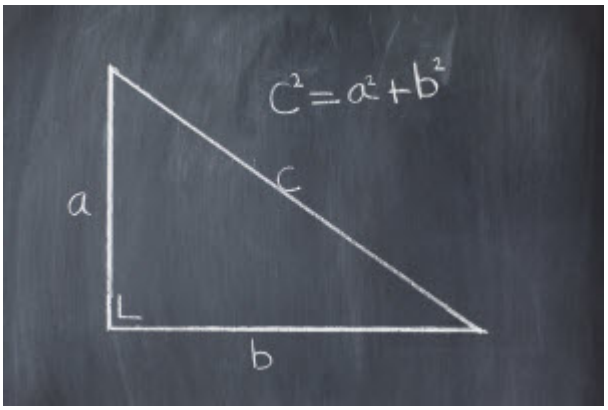
Lesson 3: Right Triangles and Trigonometric Ratios
Algebra 2 B Unit 7: Trigonometric Identities and Equations

Objectives: Find lengths of sides in a right triangle; Find measures of angles in a right triangle
Materials: *Course Materials are not available as of this time as this User has not been assigned to any Courses. Please check back once the User has been placed into a Course.*

Solving Right Triangles

You have previously used the Pythagorean Theorem to find the missing side length of a right triangle, when two side lengths are known. How can you find a missing side length of a right triangle when only one side length and at least one of the acute angles is known?



In this lesson, you will extend your understanding of trigonometric ratios beyond the unit circle, and apply them to solve right triangles.

Objectives

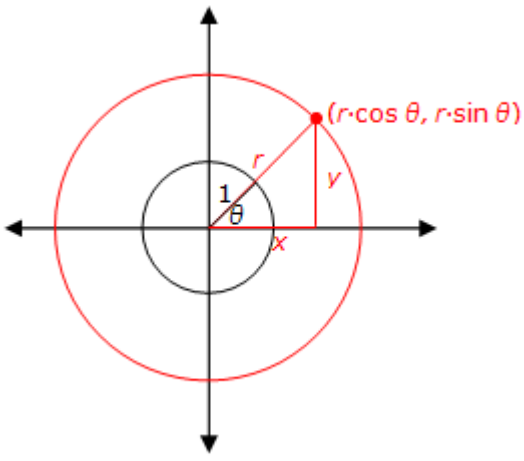
- Find lengths of sides in a right triangle
- Find measures of angles in a right triangle

Key Word

- trigonometric ratios

Right Triangle Trigonometry

You know that any point on the unit circle can be expressed as the point of intersection of the terminal arm of central angle θ with the unit circle, where $x = \cos\theta$ and $y = \sin\theta$.



In fact, the coordinates are equal to the product of the radius of the circle and the sine and cosine of the angle measure. Recall that the unit circle has a radius of 1. So, you can write the coordinates as $x = 1 \cdot \cos\theta$ and $y = 1 \cdot \sin\theta$. More generally, for the circle with radius r, the terminal arm of central angle θ will intersect the circle at the point $(x, y) = (r \cdot \cos\theta, r \cdot \sin\theta)$. You can use this point to

find the values of $\sin\theta$ and $\cos\theta$:

$$y = r \bullet \sin\theta \rightarrow \sin\theta = \frac{y}{r}$$

$$x = r \bullet \cos\theta \rightarrow \cos\theta = \frac{x}{r}$$

The other trigonometric values follow from the definitions of $\sin\theta$ and $\cos\theta$:

$$\csc\theta = \frac{r}{y} \qquad \sec\theta = \frac{r}{x}$$

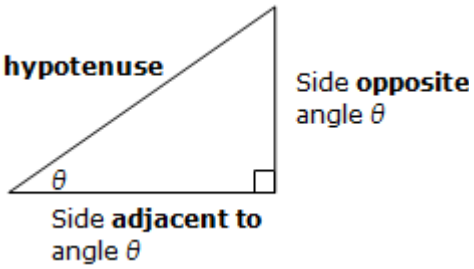
$$\tan\theta = \frac{y}{x} \qquad \cot\theta = \frac{x}{y}$$

