Algebra 2
Lesson 14-1 - Practice and Problem-Solving Exercises Answers

7.
$$\cos\theta\cot\theta = \cos\theta\left(\frac{\cos\theta}{\sin\theta}\right)$$
$$= \frac{1-\sin^2\theta}{\sin\theta}$$
$$= \frac{1}{\sin\theta} - \sin\theta$$

Domain: all real numbers except multiples of π

8.
$$\sin\theta\cot\theta = \sin\theta\left(\frac{\cos\theta}{\sin\theta}\right)$$
$$= \cos\theta$$

Domain: all real numbers except multiples of π

9.
$$\cos\theta\tan\theta = \cos\theta\left(\frac{\sin\theta}{\cos\theta}\right)$$
$$= \sin\theta$$

Domain: all real numbers except odd multiples of $\frac{\pi}{2}$

10.
$$\sin\theta\sec\theta = \sin\theta\left(\frac{1}{\cos\theta}\right)$$
$$= \frac{\sin\theta}{\cos\theta}$$
$$= \tan\theta$$

Domain: all real numbers except odd multiples of $\frac{\pi}{2}$

11.
$$\cos\theta\sec\theta = \cos\theta\left(\frac{1}{\cos\theta}\right)$$
$$= 1$$

Domain: all real numbers except odd multiples of $\frac{\pi}{2}$

12.
$$\tan\theta\cot\theta = \left(\frac{\sin\theta}{\cos\theta}\right)\left(\frac{\cos\theta}{\sin\theta}\right)$$
$$= 1$$

Domain: all real numbers except multiples of $\frac{\pi}{2}$

13.
$$\sin\theta\csc\theta = \sin\theta\left(\frac{1}{\sin\theta}\right)$$
$$= \frac{\sin\theta}{\sin\theta}$$
$$= 1$$

Domain: all real numbers except multiples of π

14.
$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$
$$= \left(\frac{1}{\sin\theta}\right)\cos\theta$$
$$= \csc\theta\cos\theta$$

Domain: all real numbers except multiples of π

15.
$$\csc\theta - \sin\theta = \frac{1}{\sin\theta} - \sin\theta$$
$$= \frac{1-\sin^2\theta}{\sin\theta}$$
$$= \frac{\cos^2\theta}{\sin\theta}$$
$$= \cot\theta\cos\theta$$

Domain: all real numbers except odd multiples of $\frac{\pi}{2}$

16. 1

17. $\sin^2\theta$.

18. $\tan^2\theta$.

19. $-\cot^2\theta$

20. $\csc^2\theta$

21. $\sin\theta$

22. $\cos\theta$

23. 1

24. $\sin\theta$

25. 1

26. **1**

27. 1

28. $\csc\theta$

29. $\sec\theta$

30. 1

31. $\sec^2\theta$

32. $\sec^2\theta$

33. $\csc\theta$

34. $\tan\theta$

35. $\sin^2\theta$

36. $\sin\theta$

37. 1

38. 1

39. 1

40. 1

41. $\pm\sqrt{1-\cos^2\theta}$

42. $\dfrac{\pm\sqrt{1-\cos^2\theta}}{\cos\theta}$

43. $\dfrac{\pm\sqrt{1-\sin^2\theta}}{\sin\theta}$

44. $\pm\sqrt{1+\cot^2\theta}$

45. $\pm\sqrt{\csc^2\theta-1}$

46. $\pm\sqrt{1+\tan^2\theta}$

47.
$$\begin{aligned}\sin^2\theta\tan^2\theta &= \sin^2\theta\left(\frac{\sin^2\theta}{\cos^2\theta}\right)\\ &= \left(1-\cos^2\theta\right)\left(\frac{\sin^2\theta}{\cos^2\theta}\right)\\ &= \frac{\sin^2\theta-\sin^2\theta\cos^2\theta}{\cos^2\theta}\\ &= \frac{\sin^2\theta}{\cos^2\theta}-\frac{\sin^2\theta\cos^2\theta}{\cos^2\theta}\\ &= \tan^2\theta-\sin^2\theta\end{aligned}$$

48.
$$\begin{aligned}\sec\theta-\sin\theta\tan\theta &= \frac{1}{\cos\theta}-\sin\theta\left(\frac{\sin\theta}{\cos\theta}\right)\\ &= \frac{1}{\cos\theta}-\frac{\sin^2\theta}{\cos\theta}\\ &= \frac{1-\sin^2\theta}{\cos\theta}\\ &= \frac{\cos^2\theta}{\cos\theta}\\ &= \cos\theta\end{aligned}$$

49.
$$\begin{aligned}\sin\theta\cos\theta\left(\tan\theta+\cot\theta\right) &= \sin\theta\cos\theta\left(\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}\right)\\ &= \frac{\sin^2\theta\cos\theta}{\cos\theta}+\frac{\cos^2\theta\sin\theta}{\sin\theta}\\ &= \sin^2\theta+\cos^2\theta=1\end{aligned}$$

50.
$$\begin{aligned}\frac{1-\sin\theta}{\cos\theta} &= \frac{1-\sin\theta}{\cos\theta}\cdot\frac{\cos\theta}{\cos\theta}\\ &= \frac{\left(1-\sin\theta\right)\cos\theta}{\cos^2\theta}\\ &= \frac{\left(1-\sin\theta\right)\cos\theta}{1-\sin^2\theta}\\ &= \frac{\left(1-\sin\theta\right)\cos\theta}{\left(1-\sin\theta\right)\left(1+\sin\theta\right)}=\frac{\cos\theta}{1+\sin\theta}\end{aligned}$$

51.
$$\begin{aligned}\frac{\sec\theta}{\cot\theta+\tan\theta} &= \frac{\frac{1}{\cos\theta}}{\frac{\cos\theta}{\sin\theta}+\frac{\sin\theta}{\cos\theta}}\cdot\frac{\sin\theta\cos\theta}{\sin\theta\cos\theta}\\ &= \frac{\sin\theta}{\cos^2\theta+\sin^2\theta}\\ &= \frac{\sin\theta}{1}\\ &= \sin\theta\end{aligned}$$

52.
$$\begin{aligned}\left(\cot\theta+1\right)^2 &= \cot^2\theta+2\cot\theta+1\\ &= \cot^2\theta+1+2\cot\theta\\ &= \csc^2\theta+2\cot\theta\end{aligned}$$

