Ans. Key - Unit 3 - Review

Score: ________/ 33 Name:______________________

Find the equation of any *vertical* tangent lines for the graph of $3y - y^3 = 5x - 2$

+1 pt each

$3\frac{dy}{dx} - 3y^2\frac{dy}{dx} = 5$

+2 → $\frac{dy}{dx} = \frac{5}{3-3y^2}$

+1 → $3 - 3y^2 = 0$

$y^2 = 1$

$y = \pm 1$

$3 - 1 = 5x - 2$

$4 = 5x$

$-3 + 1 = 5x - 2$

$0 = 5x$

$\boxed{x = 0 \quad x = 4/5}$

Let f and g be differentiable functions where $g(x) = f^{-1}(x)$ for all x. Let $f(-4) = -2$, $f(-1) = -4$, $f'(-4) = 5$, and $f'(-1) = 3$. What is the value of $g'(-4)$?

$$\frac{1}{f'(f^{-1}(-4))} = \frac{1}{f'(-1)} = \boxed{1/3}$$

x	$g(x)$	$g'(x)$	$h(x)$	$h'(x)$
-1	1	-2	5	-2
1	4	2	-1	3
4	5	-3	1	2
5	-1	3	4	-3

1. Find $\frac{d}{dx} g^{-1}(4)$.

$$\frac{1}{g'(g^{-1}(4))} = \frac{1}{g'(1)} = \boxed{1/2}$$

Find $\frac{dy}{dx}$.

$y = \tan^{-1}(3x)$

$$\boxed{\frac{3}{9x^2+1}}$$

$y = \ln(x^3 - 2)$

$$\frac{1}{x^3-2} \cdot (3x^2)$$

$$\boxed{\frac{3x^2}{x^3-2}}$$

Find the slope of the tangent line at the given point.

$y = \arctan\left(\frac{x}{2}\right)$ at $x = 2\sqrt{3}$.

$$y' = \frac{1}{\frac{x^2}{4}+1} \cdot \left(\frac{1}{2}\right)$$

$$y'(2\sqrt{3}) = \frac{1}{3+1}\left(\frac{1}{2}\right) = \boxed{\frac{1}{8}}$$

$4 = 5x + y^2$ at $(-1, 3)$

$$0 = 5 + 2y\frac{dy}{dx}$$

$$-5 = 2(3)\frac{dy}{dx}$$

$$\boxed{-\frac{5}{6}}$$

Evaluate $\frac{d^2y}{dx^2}$ at the given point.

$y = e^{3x}$ at $x = \frac{1}{3}$.

$$\frac{dy}{dx} = e^{3x} \cdot 3$$

$$\frac{d^2y}{dx^2} = e^{3x} \cdot 9$$

$$\boxed{9e}$$

Let f be a function with $f(1) = -2$. The derivative of f is given by $f'(x) = \cos\left(\frac{\pi x}{2}\right) + x^2 - 5$.

Find $f''(3)$.

$$f''(x) = -\sin\left(\tfrac{\pi}{2}x\right)\cdot\left(\tfrac{\pi}{2}\right) + 2x$$

$$f''(3) = -\tfrac{\pi}{2}\sin\left(\tfrac{3\pi}{2}\right) + 2(3)$$

$$-\tfrac{\pi}{2}(-1) + 6$$

$$\boxed{\tfrac{\pi}{2} + 6}$$

Write an equation for the line tangent to the graph of $y = \frac{1}{f(x)}$ at $x = 1$.

$$y(1) = \frac{1}{f(1)} = \frac{1}{-2} = -\frac{1}{2}$$

$+1 \rightarrow \left[\; y' = -\frac{1}{f^2}\cdot f' \right.$

$$y'(1) = -\frac{1}{(-2)^2}\left(\cos\tfrac{\pi}{2} + 1 - 5\right) = 1$$

$$\boxed{y + \tfrac{1}{2} = 1\cdot(x-1)}$$

1 pt 2 pts

Let g be the function defined by $g(x) = f\left(\sqrt{17 - 2x^2}\right)$. Find $g'(2)$.

$$g'(x) = f'\left(\sqrt{17-2x^2}\right)\cdot\left(\frac{1}{2\sqrt{17-2x^2}}\right)(-4x)$$

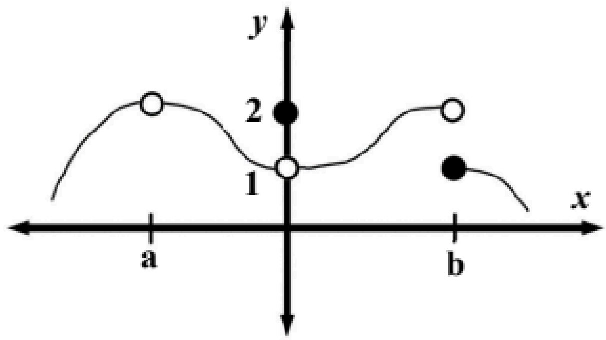
$$g'(2) = f'(3)\left(-\tfrac{4}{3}\right)$$

$$\left[\cos\left(\tfrac{3\pi}{2}\right) + 9 - 5\right]\left(-\tfrac{4}{3}\right) = \boxed{-\tfrac{16}{3}}$$

Let h be the inverse function of f. Find $h'(-2)$.

$$\frac{1}{f'(f^{-1}(-2))} = \frac{1}{f'(1)} = \frac{1}{\cos\left(\frac{\pi}{2}\right) + 1 - 5} = \boxed{-\tfrac{1}{4}}$$

B



4. The graph of the function f is shown in the figure above. Which of the following statements about f is true?

(A) $f(a)$ exists

(B) $\lim_{x \to a} f(x) = 2$

(C) $\lim_{x \to b} f(x) = 1$

(D) $\lim_{x \to b^-} f(x) = \lim_{x \to b^+} f(x)$

(E) f is continuous at $x = 0$