Ans. Key - Unit 2 - Review

Score: _______/ 28 Name:____________________

$W(x)$ is the amount of water (in gallons) in Mr. Sullivan's backyard kiddie pool and t is the number of minutes since he turned his hose on to fill up his pool. Explain the meaning of $W'(5) = 2.7$.

At 5 minutes, the pool is filling up at a rate of 2.7 gallons per minute.

The function f is defined on all real numbers such that $f(x) = \begin{cases} 2x^2 - ax + 6, & x < 3 \\ x + b, & x \geq 3 \end{cases}$. What values of a and b would make the function differentiable at $x = 3$?

$18 - 3a + 6 = 3 + b$

$-3a + 21 = b$

$-3(11) + 21 = b$

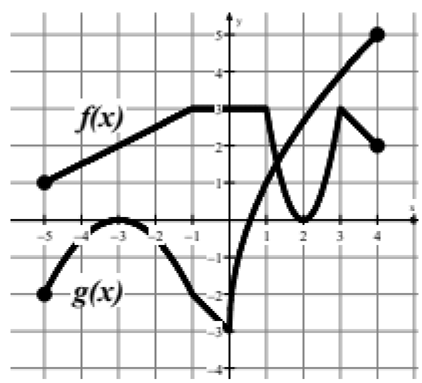
$12 - a = 1$

$-a = -11$

$\boxed{\begin{array}{l} a = 11 \\ b = -12 \end{array}}$

The graphs of f and g are shown to the right. If $j(x) = g(f(x))$. Find the average rate of change of $j(x)$ on the interval $[-3, 4]$

$\dfrac{g(f(4)) - g(f(-3))}{4 - -3} = \dfrac{0 - 0}{7} = \boxed{0}$



Find the value of the derivative at the given point. Round or truncate to three decimal places.

$f(x) = \frac{3x}{\ln(4-x)}$ at $x = 1$.

pts each

$\boxed{3.559}$

Find the equation of the tangent line to $f(x) = 4\ln x + 2x^3$ at $x = 1$.

$f(1) = 2$

$f'(x) = \frac{4}{x} + 6x^2$

$f'(1) = 4 + 6 = 10$

$\boxed{y - 2 = 10(x - 1)}$

1 pt 2 pts

The graph of $f(x)$ is shown to the right. Which of the following statements must be false?

(A) $f(a)$ exists.

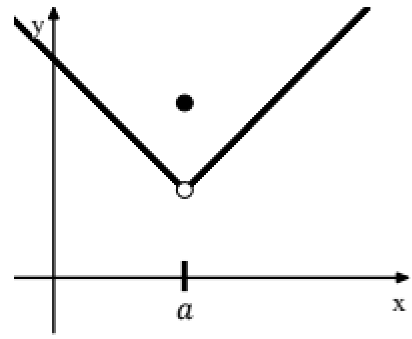
(B) $f(x)$ is defined for $0 < x < a$.

(C) f is not continuous at $x = a$.

(D) $\lim_{x \to a} f(x)$ exists.

(E) $\lim_{x \to a} f'(x)$ exists

E



Use the table to find the value of the derivatives of each function

t	$d(t)$	$d'(t)$	$q(t)$	$q'(t)$
6	-4	-6	-2	1

a. If $g(x) = \left(1 - \frac{d(t)}{2}\right)(4q(t) - 2)$, find $g'(6)$.

$g'(x) = \left(-\frac{1}{2}d'\right)(4q-2) + \left(1 - \frac{d}{2}\right)(4q')$

$(3)(-10) + (3)(4)$

$-30 + 12$

$\boxed{-18}$

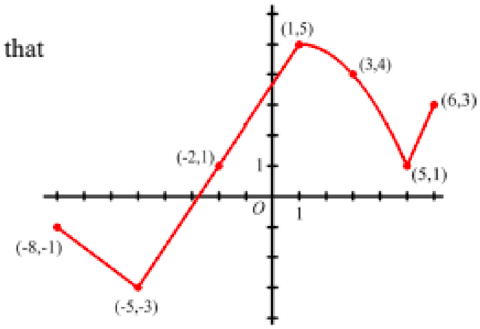
At what x-value(s) does the function $f(x) = \frac{x^4}{4} - 5x^3 + 25x^2 - 30$ have a horizontal tangent?

$x^3 - 15x^2 + 50x$

$x(x-10)(x-5) = 0$

$\boxed{x = 0, 5, 10}$

A continuous function g is defined on the closed interval $-8 \le x \le 6$ and is shown below.



(a) Find the approximate value of $g'(4)$. Show the computations that lead to your answer.

pts $\frac{1-4}{5-3} = \boxed{-3/2}$

(b) Let h be the function defined by $h(x) = \frac{g(x)}{x^2+1}$. Find $h'(-2)$.

pts $h' = \frac{g' \cdot (x^2+1) - g \cdot (2x)}{(x^2+1)^2} = \frac{4/3(5) - 1(-4)}{25} = \boxed{\frac{32}{75} \text{ or } 0.4267}$

Find the derivative of each function.

$r = 3\sqrt{t} - 7t$

pts each

$\boxed{\frac{dr}{dt} = \frac{3}{2\sqrt{t}} - 7}$

$y = 5\ln x + \csc x$

$\boxed{\frac{dy}{dx} = \frac{5}{x} - \csc x \cot x}$

$h(x) = \frac{\ln x}{3x}$

$(x) = \frac{\frac{1}{x}(3x) - \ln x \cdot (3)}{9x^2}$

$h'(x) = \frac{3 - 3\ln x}{9x^2}$

or

$\boxed{h'(x) = \frac{1 - \ln x}{3x^2}}$

$d(t) = 6e^t(4 - t^2)$

$d'(t) = 6e^t(4 - t^2) + 6e^t(-2t)$

$\boxed{d'(t) = 6e^t(4 - t^2) - 12te^t \text{ or } d'(t) = 6e^t(-t^2 - 2t + 4)}$