53. $\dfrac{1-\sin^2\theta}{\sin^2\theta}$

54. $1 - \sin\theta$

55. Errors are in the first line and the last 2 lines: first line, incorrect cancellation of $\tan^2\theta$; last 2 lines, $\frac{\sin^2\theta}{\cos^2\theta} = \tan^2\theta$, not $\cot^2\theta$. The correct identity verification is:

$$\begin{aligned}\frac{\sec^2\theta - \tan^2\theta}{\tan^2\theta} &= \frac{\frac{1}{\cos^2\theta} - \frac{\sin^2\theta}{\cos^2\theta}}{\frac{\sin^2\theta}{\cos^2\theta}} \\ &= \frac{1-\sin^2\theta}{\cos^2\theta}\cdot\frac{\cos^2\theta}{\sin^2\theta} \\ &= \frac{1-\sin^2\theta}{\sin^2\theta} \\ &= \frac{\cos^2\theta}{\sin^2\theta} \\ &= \cot^2\theta\end{aligned}$$

56. Check students' work.

57. $(x-1)^2 - 1 = x(x-2)$ is an identity since:
$(x-1)^2 - 1 = x^2 - 2x + 1 - 1 = x^2 - 2x = x(x-2)$.
$(x-1)^2 = x(x-1)$ is not an identity since it has a unique solution:

$$\begin{aligned}(x-1)^2 &= x(x-1) \\ x^2 - 2x + 1 &= x^2 - x \\ -x &= -1 \\ x &= 1\end{aligned}$$

58a. In the unit circle, the coordinates of any point are $(\cos\phi,\ \sin\phi)$.
For P, $\phi = \theta + \pi$. Thus, the y-coordinate of point P is $\sin(\theta+\pi)$.

58b. The two right triangles are congruent by HA.

58c. corresponding parts

58d. In Quadrant I, the y-coordinate is $y = \sin\theta$. The absolute values of the y-coordinates in Quadrants I and III are equal. So, the y-coordinate of P is $-\sin\theta$.

58e. Since the y-coordinate of P is $\sin(\theta+\pi)$ and is also $-\sin\theta$,
$\sin(\theta+\pi) = -\sin\theta$.

59. $\cos(\theta+\pi) = |\cos\theta|$, but is also in Quadrant III and is negative, so $\cos(\theta+\pi) = -\cos\theta$.

60.
$$\begin{aligned}\tan(\theta+\pi) &= \frac{\sin(\theta+\pi)}{\cos(\theta+\pi)} \\ &= \frac{-\sin\theta}{-\cos\theta} \\ &= \tan\theta\end{aligned}$$

61. 1

62. $\csc^2\theta$

63. If $n_2 > n_1$, then $\theta_1 > \theta_2$; if $n_2 < n_1$, then $\theta_1 < \theta_2$; if $n_2 = n_1$, then $\theta_2 = \theta_1$.

64. C

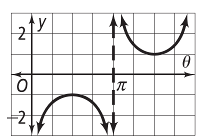
65. H

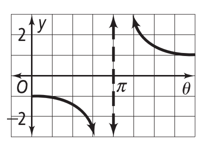
66. C

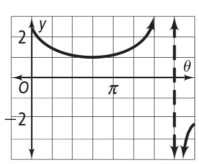
67. F

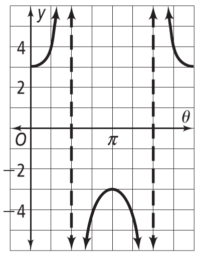
68. C

69. By the difference of Squares Property and the second Pythagorean Identity: $(\sec\theta - 1)(\sec\theta + 1) = \sec^2\theta - 1 = \tan^2\theta$

70.


71.


72.


73. 

74. 35°

75. 45°

76. 135°

77. 211°

78. $f^{-1}(x) = x - 1$

79. $f^{-1}(x) = \dfrac{x+3}{2}$

80. $f^{-1}(x) = \pm\sqrt{x-4}$

Algebra 2
Lesson 14-2 - Practice and Problem-Solving Exercises Answers

7. $90° + 360°n$

8. $30° + 360°n$ and $210° + 360°n$, or just $30° + 180°n$

9. $240° + 360°n$ and $300° + 360°n$

10. $120° + 360°n$ and $300° + 360°n$, or just $120° + 180°n$

11. $90° + 360°n$ and $270° + 360°n$, or just $90° + 180°n$

12. $135° + 360°n$ and $225° + 360°n$

13. $0.79 + 2\pi n$ and $3.93 + 2\pi n$, or just $0.79 + \pi n$

14. $0.38 + 2\pi n$ and $2.76 + 2\pi n$

15. $-0.89 + 2\pi n$ and $4.04 + 2\pi n$

16. $1.89 + 2\pi n$ and $5.03 + 2\pi n$, or $1.89 + \pi n$

17. $2.67 + 2\pi n$ and $3.62 + 2\pi n$

18. no solution

19. $\frac{\pi}{6}, \frac{5\pi}{6}$

20. $\frac{\pi}{6}, \frac{11\pi}{6}$

21. $\frac{\pi}{4}, \frac{5\pi}{4}$

22. $\frac{\pi}{4}, \frac{3\pi}{4}$

23. $0.46,\ 3.61$

24. $2.11,\ 5.25$

25. no solution

26. $\frac{4\pi}{3}, \frac{5\pi}{3}$

27. $\frac{\pi}{2}, \pi, \frac{3\pi}{2}$

28. $\frac{\pi}{2}, \frac{3\pi}{2}$

29. $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

30. $0, \frac{\pi}{4}, \pi, \frac{5\pi}{4}$

31. $0, \pi$

32. $0, \pi$

33. $\frac{7\pi}{6}, \frac{11\pi}{6}$

34. The air conditioner comes on about 4.7 hrs after midnight, 4:42 A.M., and goes off about 4.7 hrs before midnight, 7:18 P.M.

35. $30° + 360°n$ and $150° + 360°n$

36. $60° + 360°n$ and $300° + 360°n$

37. $210° + 360°n$ and $330° + 360°n$

38. $\frac{\pi}{3}, \frac{5\pi}{3}$

39. $\frac{3\pi}{2}$

40. $0.34,\ 2.80$

41. $3.04,\ 6.18$

42. 9 seconds after starting

43. The current will first reach 20 amps after 0.0028 seconds. The current will first reach -20 amps after 0.019 seconds.

44a. $\frac{\pi}{6}+2\pi n \le x \le \frac{5\pi}{6}+2\pi n$

44b. $\frac{5\pi}{6}+2\pi n \le x \le \frac{13\pi}{6}+2\pi n$

44c. Find the values of x where the graphs intersect, and then choose the appropriate interval.