If you restrict the angle measure to the first quadrant, then the values of x and y are positive and the values of the trigonometric functions are also positive. The trigonometric values are the trigonometric ratios of an acute angle θ of a right triangle, where the hypotenuse has length r, the side opposite angle θ has length y, and the side adjacent to angle θ has length x:

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \csc\theta = \frac{\text{hypotenuse}}{\text{opposite}}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} \qquad \cot\theta = \frac{\text{adjacent}}{\text{opposite}}$$



Tip: You can use a mnemonic to help you remember which sides of the triangle to use in each trigonometric ratio. Remember the word "SOHCAHTOA":

S – sine
O – opposite
H – hypotenuse
$\sin = \frac{\text{opposite}}{\text{hypotenuse}}$

C – cosine
A – adjacent
H – hypotenuse
$\cos = \frac{\text{adjacent}}{\text{hypotenuse}}$

T – tangent
O – opposite
A – adjacent
$\tan = \frac{\text{opposite}}{\text{adjacent}}$

Click on the link below to complete Solve It! activity for Chapter 14, Lesson 3 from the PowerAlgebra website. You will learn how to solve problems using right angle trigonometry.

Solve It!

Click on the links below to complete problems 1–6 from the PowerAlgebra website. Each problem below includes step-by-step instructions.

Problem 1

Problem 2

Problem 3

Problem 4

Problem 5

Problem 6

Click on the link below to watch the "Using Trigonometric Ratios" Discovery Education™ *streaming* movie. Look for examples of using right triangle trigonometry to solve problems. As you watch the movie, think about what parts of each triangle represent the hypotenuse, the side opposite the angle, and the side adjacent to the angle.

Using Trigonometric Ratios

Complete the following activities.

1. Click on the link below to watch the "Example 2: Sine, Cosine, Tangent -- Firefighting" Discovery Education™ *streaming* movie. Look for how you can use sine, cosine, and tangent ratios to solve problems.

   After viewing the movie, think of your own mnemonic sentence to help you remember "SOH CAH TOA."

   Example 2: Sine, Cosine, Tangent -- Firefighting

2. Click on the link below to complete the "Sine, Cosine, and Tangent" Gizmo to practice the concepts from today's lesson. After completing the activity, answer the assessment questions. Then click on the Check Your Answers button.

   Sine, Cosine, and Tangent

3. Click on the link below to access and complete the 14-3 Think About a Plan worksheet. You will find the length of the shadow of a tree for different angles of the sun.

   14-3 Think About a Plan

4. Continue participating in the unit discussion.

Tip: Please return to Unit 7, Lesson 1, page 4 to access the discussion link in order to add your comments.

Complete the following review activities.

1. Click on the link below to watch the "Using Trigonometric Functions in a Triangle" Discovery Education™ *streaming* movie.

   After viewing the movie, find the length of the bridge if the river were 150 feet across.

   Using Trigonometric Functions in a Triangle

2. Check your understanding of Right Triangles and Trigonometric Ratios with the Lesson Check on p. 915 in *Algebra 2*.

3. At the end of this lesson is a quiz on the lessons listed below.

   - Lesson 1: Trigonometric Identities
   - Lesson 2: Solving Trigonometric Equations Using Inverses
   - Lesson 3: Right Triangles and Trigonometric Ratios

   Be sure you understand the concepts from these lessons and review vocabulary. The activities that follow will help you review for the quiz.

4. To practice for the quiz at the end of this lesson, click on the link below to complete the Mid-Chapter Practice and Review for Chapter 14 from the PowerAlgebra website.

   Chapter 14 Mid-Chapter Practice and Review

Tip: You may choose to take the quiz on the next lesson day.

Click on the link below to access the online textbook.

Algebra 2

Lesson Answers

Click on the link below to check your answers to the 14-3 Think About a Plan worksheet.

14-3 Think About a Plan Answers

Click on the link below to check you answer to the question about the "Using Trigonometric Functions in a Triangle" Discovery Education™ *streaming* movie.

