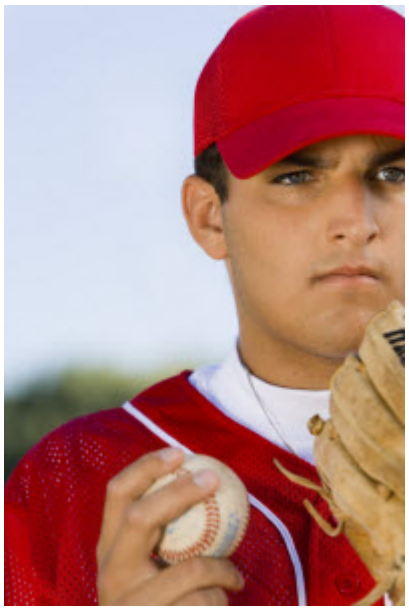
Lesson 5: The Law of Cosines
Algebra 2 B Unit 7: Trigonometric Identities and Equations

Objective: Use the Law of Cosines in finding the measures of sides and angles of a triangle
Materials: *Course Materials are not available as of this time as this User has not been assigned to any Courses. Please check back once the User has been placed into a Course.*

Finding Distances

Dylan is the pitcher for his school's baseball team. The pitcher's mound is 60 feet, 6 inches from home plate. Home plate is 90 feet from first base. The distance from the pitcher's mound to home plate forms a 45° angle with the distance from home plate to first base. If Dylan throws from the pitcher's mound to first base, how far must he throw the ball?



In this lesson, you will learn how to apply the Law of Cosines to solve triangles when you do not have enough information to apply the Law of Sines.

Objective

- Use the Law of Cosines in finding the measures of sides and angles of a triangle

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Key Word

- Law of Cosines

The Law of Cosines

There are two theorems that you can use to solve an oblique triangle:

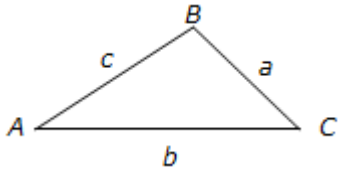
- If you know the measures of two angles of a triangle and any side, or the measures of two sides and the obtuse angle opposite one of the sides, you can use the Law of Sines to solve the triangle.

- If you know the measures of all three sides, or the measures of two sides and the included angle, you can use the Law of Cosines:

$a^2 = b^2 + c^2 - 2bc \cos A$

$b^2 = a^2 + c^2 - 2ac \cos B$

$c^2 = a^2 + b^2 - 2ab \cos C$



Click on the link below to watch the "Applying the Law of Cosines" Discovery Education™ *streaming* movie.

After viewing the movie, answer the following questions:

1. What information do you need to apply the Law of Cosines? What information can you obtain?
2. How can you use two side lengths of a triangle and the measure of the included angle to determine whether a triangle is possible?

Applying the Law of Cosines

Click on the Show Answer button below to review your answers.

Show Answer

Answers:

1. You need to know either the lengths of two sides and the measure of the included angle, or the lengths of all three sides. You can use the lengths of two sides and the measure of the included angle to find the length of the third side, or you can use all three side lengths to calculate an angle measure.
2. If the cosine value that you calculate using the two side lengths and the measure of the included angle is less than −1 or greater than 1, then the triangle is not possible.

Click on the link below to watch the "Aircraft Navigation: Learning the Laws" Discovery Education™ *streaming* movie. Look for how to use the Law of Cosines to solve an aircraft navigation problem.

After viewing the movie, answer the following questions:

1. Why is it important to draw a diagram when using an inverse trigonometric function to find an angle measure?
2. How can you use the Law of Cosines and the Law of Sines to solve a triangle?

Aircraft Navigation: Learning the Laws

Click on the Show Answer button below to review your answers.

Show Answer

Answers:

1. Since more than one angle measure may have the same value for the trigonometric function, drawing a diagram can help you determine the angle measure in the triangle.
2. If you know the lengths of two sides and the measure of the included angle, you can use the Law of Cosines to calculate the length of the third side. Then you can use the Law of Sines to calculate the remaining angle measures.