45. $0+2\pi n,\ \frac{2}{3}\pi+2\pi n,\ \frac{4}{3}\pi+2\pi n$

46. $\frac{\pi}{2}+2\pi n,\ \frac{3\pi}{2}+2\pi n$

47. $\frac{\pi}{6}+2\pi n,\ \frac{5\pi}{6}+2\pi n,\ \frac{3\pi}{2}+2\pi n$

48. $\frac{\pi}{4}+2\pi n,\ \frac{3\pi}{4}+2\pi n,\ \frac{5\pi}{4}+2\pi n,$ and $\frac{7\pi}{4}+2\pi n$
or $\frac{\pi}{4}+\frac{\pi}{2}n$

49. $\frac{\pi}{6}+2\pi n,\ \frac{5\pi}{6}+2\pi n,\ \frac{\pi}{2}+\pi n$

50. $\theta=\pi n,\ 1.25+\pi n$

51. $\frac{\pi}{2}+2\pi n,\ \frac{7\pi}{6}+2\pi n,\ \frac{11\pi}{6}+2\pi n$

52. $\frac{\pi}{6}+\pi n,\ \frac{5\pi}{6}+\pi n$

53. $\frac{\pi}{6}+2\pi n,\ \frac{5\pi}{6}+2\pi n$

54. $\frac{2\pi}{3}+2\pi n,\ \frac{4\pi}{3}+2\pi n$

55. $\frac{\pi}{4}+\frac{\pi n}{2}$

56. $0+\pi n$

57. $\frac{\pi}{4}+\frac{\pi n}{2}$

58. $0+\pi n,\ \frac{\pi}{4}+\frac{\pi n}{2}$

59. $\frac{2\pi}{3}+2\pi n,\ \frac{4\pi}{3}+2\pi n$

60. $\frac{3\pi}{2}+2\pi n$

61a. Answers may vary. Sample:
$\cos\theta=-1,\ 2\cos\theta=-2,\ 3\cos\theta=-3$

61b. Start with $\cos\theta=-1$, and then multiply each side of the equation by any nonzero number.

62. $\theta=\frac{1}{2}\cos^{-1}y$

63. $\theta=\sin^{-1}\left(\frac{y}{3}\right)-2$

64. $\theta=\frac{1}{2\pi}\cos^{-1}\left(-\frac{y}{4}\right)$

65. $\theta=\cos^{-1}\left(\frac{y-1}{2}\right)$

66a. 1:55 A.M., 11:05 A.M., and 2:55 P.M.

66b. times when the tide is at least 3 ft above the mean water level: from midnight to 1:55 A.M., and from 11:05 A.M. to 2:55 P.M.

67. D

68. H

69. C

70. G

71. D

72. $0,\ \pi,\ \frac{11\pi}{6},\ \frac{7\pi}{6}$

73. $\cot\theta$

74. $\tan^2 \theta$

75. 1

76. 1

77. $\sin \theta$

78. $\tan \theta$

79. $y = 4\cos\frac{\pi}{4}\theta$

80. $y = 3\cos\theta$

81. $y = \frac{\pi}{4}\cos\frac{2}{3}\theta$

82. 4

83. 21

84. $52\frac{1}{2}$

Algebra 2
Lesson 14-3 - Practice and Problem-Solving Exercises Answers

7. $\sin\theta = \frac{3}{5},\ \cos\theta = -\frac{4}{5},\ \tan\theta = -\frac{3}{4}$
$\csc\theta = \frac{5}{3},\ \sec\theta = -\frac{5}{4},\ \cot\theta = -\frac{4}{3}$

8. $\sin\theta = \frac{12}{13},\ \cos\theta = \frac{5}{13},\ \tan\theta = \frac{12}{5}$
$\csc\theta = \frac{13}{12}, \sec\theta = \frac{13}{5},\ \cot\theta = \frac{5}{12}$

9. $\sin\theta = -\frac{5\sqrt{26}}{26},\ \cos\theta = \frac{\sqrt{26}}{26},\ \tan\theta = -5$
$\csc\theta = -\frac{\sqrt{26}}{5},\ \sec\theta = \sqrt{26},\ \cot\theta = -\frac{1}{5}$

10. $\sin\theta = -\frac{2\sqrt{29}}{29},\ \cos\theta = -\frac{5\sqrt{29}}{29},\ \tan\theta = \frac{2,}{5}$
$\csc\theta = -\frac{\sqrt{29}}{2},\ \sec\theta = -\frac{\sqrt{29}}{5},\ \cot\theta = \frac{5}{2}$

11. $\sin\theta = \frac{\sqrt{7}}{4},\ \cos\theta = -\frac{3}{4},\ \tan\theta = -\frac{\sqrt{7}}{3}$
$\csc\theta = \frac{4\sqrt{7}}{7},\ \sec\theta = -\frac{4}{3},\ \cot\theta = -\frac{3\sqrt{7}}{7}$

12. 5 ft

13a. 0.88

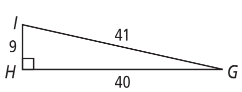
13b. 2.13

13c. 0.53

13d. 2.13

13e. 1.13

13f. 0.53

14.


14a. 0.22

14b. 0.98

14c. 4.44

14d. 4.56

14e. 0.22

14f. not defined

15. 41.8

16. 10.6

17. 25.2

18a. 300 ft

18b. 445 ft

18c. Answers may vary. Sample:
The flag pole should be straight and perpendicular to the ground, and the shadow cast on the ground should also be a straight line. You assume these things so that the flagpole and the ground form a right triangle. By having a right triangle, you can use its properties to find the missing parts.

19. $a = 8.7,\ m\angle A = 60°,\ m\angle B = 30°$

20. $c \approx 7.8,\ m\angle A \approx 39.8°,\ m\angle B \approx 50.2°$

21. $a = 9,\ m\angle A \approx 36.9°,\ m\angle B \approx 53.1°$

22. $c \approx 10.2,\ m\angle A \approx 52.6°,\ m\angle B \approx 37.4°$

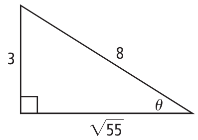
23. $a \approx 8.0,\ m\angle A \approx 61.8°,\ m\angle B \approx 28.2°$

24. $b \approx 14.0,\ m\angle A \approx 50.6°,\ m\angle B \approx 39.4°$

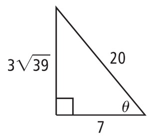
25a. $m\angle A = \cos^{-1}\left(\frac{1200}{d}\right)$

25b. 37°

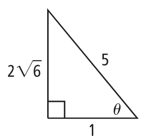
25c. 53°

26. 

