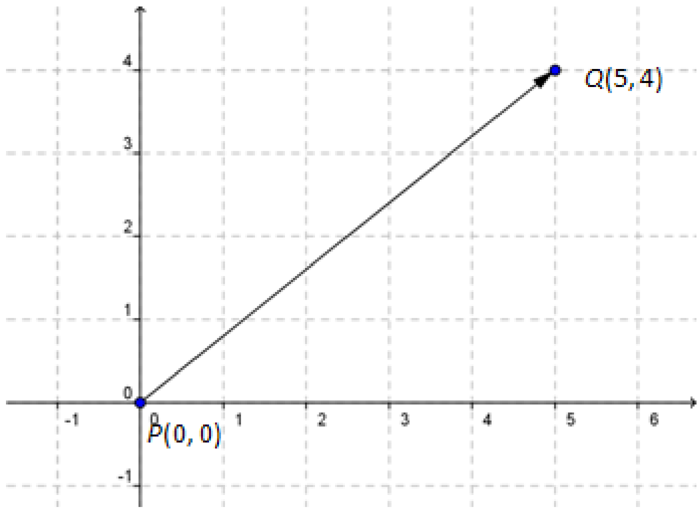
Tip: You will have two days to complete this lesson.

Drawing and Using Vectors

A vector is a quantity with magnitude, or size, and direction. If a vector is represented with an arrow, then the length of the arrow represents its magnitude, and the path from its initial point, or starting point, to its terminal point, or ending point, represents its direction.

There are different ways to notate vectors. Vectors are often notated with a non-italicized, bolded, lower-case letter, such as **v**. The magnitude of v can be shown as ||**v**||, or |**v**|. To find the magnitude of a vector, use the Distance Formula to find the distance between its initial and terminal points.

The vector v below has initial point *P*(0, 0) and terminal point *Q*(5, 4).



Use the Distance Formula to find the magnitude of the vector **v** = $\overline{PQ}$.

Let $(x_1, y_1) = (0, 0)$ and let $(x_2, y_2) = (5, 4)$.

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(5-0)^2 + (4-0)^2} \\ &= \sqrt{5^2 + 4^2} \\ &= \sqrt{25+16} \\ &= \sqrt{41} \end{aligned}$$

Click on the link below to watch the "Introduction: Sports and Vectors" Discovery Education™ *streaming* movie. Look for an illustration of the definition of a vector. Note that this movie refers to the initial point as the "tail" and the terminal point as the "end."

Introduction: Sports and Vectors

You can write a vector in component form by finding the change in *x*-values and the change in *y*-values. For the vector **v** = $\overline{PQ}$ above:

(5 – 0, 4 – 0) = (5, 4)

v = $\langle 5, 4 \rangle$

Recall that a scalar is a real number factor. To perform scalar multiplication on a vector, multiply each component by the scalar. For example, for **v** = $\langle 5, 4 \rangle$

$$\begin{aligned} 3\mathbf{v} &= \mathbf{3}\langle 5, 4 \rangle \\ &= \langle 3 \cdot 5, 3 \cdot 4 \rangle \\ &= \langle 15, 12 \rangle \end{aligned}$$

Click on the link below to complete the Solve It! activity for Chapter 12, Lesson 6 from the PowerAlgebra website.

Solve It!

Click on the links below to complete problems 1–5 from the PowerAlgebra website. Each problem below includes step-by-step instructions. You will again see the definition of a vector, as well as how to represent a vector using the component form, and how to use matrices to rotate vectors. Take notes as you complete the instruction. Be sure to include in your notes how you can use dot products to identify normal, or perpendicular, vectors.

Problem 1

Problem 2

Problem 3

Problem 4

Problem 5

Click on the link below to access and complete the 12-6 Think About a Plan worksheet. You will use vectors to represent the air speed of an airplane and wind speed, and use vector operations to calculate the airplane's resultant speed.

12-6 Think About a Plan

Click on the link below to check the 12-6 Think About a Plan worksheet answers.

