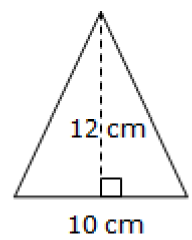
Lesson 4: Area and the Law of Sines
Algebra 2 B Unit 7: Trigonometric Identities and Equations

Objectives: Find the area of any triangle; Use the Law of Sines
Materials: *Course Materials are not available as of this time as this User has not been assigned to any Courses. Please check back once the User has been placed into a Course.*

Finding the Area of Oblique Triangles

You have previously learned that the area of a triangle is equal to one-half the product of its base and its height, where the base and height are perpendicular. How can you find the area of a triangle if you do not know its height?



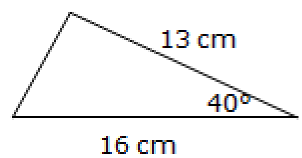
$A = \frac{1}{2}bh$

$= \frac{1}{2}(10)(12)$

$= 60\text{ cm}^2$

In this lesson, you will learn how to find the area of any triangle if you know the lengths of two sides and the measure of the included angle. You will also learn about the Law of Sines, and how you can use it to solve a triangle.

Objectives



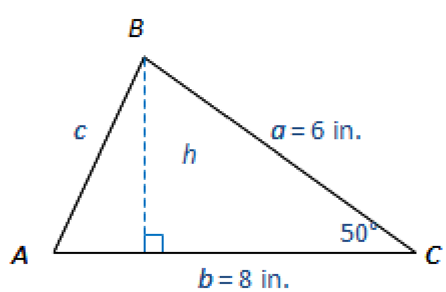
$A = ?$

- Find the area of any triangle
- Use the Law of Sines

Key Word

- Law of Sines

Applying the Sine Ratio: Area of a Triangle
Look at triangle *ABC*. You do not know its height, so you cannot calculate its area using the formula: Area = $\frac{1}{2}bh$. However, if you know the lengths of two sides and the measure of the included angle, you can use a different formula to calculate its area.



First, draw an altitude of the triangle. The altitude completes a right triangle, so $\sin A = \frac{h}{c}$, and $h = c \sin A$. Substitute for h in the formula, Area = $\frac{1}{2}bh$, to get Area = $\frac{1}{2}bc \sin A$. You can derive the formulas Area = $\frac{1}{2}ac \sin B$ and Area = $\frac{1}{2}ab \sin C$ in a similar way.

Given $a = 6$ in., $b = 8$ in., and $m\angle C = 50°$. What is the area of the triangle?

Area = $\frac{1}{2}\ ab \sin C$

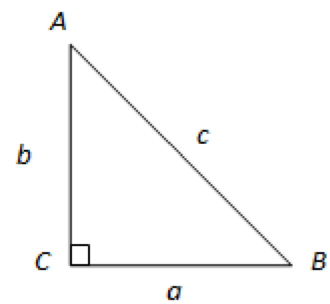
$= \frac{1}{2}(6)(8) \sin 50°$

$= 24 \sin 50°$

≈ 18.39 in.^{2}

Law of Sines

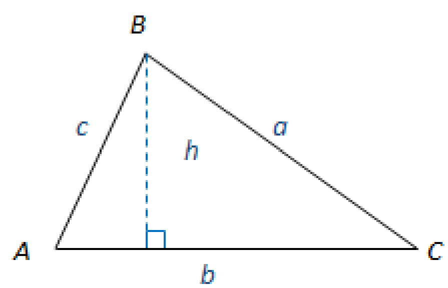
Recall that the area of a right triangle is equal to one-half the product of the lengths of its legs. Look at the triangle in more general terms. You know the measure of angle C is 90°, and sin 90° = 1. So, the expression $\frac{1}{2}ab \sin C$ becomes $\frac{1}{2}ab \sin 90° = \frac{1}{2}ab(1) = \frac{1}{2}ab$, which is the same area formula you are familiar with.



Look at oblique triangle ABC. As you have seen, altitude h is the side opposite angle A, so $\sin A = \frac{h}{c}$ and $h = c \sin A$. Notice that altitude h is also the side opposite angle C, so $\sin C = \frac{h}{a}$ and $h = a \sin C$. Therefore, $c \sin A = a \sin C$. Divide both sides by ac, and $\frac{\sin A}{a} = \frac{\sin C}{c}$. You can follow the same procedure to derive the proportions $\frac{\sin A}{a} = \frac{\sin B}{b}$ and $\frac{\sin B}{b} = \frac{\sin C}{c}$. Combining these proportions gives the Law of Sines: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.