Click on the link below to complete Solve It! for Chapter 14, Lesson 5 from the

PowerAlgebra website.

Solve It!

Now, click on the links below to complete problems 1–3 from the PowerAlgebra website. Each problem below includes step-by-step instructions. You will practice using trigonometric identities to simplify trigonometric expressions.

Problem 1

Problem 2

Problem 3

Extension: Click on the link below to access the Law of Cosines video on the Khan Academy website.

Law of Cosines

Complete the following activities.

1. Read and take notes on pp. 928–931 in *Algebra 2*. Be sure to include in your notes the three equations for the Law of Cosines and the cases in which you can apply the Law of Cosines.
2. Click on the link below to access and complete the 14-5 Think About a Plan worksheet. You will use the Law of Cosines to solve a navigation problem.

   14-5 Think About a Plan
3. Complete problems 27, 28, 34, 35, and 49 on pp. 932–933 in *Algebra 2*.
4. Think about Dylan's baseball diamond, introduced in the Getting Started portion of this lesson. The pitcher's mound is 60 feet, 6 inches from home plate. Home base is 90 feet from first base. The distance from the pitcher's mound to home plate forms a 45° angle with the distance from home plate to first base. Calculate the distance from the pitcher's mound to first base.
5. Continue participating in the unit discussion.

Tip: Please return to Unit 7, Lesson 1, page 4 to access the discussion link in order to add your comments.

Click on the link below to access the online textbook.

Algebra 2

Complete the following review activities.

1. Check your understanding of the Law of Cosines with the Lesson Check on p. 931 in *Algebra 2*.
2. In your math writing journal, complete problem 33 on p. 933.
3. At the end of this lesson is a quiz on the lessons listed below.
   - Lesson 4: The Law of Sines
   - Lesson 5: The Law of Cosines

Be sure you understand the concepts from these lessons and review the vocabulary.

Click on the link below to access the online textbook.

Algebra 2

Lesson Answers

Click on the link below to check your answers to the 14-5 Think About a Plan worksheet.

14-5 Think About a Plan Answers

Click on the link below to check your answer to question 4 from the Activity page.

Activity: Question 4 Answer

Tip: Now you will practice using WorkPad. You will use WorkPad to complete the assessment on this lesson. Select the link to access the WorkPad directions. Read the directions to understand how to use Workpad.

WorkPad

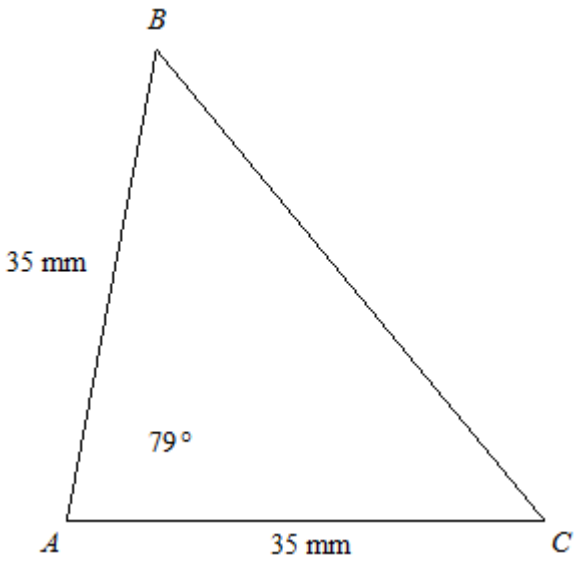
WorkPad Directions

The Law of Cosines Quiz

Multiple Choice

1.

What is the area of ΔABC to the nearest tenth of a square millimeter?



(1 point)

- ○ 3,151 mm^2
- ○ 601.2 mm^2
- ○ 612.5 mm^2
- ○ 1,202.5 mm^2

2.

Use the Law of Sines to find the missing side of the triangle.