12-6 Think About a Plan Answers

Complete the following activities.

1. Click on the link below to watch the "Example 3: Vector Addition – Billiards" Discovery Education™ *streaming* movie. Look for an example of using vector addition to solve a problem.

   After viewing the movie, explain the relationship between the direct vector from the cue ball to the 8 ball and

the vectors of the bank shot.

Example 3: Vector Addition – Billiards

2. Click on the link below to watch the "Example 2: Scalar Multiplication – Tennis" Discovery Education™ *streaming* movie. Look for an example of scalar multiplication of vectors.

   After viewing the movie, explain why you can calculate the scalar product by multiplying the component vectors by the scalar factor.

   Example 2: Scalar Multiplication – Tennis

3. Complete problems 37, 38, 39, and 44 on p. 806 in *Algebra 2*.
4. Recall the problem presented on the Getting Started screen. Zoey and Darius are pushing on a boulder in different directions and with different forces.

   Let **z** = $\langle -1, 4 \rangle$ the force that Zoey is applying to the boulder.

   Let d = $\langle 1, 6 \rangle$represent the force that Darius is applying to the boulder.

   Find the vector that represents the magnitude and direction of the resultant force.

5. Continue working on the unit portfolio and participating in the unit discussion.

Tip: Please return to Unit 5, Lesson 1, page 5 to access the discussion link in order to add your comments.

Click on the link below to access the online textbook.

Algebra 2

Complete the following review activities.

1. Click on the link below to complete the "Adding Vectors" Gizmo to practice the concepts from today's lesson. You will find the sum of two vectors on the coordinate plane. Drag the initial point of each vector to reposition it. Drag the terminal point to rotate or scale the vector.

   Adding Vectors

2. Check your understanding of Vectors with the Lesson Check on p. 805 in *Algebra 2.*
3. In your math writing journal, complete problem 45 on p. 806 in *Algebra 2*. Name the entry "Vectors."
4. To practice concepts you have learned throughout this course, complete the Mixed Review on p. 807 in *Algebra 2*. You may refer to the lessons indicated if you require additional guidance to solve the problems.
5. At the end of this lesson is a quiz on the lessons listed below.
   - Lesson 4: Inverse Matrices and Systems
   - Lesson 5: Geometric Transformations
   - Lesson 6: Vectors

   Be sure you understand the concepts from these lessons and review vocabulary.

Tip: You may want to take the quiz on the next lesson day.

Click on the link below to access the online textbook.

Algebra 2

Lesson Answers

Click on the link below to check your answer to the question about the "Example 3: Vector Addition – Billiards" Discovery Education™ *streaming* movie.

Answer

Click on the link below to check your answer to the question about the "Example 2: Scalar Multiplication – Tennis" Discovery Education™ *streaming* movie.

Answer

Click on the link below to check your answer to question 4 from the Activity page.

Activity: Question 4 Answer

Tip: Now you will practice using WorkPad. You will use WorkPad to complete the assessment on this lesson. Select the link to access the WorkPad directions. Read the directions to understand how to use Workpad.

WorkPad

WorkPad Directions

Vectors Quiz

Multiple Choice

1.

What is the solution of the matrix equation?

$$\begin{bmatrix} 5 & -2 \\ 2 & -1 \end{bmatrix} X = \begin{bmatrix} 2 \\ -4 \end{bmatrix}$$

(1 point)

- ○ $\begin{bmatrix} 2 \\ -4 \end{bmatrix}$
- ○ $\begin{bmatrix} -10 \\ -24 \end{bmatrix}$

○ $\begin{bmatrix} -10 \\ 24 \end{bmatrix}$

○ $\begin{bmatrix} 10 \\ 24 \end{bmatrix}$

2.

Write the system $\begin{cases} 6a + 5b - 5c = 6 \\ -7a + 7b + 4c = 6 \\ -7a - 4b - 9c = -1 \end{cases}$ as a matrix equation. Then identify the coefficient matrix, the variable matrix, and the constant matrix.