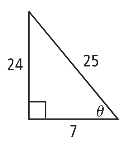
$\cos\theta = \frac{\sqrt{55}}{8}$, $\tan\theta = \frac{3\sqrt{55}}{55}$, $\csc\theta = \frac{8}{3}$,
$\sec\theta = \frac{8\sqrt{55}}{55}$, $\cot\theta = \frac{\sqrt{55}}{3}$

27. 

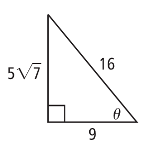
$\sin\theta = \frac{3\sqrt{39}}{20}$, $\tan\theta = \frac{3\sqrt{39}}{7}$, $\csc\theta = \frac{20\sqrt{39}}{117}$,
$\sec\theta = \frac{20}{7}$, $\cot\theta = \frac{7\sqrt{39}}{117}$

28. 

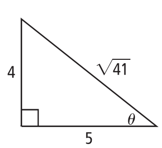
$\sin\theta = \frac{2\sqrt{6}}{5}$, $\tan\theta = 2\sqrt{6}$,
$\csc\theta = \frac{5\sqrt{6}}{12}$, $\sec\theta = 5$, $\cot\theta = \frac{\sqrt{6}}{12}$

29. 

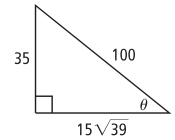
$\sin\theta = \frac{24}{25}$, $\cos\theta = \frac{7}{25}$, $\csc\theta = \frac{25}{24}$, $\sec\theta = \frac{25}{7}$, $\cot\theta = \frac{7}{24}$

30. 

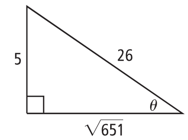
$\sin\theta = \frac{5\sqrt{7}}{16}$, $\cos\theta = \frac{9}{16}$, $\tan\theta = \frac{5\sqrt{7}}{9}$,
$\csc\theta = \frac{16\sqrt{7}}{35}$, $\cot\theta = \frac{9\sqrt{7}}{35}$

31. 

$\sin\theta = \frac{4\sqrt{41}}{41}$, $\cos\theta = \frac{5\sqrt{41}}{41}$, $\tan\theta = \frac{4}{5}$,
$\csc\theta = \frac{\sqrt{41}}{4}$, $\sec\theta = \frac{\sqrt{41}}{5}$

32. 

$\cos\theta = \frac{3\sqrt{39}}{20}$, $\tan\theta = \frac{7\sqrt{39}}{117}$, $\csc\theta = \frac{20}{7}$,
$\sec\theta = \frac{20\sqrt{39}}{117}$, $\cot\theta = \frac{3\sqrt{39}}{7}$

33. 

$\sin\theta = \frac{5}{26}$, $\cos\theta = \frac{\sqrt{651}}{26}$, $\tan\theta = \frac{5\sqrt{651}}{651}$,
$\sec\theta = \frac{26\sqrt{651}}{651}$, $\cot\theta = \frac{\sqrt{651}}{5}$

34. $d = \frac{100}{\sin\theta}$; 115.5 ft; 130.5 ft

35. 33.4 ft

36. $x = \frac{150}{\tan\theta}$; 214.2 ft; 54.6 ft

37. 20.3 m^2

38. $a = 15$, $m\angle A \approx 61.9°$, $m\angle B \approx 28.1°$

39. $c \approx 12.2$, $m\angle A \approx 35.0°$, $m\angle B \approx 55.0°$

40. $a \approx 7.9$, $b \approx 6.2$, $m\angle B = 38°$

41. $a \approx 3.9$, $c \approx 6.9$, $m\angle B = 55.8°$

42. $a \approx 26.8$, $c \approx 28.1$, $m\angle A = 72.8°$

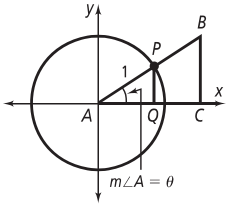
43. $a \approx 19.8$, $b \approx 2.9$, $m\angle A = 81.7°$

44a. 12

44b. 12

45. Using inverse sine, you can find that $\theta = 30°$. Since sine is positive in the first and second quadrants, another solution is 150°. All the solutions would be $30° + 360°n$ and $150° + 360°n$.

46.



Since $\triangle APQ$ and $\triangle ABC$ are similar triangles, $\dfrac{AQ}{AP} = \dfrac{AC}{AB}$.

So, $\cos\theta = AQ = \dfrac{AQ}{1} = \dfrac{AQ}{AP} = \dfrac{AC}{AB} = \cos A$.

47. $\sec A = \dfrac{c}{b} = \dfrac{1}{\left(\dfrac{b}{c}\right)} = \dfrac{1}{\cos A}$

48. $\tan A = \dfrac{a}{b} = \dfrac{\left(\dfrac{a}{c}\right)}{\left(\dfrac{b}{c}\right)} = \dfrac{\sin A}{\cos A}$

49. $\cos^2 A + \sin^2 A = \left(\dfrac{b}{c}\right)^2 + \left(\dfrac{a}{c}\right)^2 \stackrel{?}{=} 1$

$$b^2 + a^2 \stackrel{?}{=} c^2$$
$$c^2 = c^2$$

50a. 72°

50b. 19.0 cm

51. 61.7 m

52. A

53. G

54. C

55. I

56. 54.9°, 35.1°

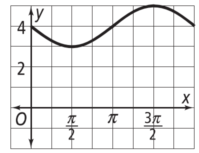
57. $180° + 360°n$

58. $90° + 180°n$ or $45° + 180°n$

59. $0° + 360°n$ and $180° + 360°n$, or $0° + 180°n$

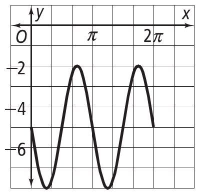
60. $y = \sin\left(x - \pi\right) + 4$

graph $y = \sin x$, translated π units right and 4 units up



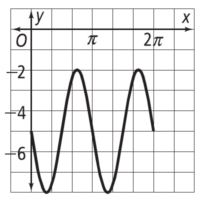
61. $y = 3\sin 2\left(x + \dfrac{\pi}{2}\right) - 5$

graph $y = 3\sin 2x$, translated $\dfrac{\pi}{2}$ units left and 5 units down



62. $y = 3\sin 2\left(x + \dfrac{\pi}{2}\right) - 5$

graph $y = 3\sin 2x$, translated $\dfrac{\pi}{2}$ units left and 5 units down



63. 6 cm^2

64. 45 in.^2

65. 32.76 mm^2

Algebra 2
Lesson 14-4 - Practice and Problem-Solving Exercises Answers

6. 18.7 cm^2

7. 9.1 $in.^2$

8. 81.9 m^2

9. 10.9

10. 9.2

11. 7.4

12. 41.1°

13. 33.5°

14. 27.0°

15. 31.7°

16. 340 ft

17. 32 cm

18. 7.5 mi, 7.9 mi

19. 66°

20. $m\angle F = 72°, e = 20$ in., $\frac{\sin 54°}{20} = \frac{\sin 72°}{f}$, $f \approx 23.5$

21. $m\angle E \approx 40.3°$, $m\angle F \approx 85.7°$, $f \approx 12.3$

22a. when $m\angle C = 50°$

22b. when $m\angle C = 50°$

22c. 90°; that is the angle measure at which $\sin\theta$ is greatest.