Answer

Tip: Now you will practice using WorkPad. You will use WorkPad to complete the assessment on this lesson. Select the link to access the WorkPad directions. Read the directions to understand how to use Workpad.

WorkPad

WorkPad Directions

Right Triangles and Trigonometric Ratios Quiz

1.

Simplify the trigonometric expression.

$\sec\theta\cos\theta$

(1 point)

- ○ $\tan\theta$
- ○ 1
- ○ $\cot\theta$
- ○ $\sin\theta$

2.

Use the unit circle to find the inverse function value in degrees.

$\tan^{-1}\sqrt{3}$

(1 point)

- ○ 120º
- ○ 90º
- ○ 60º
- ○ 30º

For questions 3 and 4, what values for $\theta(0 \le \theta \le 2\pi)$ satisfy the equation?

3.

$3\sin\theta = \sin\theta - 1$

(1 point)

- ○ $\frac{\pi}{3}, \frac{5\pi}{3}$
- ○ $\frac{2\pi}{3}, \frac{4\pi}{3}$
- ○ $\frac{\pi}{6}, \frac{3\pi}{6}$
- ○ $\frac{7\pi}{6}, \frac{11\pi}{6}$

4.

$\tan^2\theta = -\frac{3}{2}\sec\theta$

(1 point)

- ○ $\frac{\pi}{2}, \frac{5\pi}{2}$
- ○ $\frac{3\pi}{4}, \frac{5\pi}{4}$
- ○ $\frac{\pi}{3}, \frac{5\pi}{3}$
- ○ $\frac{2\pi}{3}, \frac{4\pi}{3}$

5.

For a standard-position angle determined by the point (x, y), what are the values of the trigonometric functions?

For the point (16, 12), find $\sin\theta$ and $\cos\theta$.

(1 point)

- ○ $\cos\theta = \frac{16}{20}$

  $\sin\theta = \frac{12}{20}$
- ○ $\cos\theta = \frac{20}{12}$

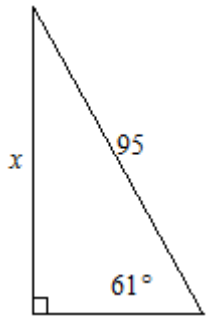
  $\sin\theta = \frac{12}{20}$
- ○ $\cos\theta = \frac{20}{16}$

  $\sin\theta = \frac{16}{12}$
- ○ $\cos\theta = \frac{16}{12}$

  $\sin\theta = \frac{16}{20}$

6.

Find the height of the triangle.



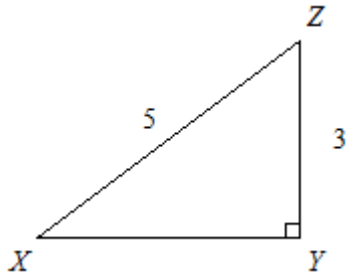
(1 point)

- ○ 83.1
- ○ 46.1
- ○ 171.4
- ○ 108.6

7.

In ΔXYZ, $\angle Y$ is a right angle and the value of sin X is given.

In ΔXYZ, $\angle Y$ is a right angle and $\sin X = \frac{3}{5}$. Find cos X, cot X, and sin Z in fraction and in decimal form. Round to the nearest hundredth, if necessary.



(1 point)