Click on the link below to watch the "The Law of Sines" Discovery Education™ *streaming* movie.

After viewing the movie, explain why $a \sin C = c \sin A$ in the triangle shown.



The Law of Sines

Click on the Show Answer button below to check your answer.

Show Answer

Answer:

Answers may vary but should be similar to:

When triangle *ABC* is divided into two right triangles, they share an altitude. In one triangle, the altitude is equal to $a \sin C$, and in the other, it is equal to $c \sin A$. Since the altitudes are the same line, $a \sin C = c \sin A$.

Law of Sines (continued)

You can use the Law of Sines to find missing measure of a triangle if you know the measures of any two angles and any one side, or two sides and an obtuse angle opposite them.

Click on the link below to watch the "Applying the Law of Sines" Discovery Education™ *streaming* movie. Look for how to apply the Law of Sines to solve a triangle.

After viewing the movie, answer the following questions:

1. Why can side a not be less than $b \sin A$?
2. What does it mean if a is equal to $b \sin A$?
3. How many measures are possible for angle B if a is greater than $b \sin A$?

Applying the Law of Sines

Click on the Show Answer button below to check your answers.

Show Answer

Answers:

Answers may vary but should be similar to:

1. If side a is less than $b \sin A$, the ratio $\frac{b \sin A}{a}$ will be greater than 1, and there is no angle measure for which $\sin B > 1$.
2. If a is equal to $b \sin A$, angle B is a right angle.
3. If a is greater than $b \sin A$, there are two possible, supplementary, measures for angle B.

Click on the link below to complete Solve It! activity for Chapter 14, Lesson 4 from the PowerAlgebra website. You will find the area of triangles and use the Law of Sines to solve triangles.

Solve It!

Now, click on the links below to complete problems 1–4 from the PowerAlgebra website. Each problem below includes step-by-step instructions.

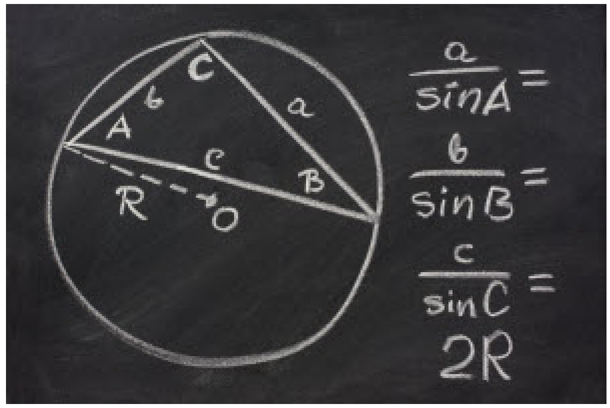
Problem 1

Problem 2

Problem 3

Problem 4

Complete the following activities.



1. Read and take notes on pp. 920–923 in *Algebra 2*. Be sure to include in your notes the formula for the area of a triangle and the Law of Sines.
2. Click on the link below to access and complete the 14-4 Think About a Plan worksheet. You will apply the Law of Sines to find the measure of the largest angle of a triangle.

   14-4 Think About a Plan

3. Complete problems 16, 18, and 23 on pp. 924–925 in *Algebra 2*.

Click on the link below to access the online textbook.

Algebra 2

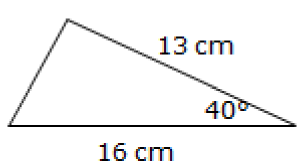
Complete the following review activities.

1. Click on the link below to complete the Self-Assessment 14-4 activity from the PowerAlgebra website.

   Self-Assessment 14-4

2. To practice concepts you have learned throughout this course, complete the Mixed Review on p. 926 in *Algebra 2*. You may refer to the lessons indicated if you require additional guidance to solve the problems.

3. Find the area of the triangle that was presented on the Getting Started page. Round your answer to the nearest hundredth.



Click on the link below to access the online textbook.

Algebra 2

Lesson Answers

Click on the link below to check your answers to the 14-4 Think About a Plan worksheet.

14-4 Think About a Plan Answers

Click on the link below to check your answer to question 3 from the Review page.