23. Check students' work.

24. 85.0

25. 44.4

26. 29.7

27. 49.4

28. 8.2 m

29. 28.0 ft

30. 43.2 yd

31. 4.0 cm

32. 504 ft

33a. 56.4°, 93.6°, 26.4°

33b. No; $\triangle EFG$ could be congruent to $\triangle ABC$ instead of $\triangle ABD$.

34. 2.4 mi

35. No; you need at least one side length in order to set up a proportion you can solve.

36. A

37. I

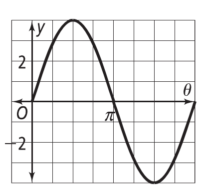
38. C

39. Area $= \frac{1}{2}ab\sin C$

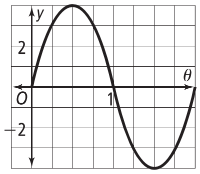
$\sin C = \frac{2(\text{Area})}{ab} = \frac{2(31.5)}{9(14)} = 0.5$

$m\angle C = \sin^{-1} 0.5 = 30°$

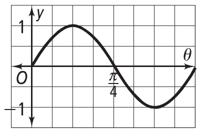
So, the measure of the included angle for the given sides is 30°, or 150°.

40. 

41.



42.



43. $\langle -1,\ 7 \rangle$

44. $\langle 5,\ 3 \rangle$

45. $\langle -3,\ -1 \rangle$

46. $\langle -6,\ 4 \rangle$

47. $53.1°$

48. $24.6°$

49. $38.7°$

50. $54.7°$

Algebra 2
Lesson 14-5 - Practice and Problem-Solving Exercises Answers

7. 37.1

8. 27.9

9. 13.7

10. 47.3°

11. 27.0°

12. 125.1°

13. 33.7°

14. 83.3°

15. 47.2°

16. 60.5°

17. 50.8°

18. 71.7°

19. 27.0°

20. 35.5°

21. $b^2 = a^2 + c^2 - 2ac\cos B$

22. $c^2 = a^2 + b^2 - 2ab\cos C$

23. $\frac{\sin B}{b} = \frac{\sin C}{c}$

24. $\frac{\sin A}{a} = \frac{\sin B}{b}$

25. $\frac{\sin C}{c} = \frac{\sin A}{a}$

26. $a^2 = b^2 + c^2 - 2bc\cos A$

27. about 59.1 nautical miles

28. 46 ft

29. $b \approx 34.7,\ m\angle A \approx 26.7°,\ m\angle C \approx 33.3°$

30. $g \approx 53.3,\ m\angle F \approx 31.9°,\ m\angle H \approx 38.1°$

31. $m\angle A \approx 56.1°,\ m\angle B \approx 70.0°,\ m\angle C \approx 53.9°$

32a. Check students' work.

32b. Check students' work.

33. For any two side lengths a and b, the ratio $\frac{a}{b}$ is equal to the ratio $\frac{\sin A}{\sin B}$, which can be found since A and B are given.

34. 77.2°

35a. about 45.5 mi

35b. 14.4° left; 4.4° west of north

36. 56.6°

37. 11.0 cm

38. 8.8 cm

39. 27.0°

40. 117.3°

41. 13.0 cm

42. 32.6°

43. 21.5°

44. 67.2°

45. 8.3 ft

46. 64.0°

47. 79.6°

48. 109 cm

49. 18 cm

50. Yes; since $\cos 90° = 0$, $c^2 = a^2 + b^2 - 2ab\cos C$ reduces to $c^2 = a^2 + b^2$.

51. 4.8 in.

52a. 2.1 m

52b. 9.8 m^2

53a. $\cos A > 0$ if $b^2 + c^2 > a^2$
$\cos A = 0$ if $b^2 + c^2 = a^2$
$\cos A < 0$ if $b^2 + c^2 < a^2$

53b. acute $\triangle$ if $\cos A > 0$
right $\triangle$ if $\cos A = 0$
obtuse $\triangle$ if $\cos A < 0$

54. 15.6

55. 85.4°

56. 61.4°

57. 27.1°

58. 11.0

59. 24.1 units^2

60. 17.1 in.

61. 8.9 m

62. 26.3 in.

63. 54.0°

64. 2π; $x = \pm\pi$

65. $\frac{2}{3}$; $x = \pm\frac{1}{3}$

66. $\frac{\pi}{3}$; $x = \pm\frac{\pi}{6}$

67. 1; $x = \pm\frac{1}{2}$

68. $\sin\theta$

69. $\cos\theta$

70. 1

71. $\csc^2\theta$

Algebra 2
Lesson 14-6 - Practice and Problem-Solving Exercises Answers

7.
$$\csc\left(\theta-\frac{\pi}{2}\right)=\frac{1}{\sin\left(\theta-\frac{\pi}{2}\right)}$$
$$=\frac{1}{\sin\left(-\left(\frac{\pi}{2}-\theta\right)\right)}$$
$$=\frac{1}{-\sin\left(\frac{\pi}{2}-\theta\right)}$$
$$=\frac{1}{-\cos\theta}$$
$$=-\sec\theta$$

8.
$$\sec\left(\theta-\frac{\pi}{2}\right)=\frac{1}{\cos\left(\theta-\frac{\pi}{2}\right)}$$
$$=\frac{1}{\cos\left(-\left(\frac{\pi}{2}-\theta\right)\right)}$$
$$=\frac{1}{\cos\left(\frac{\pi}{2}-\theta\right)}$$
$$=\frac{1}{\sin\theta}$$
$$=\csc\theta$$

9.
$$\cot\left(\frac{\pi}{2}-\theta\right)=\frac{\cos\left(\frac{\pi}{2}-\theta\right)}{\sin\left(\frac{\pi}{2}-\theta\right)}$$
$$=\frac{\sin\theta}{\cos\theta}$$
$$=\tan\theta$$