(1 point)

○ $\begin{bmatrix} 6 & 5 & -5 \\ -7 & 7 & 4 \\ -7 & -4 & -9 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 6 \\ 6 \\ -1 \end{bmatrix}$

variable matrix: $\begin{bmatrix} 6 & 5 & -5 \\ -7 & 7 & 4 \\ -7 & -4 & -9 \end{bmatrix}$

constant matrix: $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$

coefficient matrix: $\begin{bmatrix} 6 \\ 6 \\ -1 \end{bmatrix}$

○

$$\begin{bmatrix} 6 & -7 & -7 \\ 5 & 7 & -4 \\ -5 & 4 & -9 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 6 \\ 6 \\ -1 \end{bmatrix}$$

coefficient matrix: $\begin{bmatrix} 6 & 5 & -5 \\ -7 & 7 & 4 \\ -7 & -4 & -9 \end{bmatrix}$

constant matrix: $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$

variable matrix: $\begin{bmatrix} 6 \\ 6 \\ -1 \end{bmatrix}$

○

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variable matrix: $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$

constant matrix: $\begin{bmatrix} 6 \\ 6 \\ -1 \end{bmatrix}$

$$\begin{bmatrix} 6 & 5 & -5 \\ -7 & 7 & 4 \\ -7 & -4 & -9 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 6 \\ 6 \\ -1 \end{bmatrix}$$

variable matrix: $\begin{bmatrix} 6 & 5 & -5 \\ -7 & 7 & 4 \\ -7 & -4 & -9 \end{bmatrix}$

coefficient matrix: $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$

constant matrix: $\begin{bmatrix} 6 \\ 6 \\ -1 \end{bmatrix}$

3.

Solve the system.

$$\begin{cases} -5x - y = 21 \\ 3x - 4y + 5z = -8 \\ -3x - z = 12 \end{cases}$$

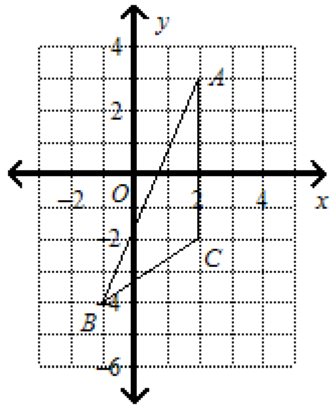
(1 point)

- (–4, 1, –1)
- (21, –8, 12)
- (4, –1, 0)
- (–4, –1, 0)

The points represent the vertices of a polygon. Use a matrix to find the coordinates of the image after the given transformation. Graph the preimage and the image.

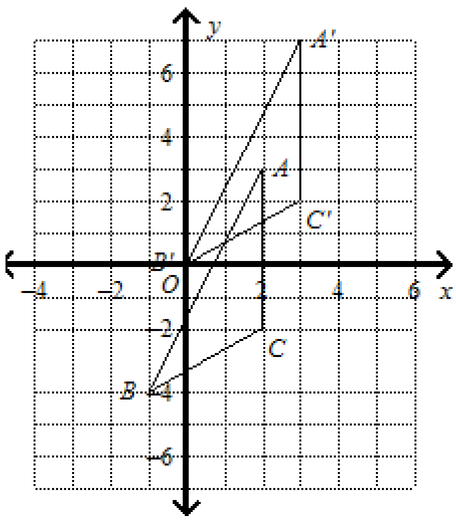
4.

