Unit 5 - Review

Score: _____/ 31 Name:_____

The first derivative of the function $f$ is defined by $f'(x) = \sin(2x)$ for $0 < x \leq 3$. On what intervals is $f$ decreasing?

An ant is walking along the curve $x^2 + xy + y^2 = 19$. If the ant is moving to the **right** at the rate of 3 cm/sec, how fast is the ant moving **up or down** when the ant reaches the point $(2, 3)$. Be sure you specify direction.

Let $f$ be a function such that $f''(x) > 0$ for all $x$ in the closed interval $[0, 1]$. Selected values of $f$ are shown in the table below.

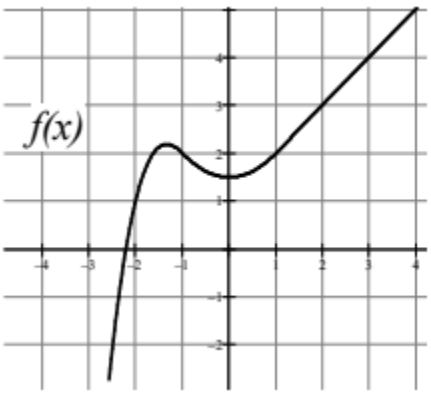
| $x$ | 0.4 | 0.5 | 0.6 | 0.7 |
|---|---|---|---|---|
| $f(x)$ | 5.76 | 5.46 | 5.29 | 5.14 |

Which of the following must be true about $f'(0.5)$?

(A) $f'(0.5) < -3$ (B) $-3 < f'(0.5) < -1.7$ (C) $-1.7 < f'(0.5) < -1.4$

(D) $-1.4 < f'(0.5) < 0$ (E) $f'(0.5) > 0$

Sketch the derivative of the given function.



A particle moves along the $x$-axis with velocity given by $v(t) = \frac{10\sin(0.4t^2)}{t^2-t+3}$ for time $0 \le t \le 3.5$ where $t$ is measured in minutes, and $v$ is measured in feet per minute.

a. Find the acceleration of the particle at time $t = 2$.

b. Is the particle moving to the left or right at $t = 2$? Justify your answer with specific values.

At time $t \ge 0$, the position of a particle moving along the $x$-axis is given by $x(t) = \frac{t^3}{3} + 2t + 2$. For what value of $t$ in the interval $[0,3]$ will the instantaneous velocity of the particle equal the average velocity of the particle from time $t = 0$ to time $t = 3$?

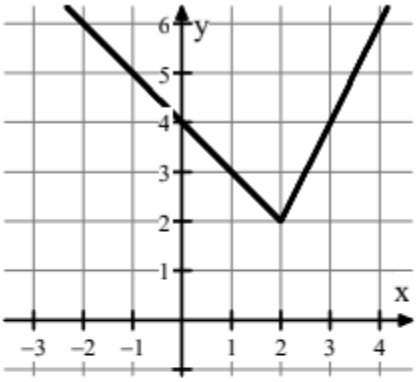
(A) 1 (B) $\sqrt{3}$ (C) $\sqrt{7}$ (D) 3 (E) 5

A rectangle is formed in Quadrant I with one side on the $x$-axis, another on the $y$-axis, and the corner opposite the origin on the graph $y = 9 - x^2$. Find the dimensions of the rectangle with the largest area.

The derivative of $g$ is given by $g'(x) = x^3(4-x)(x-2)$. Find all relative extrema and justify your conclusions.

A particle's position along the $y$-axis is measured by $y(t) = t^3 - 2t^2 - 4t$ where $t > 0$. Find the intervals where the particle is slowing down.

. The graph of the function $h$ is shown below.




Graph of $h$

If $f$ is the function given by $f(x) = h\big(h(x)\big)$, what is the value of $f'(1)$?

(A) 3 (B) $-1$ (C) 5 (D) $-4$ (E) $-2$

. The third derivative of the function $f$ is continuous on the interval $(1, 6)$. Values for $f$ and its first three derivatives at $x = 5$ are given in the table below. What is $\lim_{x\to 5} \frac{f(x)}{(x-5)^2}$?

| $x$ | $f(x)$ | $f'(x)$ | $f''(x)$ | $f'''(x)$ |
|---|---|---|---|---|
| 5 | 0 | 0 | $-1$ | 6 |

(A) $-\frac{1}{2}$ (B) $-\frac{1}{5}$ (C) $-1$ (D) 6 (E) The limit does not exist.