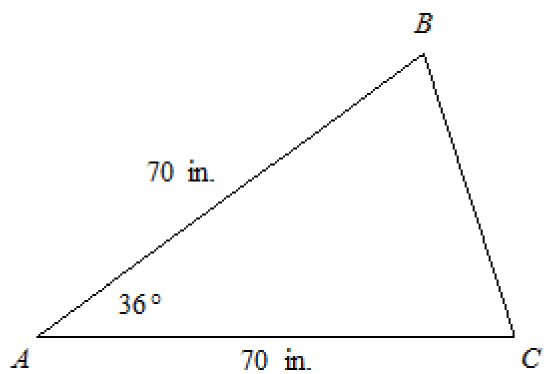
Review: Question 3 Answer

Area and the Law of Sines

Multiple Choice

1.

What is the area of ΔABC to the nearest tenth of a square inch?



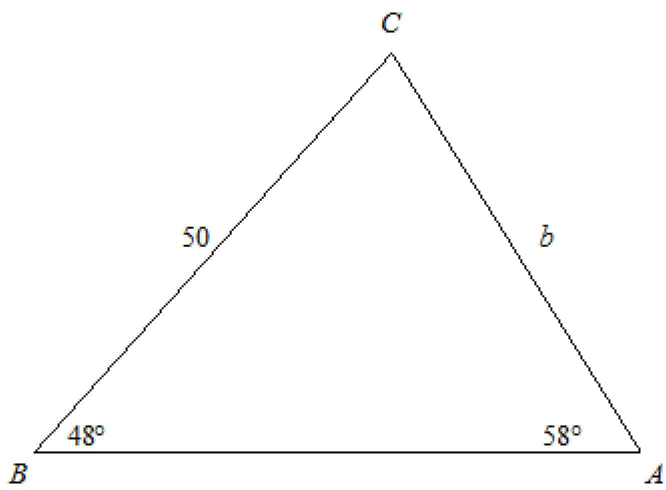
(1 point)

- ○ 2,450 $in.^2$
- ○ 1,780 $in.^2$
- ○ 1,440.1 $in.^2$
- ○ 2,880.1 $in.^2$

2.

Use the Law of Sines to find the missing side of the triangle.

Find *b*.



(1 point)

- 70.1
- 43.8
- 57.1
- 31.5

3.

For a triangle *ABC*, find the measure of $\overline{AB}$ given $m\angle A = 55°$, $m\angle B = 44°$, and $b = 68$.

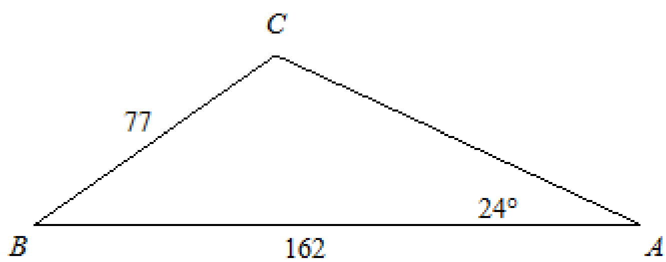
(1 point)

- 45.22
- 96.68
- 88.19
- 81.12

4.

Use the Law of Sines to find the missing angle of the triangle.

Find $m\angle C$ to the nearest tenth.

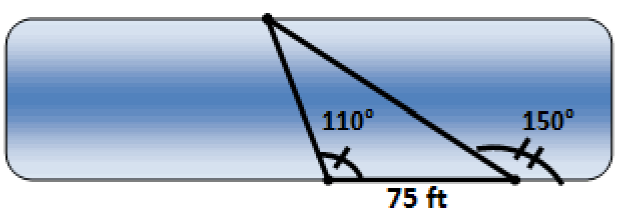


(1 point)

- 11.1º
- 58.8º
- 121.2º
- 168.9º

5.

A surveyor sights the far bank of a river at an angle of 110° to the near bank. She then moves 75 feet upriver and sights the same point on the far bank of the river at an angle of 150°. What is the shortest distance across the river?



(1 point)

- 54.82 ft
- 58.34 ft
- 94.95 ft
- 96.42 ft

Review: Question 3 Answer

$$A = \frac{1}{2}\, bc \sin A$$

$$= \frac{1}{2}(13)(16) \sin 40°$$

$= 104 \sin 40°$

$\approx 66.85 \text{ cm}^2$