A(2, 3), *B*(–1, –4), and *C*(2, –2); a translation 1 unit right and 4 units up

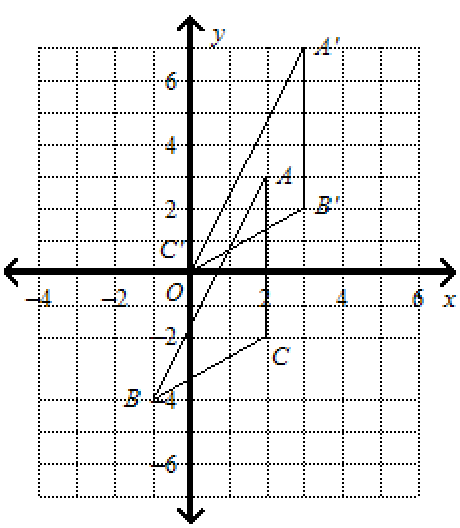


(1 point)

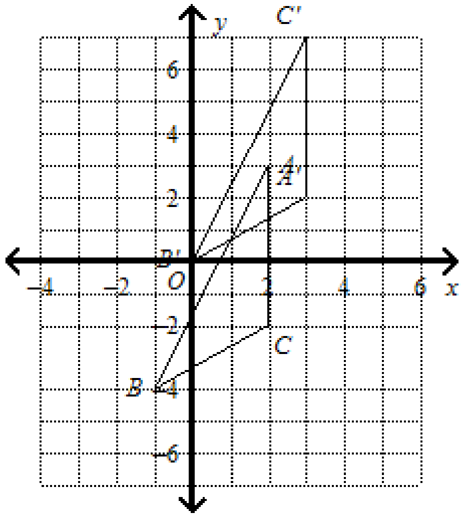
- ○ *A*'(3, 7), *B*'(0, 0), *C*'(3, 2)



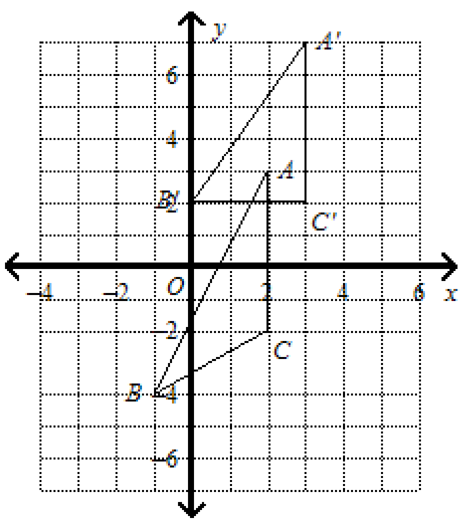
- ○ *A*'(3, 7), *B*'(3, 2), *C*'(0, 0)



- ○ *A*'(3, 2), *B*'(0, 0), *C*'(3, 7)

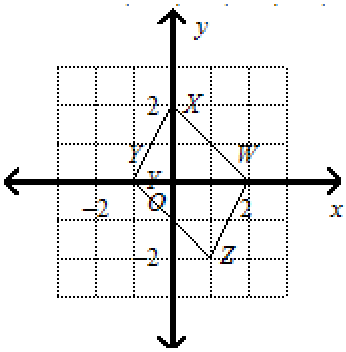


◯ $A'(3, 7)$, $B'(0, 2)$, $C'(3, 2)$



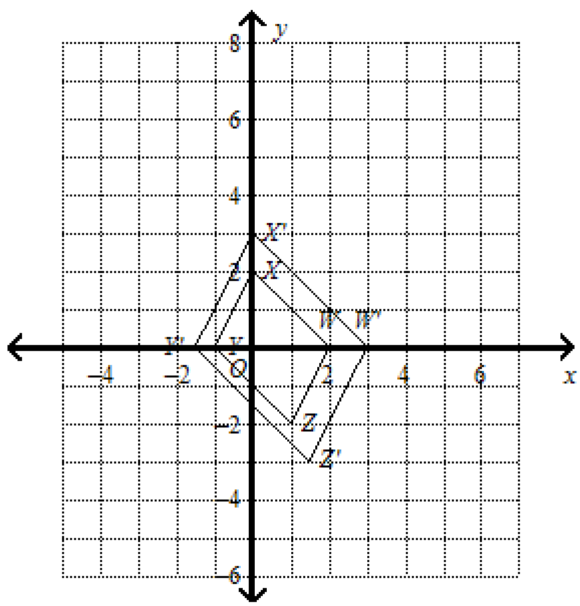
5.