10.
$$\csc\left(\frac{\pi}{2}-\theta\right)=\frac{1}{\sin\left(\frac{\pi}{2}-\theta\right)}$$
$$=\frac{1}{\cos\theta}$$
$$=\sec\theta$$

11.
$$\tan\left(\theta-\frac{\pi}{2}\right)=\tan\left(-\left(\frac{\pi}{2}-\theta\right)\right)$$
$$=-\tan\left(\frac{\pi}{2}-\theta\right)$$
$$=-\cot\theta$$

12.
$$\sec\left(\frac{\pi}{2}-\theta\right)=\frac{1}{\cos\left(\frac{\pi}{2}-\theta\right)}$$
$$=\frac{1}{\sin\theta}$$
$$=\csc\theta$$

13. $\tan(90^\circ-A)=\cot A$

14. $\csc(90^\circ-A)=\sec A$

15. $\cot(90^\circ-A)=\tan A$

16. $\frac{\pi}{2},\ \frac{3\pi}{2}$

17. $\frac{\pi}{2},\ \frac{3\pi}{2}$

18. $\frac{\pi}{4},\ \frac{3\pi}{4},\ \frac{5\pi}{4},\ \frac{7\pi}{4}$

19. π

20. 2.034, 5.176

21. $\frac{\pi}{2},\ \frac{3\pi}{2}$

22. 0

23. $\frac{\sqrt{2}}{2}$

24. -1

25. 0

26. $\frac{\sqrt{2}-\sqrt{6}}{4}$

27. $-\sqrt{3}-2$

28. $2-\sqrt{3}$

29. $\dfrac{\sqrt{2}+\sqrt{6}}{4}$

30. $\dfrac{\sqrt{6}-\sqrt{2}}{4}$

31. $-2+\sqrt{3}$

32. $-\dfrac{\sqrt{2}}{2}$

33. $-\dfrac{1}{2}$

34. $\dfrac{1}{2}$

35. $\dfrac{1}{2}$

36. $Q\left(\dfrac{\sqrt{2}}{2}\left(\cos\theta-\sin\theta\right),\ -\dfrac{\sqrt{2}}{2}\left(\sin\theta+\cos\theta\right)\right)$

37.
$$\begin{aligned}\sin\left(A-B\right) &= \cos\left[\frac{\pi}{2}-\left(A-B\right)\right]\\ &= \cos\left[\left(\frac{\pi}{2}-A\right)+B\right]\\ &= \cos\left(\frac{\pi}{2}-A\right)\cos B-\sin\left(\frac{\pi}{2}-A\right)\sin B\\ &= \sin A\cos B-\cos A\sin B\end{aligned}$$

38.
$$\begin{aligned}\tan\left(A-B\right) &= \frac{\sin\left(A-B\right)}{\cos\left(A-B\right)}\\ &= \frac{\sin A\cos B-\cos A\sin B}{\cos A\cos B+\sin A\sin B}\\ &= \frac{\dfrac{\sin A\cos B-\cos A\sin B}{\cos A\cos B}}{\dfrac{\cos A\cos B+\sin A\sin B}{\cos A\cos B}}\\ &= \frac{\dfrac{\sin A\cos B}{\cos A\cos B}-\dfrac{\cos A\sin B}{\cos A\cos B}}{\dfrac{\cos A\cos B}{\cos A\cos B}+\dfrac{\sin A\sin B}{\cos A\cos B}}\\ &= \frac{\tan A-\tan B}{1+\tan A\tan B}\end{aligned}$$

39.
$$\begin{aligned}\tan\left(A+B\right) &= \frac{\sin\left(A+B\right)}{\cos\left(A+B\right)}\\ &= \frac{\sin A\cos B+\cos A\sin B}{\cos A\cos B-\sin A\sin B}\\ &= \frac{\dfrac{\sin A\cos B+\cos A\sin B}{\cos A\cos B}}{\dfrac{\cos A\cos B-\sin A\sin B}{\cos A\cos B}}\\ &= \frac{\dfrac{\sin A\cos B}{\cos A\cos B}+\dfrac{\cos A\sin B}{\cos A\cos B}}{\dfrac{\cos A\cos B}{\cos A\cos B}-\dfrac{\sin A\sin B}{\cos A\cos B}}\\ &= \frac{\tan A+\tan B}{1-\tan A\tan B}\end{aligned}$$

40.
$$\begin{aligned}&\sin\left(x+\frac{\pi}{3}\right)+\sin\left(x-\frac{\pi}{3}\right)\\ &= \sin x\cos\frac{\pi}{3}+\cos x\sin\frac{\pi}{3}+\sin x\cos\frac{\pi}{3}-\cos x\sin\frac{\pi}{3}\\ &= 2\sin x\cos\frac{\pi}{3}\\ &= 2\sin x\cdot\frac{1}{2}\\ &= \sin x\end{aligned}$$

41. $\left(5\cos\theta-5\sqrt{3}\sin\theta,\ 5\sin\theta+5\sqrt{3}\cos\theta\right)$

42. $\sin 3\theta$

43. $\sin\ 5\theta$

44. $\cos 7\theta$

45. $\cos 5\theta$

46. $\tan 11\theta$

47. $\tan 2\theta$

48. Let $A = 30°$ and $B = 60°$,

$\sin(30° + 60°) = \sin 90° = 1$

$\sin 30° + \sin 60° = \frac{1}{2} + \frac{\sqrt{3}}{2} \neq 1$

49a. even: cosine, secant
odd: sine, cosecant, tangent, cotangent

49b. No; answers may vary. Sample:

$y = \sin x - \cos x$. For $x = \frac{\pi}{4}$, $f(x) = \sin\frac{\pi}{4} - \cos\frac{\pi}{4} = 0$, and

$f(x) = \sin\left(-\frac{\pi}{4}\right) - \cos\left(-\frac{\pi}{4}\right) = -\sqrt{2}$. Because $f(x) \neq f(-x)$

and $f(x) \neq -f(-x)$, the function is neither even nor odd.