- ○ $\cos X = \frac{5}{3}$; 1.67

$\cot X = \frac{3}{4}$; 1.67

$\sin Z = \frac{4}{3}$; 0.8

○ $\cos X = \frac{5}{4}$; 1.25

$\cot X = \frac{5}{3}$; 0.8

$\sin Z = \frac{5}{4}$; 1.33

○ $\cos X = \frac{4}{5}$, 0.8

$\cot X = \frac{4}{3}$; $1.\overline{3}$

$\sin Z = \frac{4}{5}$; 0.8

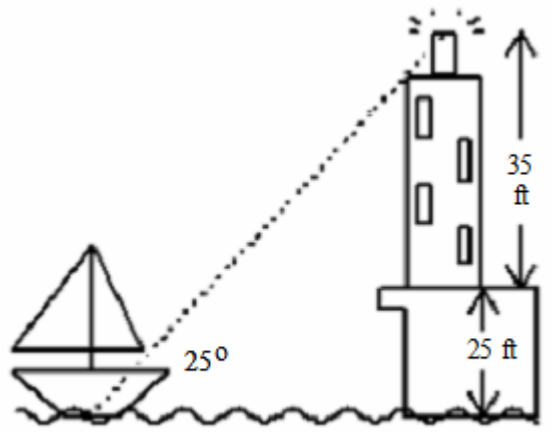
○ $\cos X = \frac{4}{3}$; $1.\overline{3}$

$\cot X = \frac{3}{4}$; 0.8

$\sin Z = \frac{5}{3}$; 1.67

8.

The line of sight from a small boat to the light at the top of a 35-foot lighthouse built on a cliff 25 feet above the water makes a 25° angle with the water. To the nearest foot, how far is the boat from the cliff?



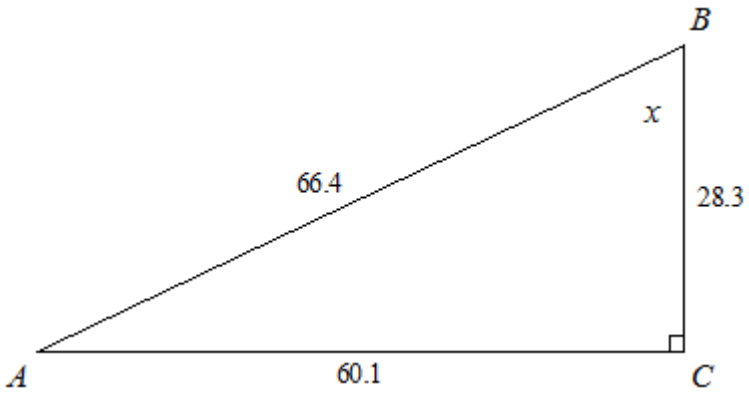
Drawing is not to scale.

(1 point)

- ◯ 141 feet
- ◯ 128 feet
- ◯ 27 feet
- ◯ 75 feet

9.

In ΔABC, $\angle C$ is a right angle, what is the measure of x?



(1 point)

- ◯ 42.1
- ◯ 64.8
- ◯ 23.1
- ◯ 25.2

10.

Work Pad

Note: For 10–12, remember to show all the steps you used to complete the problem. You can use the comments field to explain your work. Your teacher will review each step of your response to ensure you receive proper credit for your answer.

Verify the basic identity. What is the domain of validity?

$\cot\theta = \cos\theta\csc\theta$

(2 points)

11.

Verify the identity. Justify each step.

$$\tan\theta + \cot\theta = \frac{1}{\sin\theta\cos\theta}$$

(2 points)

12.

A weight is attached to a spring that is fixed to the floor. The equation $h = 7\cos\left(\frac{\pi}{3}t\right)$ models the height, *h,* in centimeters after *t* seconds of the weight being stretched and released.

a. Solve the equation fo *t*.

b. Find the times at which the weight is first at a height of 1 cm, of 3 cm, and of 5 cm above the rest position. Round your answers to the nearest hundredth.

(4 points)

13.

Martin is painting his house and needs to reach a point 9 feet above the ground. He places a 10.5-foot-long ladder against the house so that it forms an angle of θ with the ground.

a. Describe the triangle that is formed by the ground, the house, and the ladde Identify the hypotenuse, the adjacent side, and the opposite side.

b. To the nearest tenth of a degree, find the angle the ladder forms with the groun Show your work.

(2 points)

Answer:

The bridge is the hypotenuse of the right triangle, and the width of the river is the side opposite the given angle.

$$\cos\frac{\pi}{4} = \frac{150}{h}$$

$$\frac{\sqrt{2}}{2} = \frac{150}{h}$$

$$h = \frac{150}{\frac{\sqrt{2}}{2}}$$

$$= \frac{300}{\sqrt{2}}$$

$$\approx 212.13 \text{ ft}$$