W(2, 0), *X*(0, 2), *Y*(–1, 0), and Z(1, –2); a dilation of 2.5

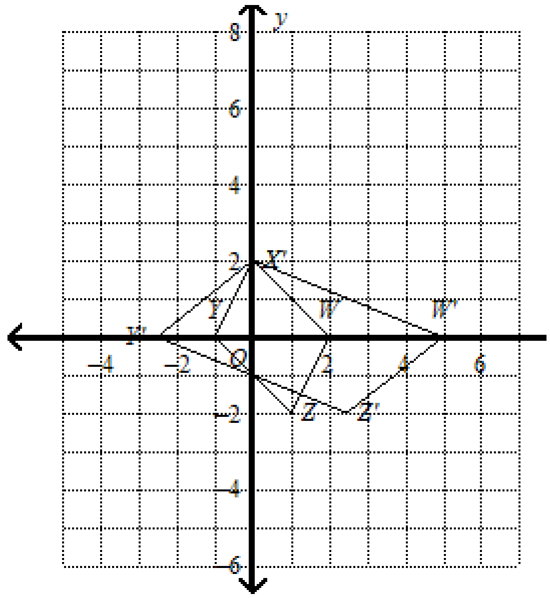


(1 point)

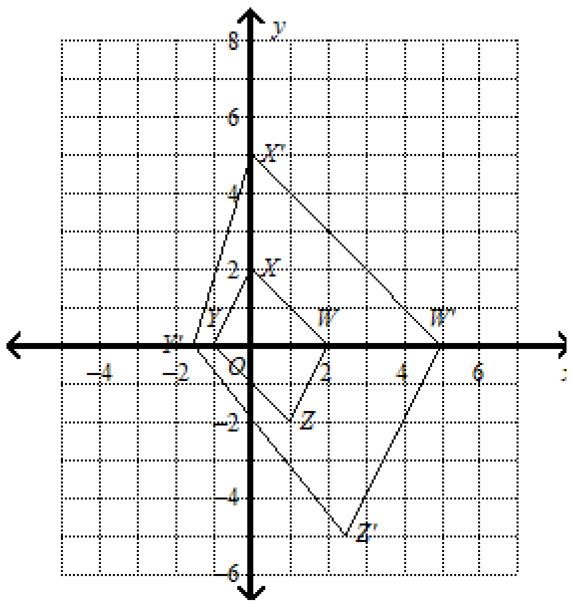
○ *W′*(3, 0), *X′*(0, 3), *Y′*(–1.5, 0), and Z'(1.5, –3)



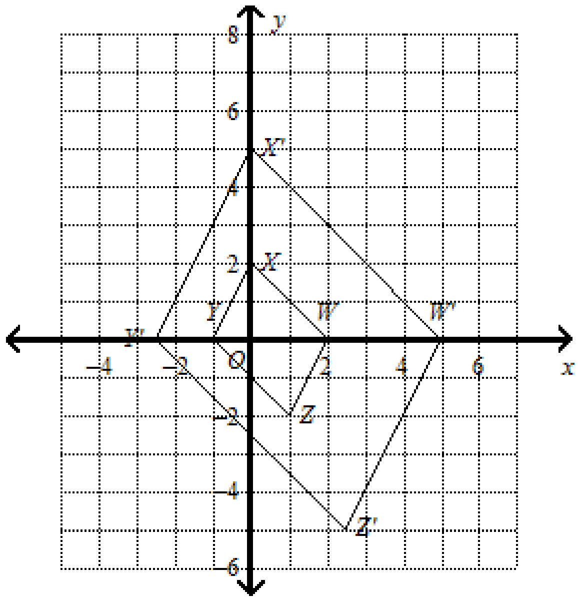
○ *W′*(2, 0), *X′*(0, 3), *Y′*(–1, 0), and Z'(2.5, –3)



○ *W'*(3, 0), *X'*(0, 5), *Y'*(–1.5, 0), and Z'(2.5, –5)