50.
$$\begin{aligned}\cos(\pi - \theta) &= \cos\pi\cos\theta + \sin\pi\sin\theta \\ &= -1\cdot\cos\theta + 0\cdot\sin\theta \\ &= -\cos\theta + 0 \\ &= -\cos\theta\end{aligned}$$

51.
$$\begin{aligned}\sin(\pi - \theta) &= \sin\pi\cos\theta - \cos\pi\sin\theta \\ &= 0\cdot\cos\theta - (-1)\cdot\sin\theta \\ &= 0 + \sin\theta \\ &= \sin\theta\end{aligned}$$

52.
$$\begin{aligned}\sin(\pi + \theta) &= \sin\pi\cos\theta + \cos\pi\sin\theta \\ &= 0\cdot\cos\theta + (-1)\cdot\sin\theta \\ &= 0 - \sin\theta \\ &= -\sin\theta\end{aligned}$$

53.
$$\begin{aligned}\cos(\pi + \theta) &= \cos\pi\cos\theta - \sin\pi\sin\theta \\ &= -1\cdot\cos\theta\cdot\ 0\cdot\sin\theta \\ &= -\cos\theta - 0 \\ &= -\cos\theta\end{aligned}$$

54.
$$\begin{aligned}\sin\left(\frac{3\pi}{2} - \theta\right) &= \sin\frac{3\pi}{2}\cos\theta - \cos\frac{3\pi}{2}\sin\theta \\ &= -1\cdot\cos\theta - 0\cdot\sin\theta \\ &= -\cos\theta - 0 \\ &= -\cos\theta\end{aligned}$$

55.
$$\begin{aligned}\cos\left(\theta + \frac{3\pi}{2}\right) &= \cos\theta\cos\frac{3\pi}{2} - \sin\theta\sin\frac{3\pi}{2} \\ &= \cos\theta\cdot 0 - \sin\theta\cdot(-1) \\ &= 0 + \sin\theta \\ &= \sin\theta\end{aligned}$$

56. D

57. I

58. B

59.
$$\begin{aligned}\sin(165°) &= \sin(15°) \\ &= \sin(45° - 30°) \\ &= \sin 45°\cdot\cos 30° - \cos 45°\cdot\sin 30° \\ &= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2} \\ &= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} \\ &= \frac{\sqrt{6} - \sqrt{2}}{4}\end{aligned}$$

60. ≈ 16.3 ft

61. ≈ 10.0 cm

62. $\frac{4\pi}{9}$ and 1.40

63. $-\frac{5\pi}{18}$ and -0.87

64. $-\frac{\pi}{12}$ and -0.26

65. $\frac{7\pi}{18}$ and 1.22

66. $\frac{19\pi}{18}$ and 3.32

67. $\cos A\cos B - \sin A\sin B$

68. $\sin A\cos B + \cos A\sin B$

69. $\frac{\tan A + \tan B}{1 - \tan A\tan B}$

Algebra 2
Lesson 14-7 - Practice and Problem-Solving Exercises Answers

7. $$\begin{aligned}\sin 2\theta &= \sin(\theta+\theta)\\ &= \sin\theta\cos\theta+\cos\theta\sin\theta\\ &= 2\sin\theta\cos\theta\end{aligned}$$

8. $$\begin{aligned}\tan 2\theta &= \tan(\theta+\theta)\\ &= \frac{\tan\theta+\tan\theta}{1-\tan\theta\tan\theta}\\ &= \frac{2\tan\theta}{1-\tan^2\theta}\end{aligned}$$

9. $-\frac{\sqrt{3}}{2}$

10. $-\frac{1}{2}$

11. $-\sqrt{3}$

12. 1

13. $-\frac{1}{2}$

14. $\sqrt{3}$

15. $-\frac{1}{2}$

16. $-\frac{\sqrt{3}}{2}$

17. $\frac{\sqrt{2+\sqrt{3}}}{2}$

18. $\sqrt{7-4\sqrt{3}}$ or $2-\sqrt{3}$

19. $\frac{\sqrt{2-\sqrt{3}}}{2}$

20. $\frac{\sqrt{2-\sqrt{2}}}{2}$

21. $\frac{\sqrt{2+\sqrt{2}}}{2}$

22. $\sqrt{3-2\sqrt{2}}$ or $\sqrt{2}-1$

23. 0

24. $\frac{\sqrt{2-\sqrt{2+\sqrt{3}}}}{2}$

25. $\frac{3\sqrt{10}}{10}$

26. $\frac{\sqrt{10}}{10}$

27. 3

28. $\frac{1}{3}$

29. $\frac{4\sqrt{17}}{17}$

30. $-\frac{\sqrt{17}}{17}$

31. -4

32. $-\sqrt{17}$

33. $$\begin{aligned}\cos B &= 2\cos^2\frac{B}{2}-1\\ 2\cos^2\frac{B}{2} &= \cos B+1\\ \cos^2\frac{B}{2} &= \frac{\cos B+1}{2}\end{aligned}$$

Since $\cos B = \frac{a}{c}$,

$$\cos^2\frac{B}{2} = \frac{\frac{a}{c}+1}{2} = \frac{a+c}{2c}.$$

34. $\sin 2R = 2\sin R\cos R$
$$= 2\cdot\frac{r}{t}\cdot\frac{s}{t}$$
$$= \frac{2rs}{t^2}$$

35. $\cos 2R = \cos^2 R - \sin^2 R$
$$= \left(\frac{s}{t}\right)^2 - \left(\frac{r}{t}\right)^2$$
$$= \frac{s^2}{t^2} - \frac{r^2}{t^2}$$
$$= \frac{s^2 - r^2}{t^2}$$

36. $\sin 2S = 2\sin S\cos S$
$$= 2\cdot\frac{s}{t}\cdot\frac{r}{t}$$
$$= \frac{2sr}{t^2}$$
$$= 2\sin R\cos R$$
$$= \sin 2R$$

37. $\sin^2\frac{S}{2} = \left(\sin\frac{S}{2}\right)^2$
$$= \left(\pm\sqrt{\frac{1-\cos S}{2}}\right)^2$$
$$= \frac{1-\cos S}{2}$$
$$= \frac{1-\frac{r}{t}}{2}$$
$$= \frac{1}{2} - \frac{r}{2t}$$
$$= \frac{t-r}{2t}$$

38. $\tan\frac{R}{2} = \sqrt{\frac{1-\cos R}{1+\cos R}}$
$$= \sqrt{\frac{1-\frac{s}{t}}{1+\frac{s}{t}}}$$
$$= \sqrt{\frac{(t-s)}{(t+s)}\cdot\frac{(t+s)}{(t+s)}}$$
$$= \sqrt{\frac{t^2-s^2}{(t+s)^2}}$$
$$= \sqrt{\frac{r^2}{(t+s)^2}}$$
$$= \frac{r}{t+s}$$

