Ans. Key - Unit 4 - Review

Score: _____/ 31 Name:_____

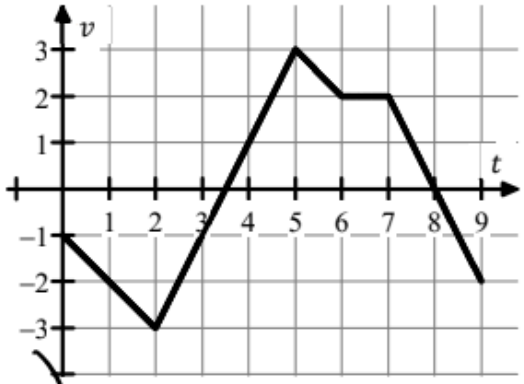
. The velocity of a particle moving along the x-axis is given by $v(t)$ for $t \geq 0$. The following table gives information about $v(t)$ and $v'(t)$.

t	$0 < t < 1$	1	$1 < t < 2$	2	$2 < t < 3$	3	$t > 3$
$v(t)$	Negative	0	Positive	Positive	Positive	0	Negative
$v'(t)$	Positive	Positive	Positive	0	Negative	Negative	Negative

a. Suppose that on the interval $(0, 3)$, the particle lies in the positive ray of the x-axis. At what times in the interval $(0, 3)$ is the particle moving away from the origin?

2 pts $(1, 3)$

The graph shows the velocity $v(t)$, of a particle moving along the x-axis for $0 \leq t \leq 9$.



a. At what time(s) is the speed of the particle greatest?

t=2 and t=5

b. When does the particle reverse direction?

2 t=3.5 and t=8

c. What time interval(s) is the particle speeding up?

3 $(0,2)$ $(3.5, 5)$, and $(8,9)$

d. What time interval(s) is the particle moving left?

2 $(0, 3.5)$ and $(8,9)$

e. At which of the times, $t = 1, t = 3, t = 5$ or $t = 6.5$, is the acceleration of the particle greatest? Justify.

+3 $t=3$ because $V'(3) >$ than all other values.

Consider the differential equation $\frac{dy}{dx} = 2xy$. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(4) = 1$.

a. Write an equation for the line tangent to the graph of f at the point (4,1).

$m = 2(4)(1) = 8$

2 $y - 1 = 8(x - 4)$

b. Use the tangent line to approximate $f(4.1)$.

+2 1.8

A railroad track and a road cross at right angles. An observer stands on the road 70 meters south of the crossing and watches an eastbound train traveling at 60 meters per second. At how many meters per second is the train moving away from the observer 4 seconds after it passes through the intersection?

3 A

(A) 57.60 (B) 57.88 (C) 59.20 (D) 60.00 (E) 67.40

Find the following.

$\lim_{x\to 0} \frac{2e^x-2-2x}{x^2}$

$\frac{2e^x-2}{2x}$

$\lim_{x\to\infty} \frac{2e^x}{2} = 1$

$\lim_{x\to -2} \frac{x^2+6x+8}{x+2}$

$\lim_{x\to -2} \frac{2x+6}{1} = 2$

A right triangle is changing shape. The base is increasing at the rate of 5 inches/sec while the height is increasing at a rate of 2 inches/sec. When the base is 4 inches and the height is 6 inches, how fast is the area changing? $\left(A = \frac{1}{2}bh\right)$

5

$\frac{db}{dt} = 5$

$\frac{dh}{dt} = 2$

$b = 4$

$h = 6$

$\frac{dA}{dt} = ?$

$\frac{dA}{dt} = \frac{1}{2}\frac{db}{dt}h + \frac{1}{2}b\frac{dh}{dt}$ ← 2

$= \frac{1}{2}(5)(6) + \frac{1}{2}(4)(2)$

19 in^2/sec

Let f be the function defined by $f(x) = \begin{cases} \frac{x^2-5x-14}{(x-7)}, & x \neq 7 \\ c, & x = 7 \end{cases}$. For what value of c is f continuous at $x = 7$?

2

$\frac{(x-7)(x+2)}{(x-7)} = c$

$x+2 = c$

$9 = c$

A circular oil slick of uniform height contains 100 cm^3 of oil. Recall that the volume of a cylindrical prism is modeled by $V = \pi r^2 h$. As the oil spreads, the height is decreasing at the rate of 0.01 cm/min. At what rate is the radius of the slick increasing when the **diameter** is 20 cm?

$V = 100$ $\quad \frac{dh}{dt} = 0.01$ $\quad r = 10$

$100 = 100\pi h$ $\quad \frac{1}{\pi} = h$ (1 pt)

2 pts $\quad \frac{dV}{dt} = 2\pi r \frac{dr}{dt} h + \pi r^2 \frac{dh}{dt}$

$0 = 2\pi(10)\frac{dr}{dt}\left(\frac{1}{\pi}\right) + \pi(100)(-0.01)$

$\frac{dr}{dt} = \frac{\pi}{20}$ cm/min (1 pt)

A particle moves along the x-axis with velocity given by $v(t) = \frac{10\sin(0.3t^2)}{t^2 - t + 3}$ for time $0 \le t \le 3.5$.

a. Find the acceleration of the particle at time $t = 2$.

·2 $\quad v'(2) = -0.2487$

b. Is the particle moving to the left or right at $t = 2$? Justify your answer with specific values.

·3 $v(2) = 1.864$ $\quad$ Right because $v(2) > 0$.