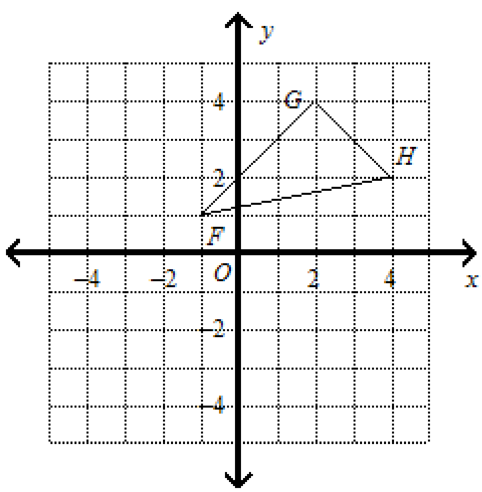
○ *W'*(5, 0), *X'*(0, 5), *Y'*(–2.5, 0), and Z'(2.5, –5)



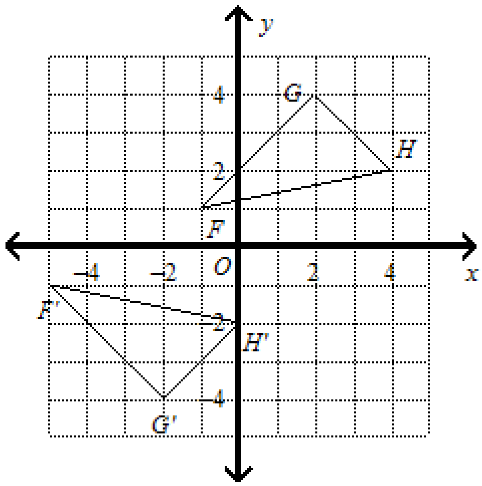
6.

Rotate the triangle with the given vertices by the indicated amount. What are the vertices of the image? Graph the preimage and the image in the same coordinate plane.

F(–1, 1), G(2, 4), and *H*(4, 2); a rotation of 180º



(1 point)

○ 
y
x
O
G
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F
F'
H'
G'
4
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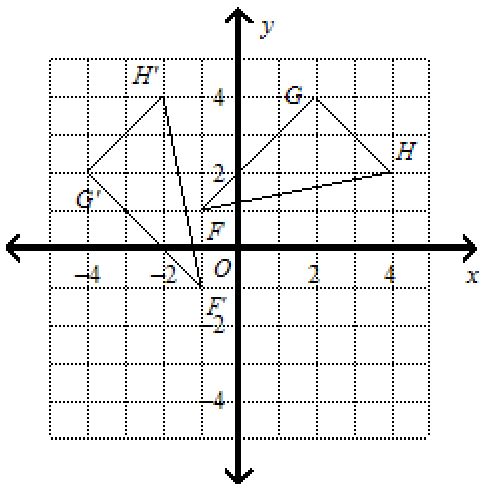
–4
–2
2
4
–2
–4

○ 
y
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2
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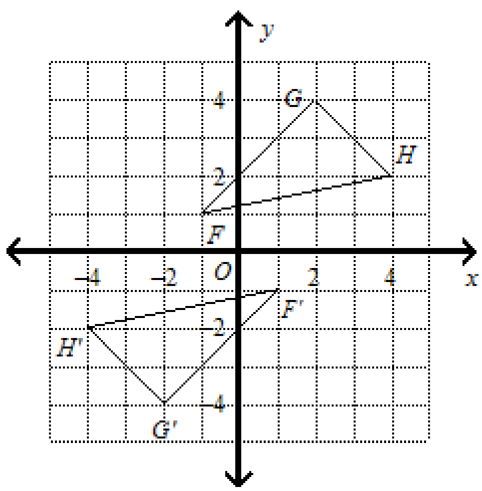
–2
–4