39. $\tan^2\frac{S}{2} = \left(\tan\frac{S}{2}\right)^2$
$$= \left(\pm\sqrt{\frac{1-\cos S}{1+\cos S}}\right)^2$$
$$= \frac{1-\cos S}{1+\cos S}$$
$$= \frac{1-\frac{r}{t}}{1+\frac{r}{t}}$$
$$= \frac{t-r}{t+r}$$

40. No; since the sine function is periodic, A and B can have many different values.

41. $-\frac{24}{25}$

42. $-\frac{7}{25}$

43. $\frac{24}{7}$

44. $-\frac{25}{24}$

45. $\frac{\sqrt{5}}{5}$

46. $-\frac{2\sqrt{5}}{5}$

47. $-\frac{1}{2}$

48. -2

49. $\cos\theta(8\sin\theta - 3) = 0;\ \frac{\pi}{2},\ \frac{3\pi}{2},\ 0.384,\ 2.757$

50. $\sin\theta(4\cos\theta - 3) = 0;\ 0,\ \pi,\ 0.723,\ 5.560$

51. $\cos\theta(2\sin^2\theta - 1) = 0;\ \frac{\pi}{2},\ \frac{3\pi}{2},\ \frac{\pi}{4},\ \frac{3\pi}{4},\ \frac{5\pi}{4},\ \frac{7\pi}{4}$

52. $4\cos^2\theta - 1 = 0;\ \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

53. 1

54. $-\cos\theta$

55. $\cos\theta - \sin\theta$

56a. $\sin^2\theta = \frac{1-\cos 2\theta}{2}$

56b. $\cos^2\theta = \frac{1+\cos 2\theta}{2}$

57a. Answers may vary. Sample:

$$A = 60^\circ$$
$$\sin 60^\circ = \frac{\sqrt{3}}{2}$$
$$\cos 60^\circ = \frac{1}{2}$$

57b. Answers may vary. Sample:

$$\begin{aligned} A &= 60^\circ \\ \sin 2A &= 2\sin 60^\circ \cos 60^\circ \\ &= 2\cdot\frac{\sqrt{3}}{2}\cdot\frac{1}{2} \\ &= \frac{\sqrt{3}}{2} \end{aligned}$$

57c. Answers may vary. Sample:

$$\begin{aligned} A &= 60^\circ \\ \cos\frac{60^\circ}{2} &= \sqrt{\frac{1+\cos 60^\circ}{2}} \\ &= \sqrt{\frac{1+\frac{1}{2}}{2}} \\ &= \sqrt{\frac{3}{4}} \\ &= \frac{\sqrt{3}}{2} \end{aligned}$$

58. The graph of $\sqrt{\frac{1-\cos A}{1+\cos A}}$ represents the positive tangent function for $\tan\frac{A}{2}$ with a period of 2π and vertical asymptotes at $-\pi$ and π $(n\pi$ for odd $n)$. There is no amplitude because the values of the tangent are unbounded.

59. Answers may vary. Sample:

$$\begin{aligned} \sin 4\theta &= \sin 2(2\theta) \\ &= 2\sin 2\theta\cos 2\theta \\ &= 2\cdot 2\sin\theta\cos\theta\cdot(\cos^2\theta - \sin^2\theta) \\ &= 4\sin\theta\cos\theta(\cos^2\theta - \sin^2\theta) \end{aligned}$$

60. Answers may vary. Sample:

$$\begin{aligned} \cos 4\theta &= \cos 2(2\theta) \\ &= 2\cos^2 2\theta - 1 = 2(2\cos^2\theta - 1)^2 - 1 \\ &= 2(4\cos^4\theta - 4\cos^2\theta + 1) - 1 \\ &= 8\cos^4\theta - 8\cos^2\theta + 2 - 1 \\ &= 8\cos^4\theta - 8\cos^2\theta + 1 \end{aligned}$$

61. $\frac{4\tan\theta(1-\tan^2\theta)}{\tan^4\theta - 6\tan^2\theta + 1}$

62. $\pm\sqrt{\frac{1}{2} \pm \frac{1}{2}\sqrt{\frac{1}{2}+\frac{1}{2}\cos\theta}}$

63. $\pm\sqrt{\frac{1}{2} \pm \frac{1}{2}\sqrt{\frac{1}{2}+\frac{1}{2}\cos\theta}}$

64. $\pm\sqrt{\frac{1\pm\sqrt{\frac{1}{2}+\frac{1}{2}\cos\theta}}{1\pm\sqrt{\frac{1}{2}+\frac{1}{2}\cos\theta}}}$

65a.

$$\begin{aligned} \tan\frac{A}{2} &= \pm\sqrt{\frac{1-\cos A}{1+\cos A}} \\ &= \pm\sqrt{\frac{1-\cos A}{1+\cos A}}\cdot\sqrt{\frac{1+\cos A}{1+\cos A}} \\ &= \pm\sqrt{\frac{1-\cos^2 A}{(1+\cos A)^2}} \\ &= \pm\sqrt{\frac{\sin^2 A}{(1+\cos A)^2}} \\ &= \frac{\sin A}{1+\cos A} \end{aligned}$$

Since $\tan\frac{A}{2}$ and $\sin A$ have the same sign wherever $\tan\frac{A}{2}$ is defined, only the positive sign occurs.

65b.
$$\tan\frac{A}{2} = \pm\sqrt{\frac{1-\cos A}{1+\cos A}}$$
$$= \pm\sqrt{\frac{1-\cos A}{1+\cos A}}\cdot\sqrt{\frac{1-\cos A}{1-\cos A}}$$
$$= \pm\sqrt{\frac{(1-\cos A)^2}{1-\cos^2 A}}$$
$$= \pm\sqrt{\frac{(1-\cos A)^2}{\sin^2 A}}$$
$$= \frac{1-\cos A}{\sin A}$$

Since $\tan\frac{A}{2}$ and $\sin A$ have the same sign wherever $\tan\frac{A}{2}$ is defined, only the positive sign occurs.

66. D

67. G

68. A

69. I

70.
$$\sin\frac{135°}{2} = \sqrt{\frac{1-\cos 135°}{2}}$$
$$= \sqrt{\frac{1-\left(-\frac{\sqrt{2}}{2}\right)}{2}}$$
$$= \sqrt{\frac{\frac{2+\sqrt{2}}{2}}{2}}$$
$$= \frac{\sqrt{2+\sqrt{2}}}{2}$$

71. $\frac{\sqrt{2}}{2}$

72. $\frac{\sqrt{3}}{2}$

73. $\sqrt{3}$

74. about 4.1; 12

75. 2; 4

76. 45.2

77. 26.6

78. 57.6