Find the measure of b, given $m\angle A = 38°$, $m\angle B = 74°$, and $a = 31$.

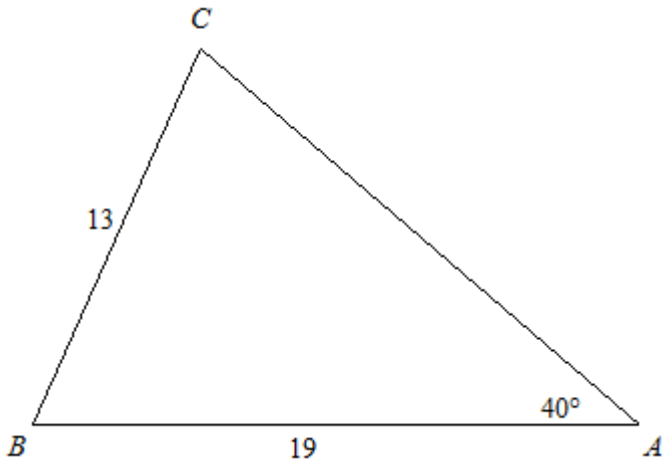
(1 point)

- ○ 19.9
- ○ 18.3
- ○ 37.8
- ○ 48.4

3.

Use the Law of Sines to find the missing angle of the triangle.

Find $m\angle B$ to the nearest tenth.



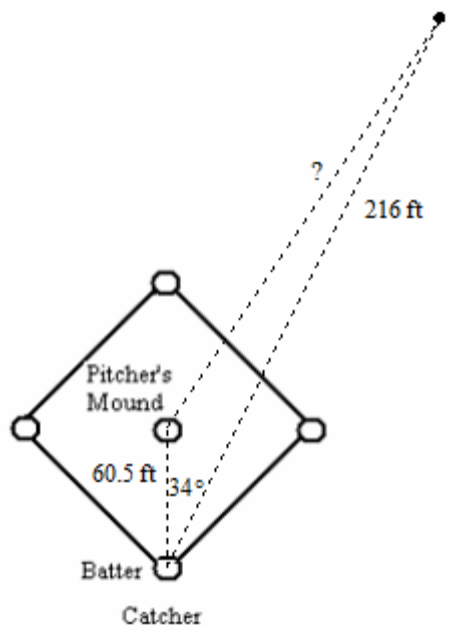
(1 point)

- ○ 110.0º
- ○ 153.9º
- ○ 26.1º
- ○ 70.0º

4.

Use the Law of Cosines to solve the problem.

On a baseball field, the pitcher's mound is 60.5 feet from home plate. During practice, a batter hits a ball 216 feet. The path of the ball makes a 34° angle with the line connecting the pitcher and the catcher, to the right of the pitcher's mound. An outfielder catches the ball and throws it to the pitcher. How far does the outfielder throw the ball?



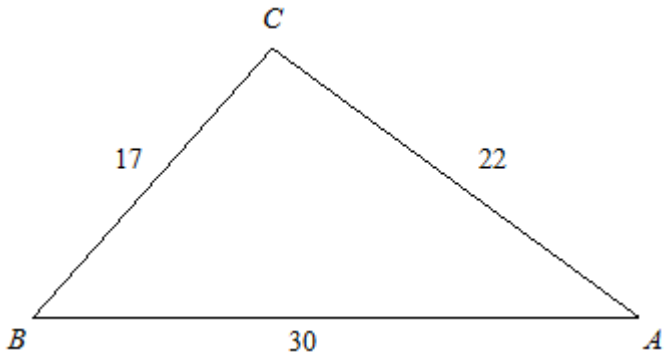
(1 point)

- ○ 207.4 ft
- ○ 224.3 ft
- ○ 169.3 ft
- ○ 198.7 ft

5.

Use the Law of Cosines to find the missing angle.

Find $m\angle A$ to the nearest tenth of a degree.



(1 point)

- ◯ 33.9º
- ◯ 57.7º
- ◯ 46.3º
- ◯ 85.7º

6.