○ 
y
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–2
2
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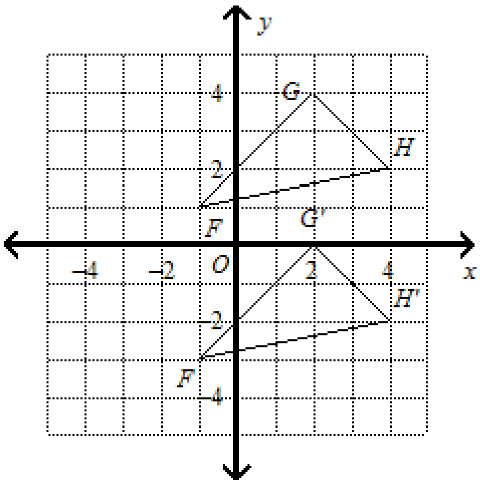
–4

○ 

7.

Find the coordinates of the image after a reflection in the given line.

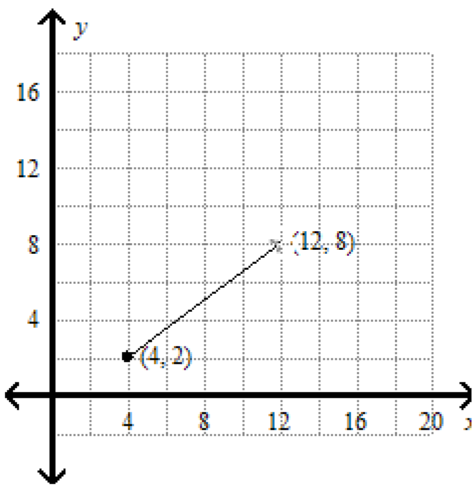
$\begin{bmatrix} 1 & 3 & -2 \\ 1 & 9 & 5 \end{bmatrix}$; $y = -x$

(1 point)

- ○ $\begin{bmatrix} -1 & -9 & -5 \\ -1 & -3 & 2 \end{bmatrix}$
- ○ $\begin{bmatrix} 1 & 3 & -2 \\ 2 & -3 & -1 \end{bmatrix}$
- ○ $\begin{bmatrix} 5 & 9 & 1 \\ 1 & 3 & -2 \end{bmatrix}$
- ○ $\begin{bmatrix} -5 & -9 & -1 \\ 2 & -3 & -1 \end{bmatrix}$

8.

What is the component form of the vector shown?



(1 point)

- ○ $\langle 6, 8\rangle$
- ○ $\langle 8, 6\rangle$
- ○ $\langle 10, 4\rangle$
- ○ $\langle 4, 10\rangle$

9.

Rotate the vector by the angle given. What is the component form of the resulting vector?

$w = \langle 3, 7\rangle$ by 180°

(1 point)

- ○ $\langle -3, -7\rangle$
- ○ $\langle 7, -3\rangle$
- ○ $\langle 3, -7\rangle$
- ○ $\langle -7, 3\rangle$

10.

Write the sum of the two vectors as an ordered pair.
$\langle 0, 6\rangle$ and $\langle 2, -2\rangle$

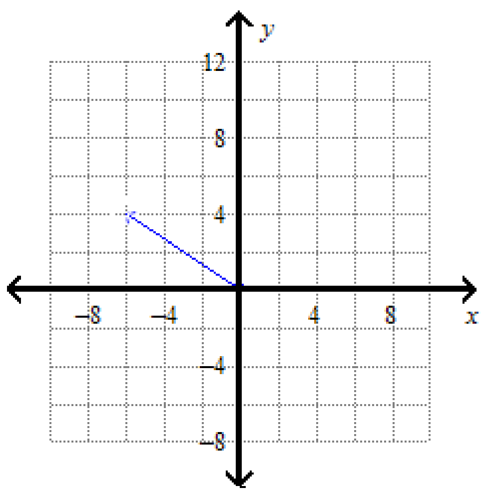
(1 point)

- ○ $\langle 2, 4\rangle$
- ○ $\langle 0, 6\rangle$
- ○ $\langle 6, 0\rangle$
- ○ $\langle 4, 2\rangle$

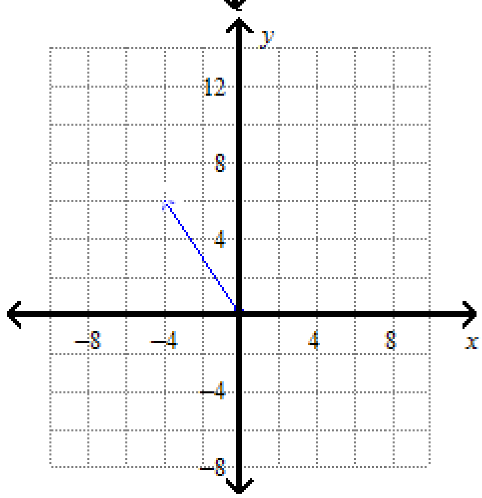
11.

For $v = \langle 2, -3\rangle$, what is the graph of $2v$?

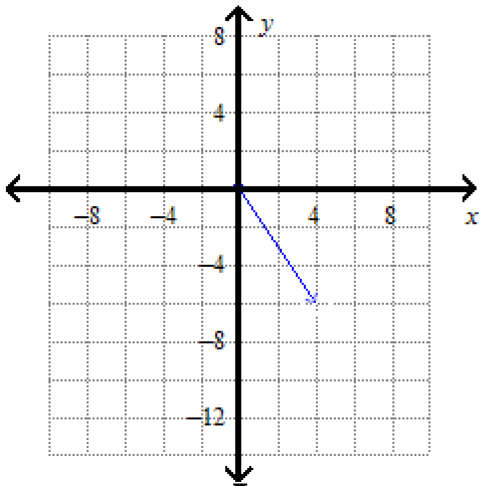
(1 point)

○ 

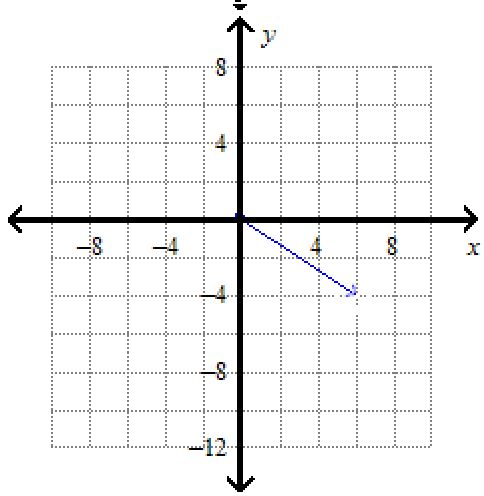
$\langle -6, 4\rangle$

○ 

$\langle -4, 6\rangle$



○ $\langle 4, -6 \rangle$



○ $\langle 6, -4 \rangle$

12.

WorkPad

Note: Remember to show all of the steps that you use to solve the problem. You can use the comments field to explain your work. Your teacher will review each step of your response to ensure you receive proper credit for your answer.

What is the solution of the system? Solve using matrices.

$$\begin{cases} x + 2y = 9 \\ x + y = 1 \end{cases}$$

(2 points)

13.

Use matrices to rotate the vector **v** = $\langle -4, 1 \rangle$= by 270º. Write the resulting vector in component form.

(2 points)

14.

Are the given vectors normal?

a = $\langle 5, -2 \rangle$ and b = $\langle 6, 15 \rangle$

(2 points)

Answer:

Answers may vary, but should be similar to:

The direct vector is the sum of the vectors of the bank shot, because the starting point of the direct vector is the starting point of one of the bank shot vectors, and the ending point of the direct vector is the ending point of the other bank shot vector.

Answer:

Answers may vary but should be similar to:

Since the sum of the component vectors is equal to the original vector, the sum of the component vectors multiplied by the scalar factor will equal the original vector multiplied by the scalar factor.

Activity: Question 4 Answer

Type your answer here. **z** + **d** = $\langle -1+1, 4+6 \rangle = \langle 0, 10 \rangle$