Find $m\angle B$, given $a = 11$, $b = 12$, and $c = 17$.

(1 point)

- ◯ $m\angle B = 49.9º$
- ◯ $m\angle B = 40.1º$
- ◯ $m\angle B = 45.3º$
- ◯ $m\angle B = 44.7º$

7.

In ΔJKL, $j = 10$ in., $k = 7$ in., and $l = 6.58$ in. Find $m\angle J$.

(1 point)

- ◯ 38º
- ◯ 95º
- ◯ 44º

○ 25°

8.

In triangle STU, $s = 9$ cm, $t = 15$ cm, and $m\angle U = 37°$. Find u.

(1 point)

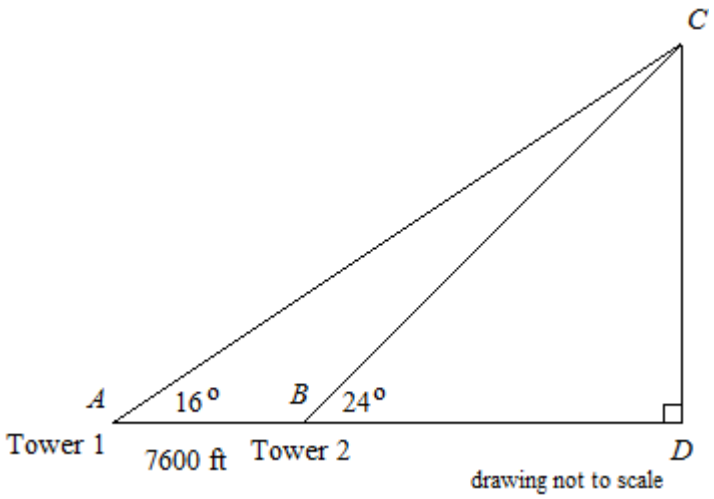
○ 7.4 cm
○ 8.6 cm
○ 9.5 cm
○ 10.3 cm

9.

Work Pad

Note: Remember to show all the steps you used to complete problems 9 and 10. You can use the comments field to explain your work. Your teacher will review each step of your response to ensure you receive proper credit for your answer.

A plane is located at C on the diagram. There are two towers located at A and B. The distance between the towers is 7,600 feet, and the angles of elevation are given.



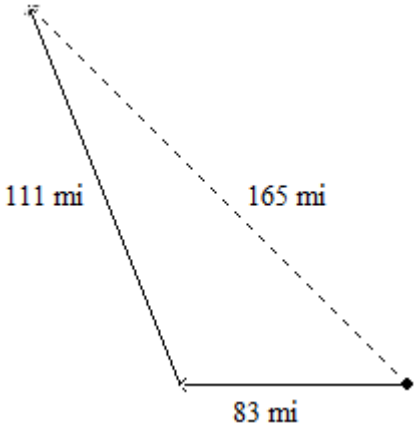
a. Find BC, the distance from Tower 2 to the plane, to the nearest foot.
b. Find CD, the height of the plane from the ground, to the nearest foot.

(3 points)

10.

Use the Law of Cosines to solve the problem.

A ship travels due west for 83 miles. It then travels in a northwest direction for 111 miles and ends up 165 miles from its original position. To the nearest tenth of a degree, how many degrees north of west did it turn when it changed direction? Show your work.



(4 points)

Activity: Question 4 Answer

Let the distance from home plate to the pitcher's mound be a .

Let the distance from home plate to first base be b .

Let the angle between the two distance be C .

$c^2 = a^2 + b^2 - 2\,ab \cos C$

$= 60.5^2 + 90^2 - 2(60.5)(90) \cos 45°$

$\approx 3{,}660.25 + 8{,}100 - 10{,}890(0.7071)$

$\approx 3{,}660.25 + 8{,}100 - 7700.319$

$\approx 4{,}059.931$

$c \approx \sqrt{4{,}059.931} \approx 63.72$ ft ≈ 63 ft 9 in.