Ans. Key - Unit 5 - Review

Score: _______/ 31 Name:____________________

The first derivative of the function f is defined by $f'(x) = \sin(2x)$ for $0 < x \le 3$. On what intervals is f decreasing?

$(\pi/2, 3)$ or $(1.5707, 3)$

An ant is walking along the curve $x^2 + xy + y^2 = 19$. If the ant is moving to the **right** at the rate of 3 cm/sec, how fast is the ant moving **up or down** when the ant reaches the point $(2, 3)$. Be sure you specify direction.

+2 pts

$$2x\frac{dx}{dt} + \frac{dx}{dt}y + x\frac{dy}{dt} + 2y\frac{dy}{dt} = 0$$

$$2(2)(3) + (3)(3) + (2)\frac{dy}{dt} + 6\frac{dy}{dt} = 0$$

$$8\frac{dy}{dt} = -21$$

+1 pt 1 pt

$\boxed{\text{Down at the rate of } 21/8 \text{ cm/sec.}}$

Let f be a function such that $f''(x) > 0$ for all x in the closed interval $[0, 1]$. Selected values of f are shown in the table below.

x	0.4	0.5	0.6	0.7
$f(x)$	5.76	5.46	5.29	5.14

Which of the following must be true about $f'(0.5)$?

B $\quad \dfrac{5.46 - 5.76}{0.5 - 0.4} = -3 \qquad \dfrac{5.29 - 5.46}{0.6 - 0.5} = -1.7$

(A) $f'(0.5) < -3$ (B) $-3 < f'(0.5) < -1.7$ (C) $-1.7 < f'(0.5) < -1.4$

(D) $-1.4 < f'(0.5) < 0$ (E) $f'(0.5) > 0$

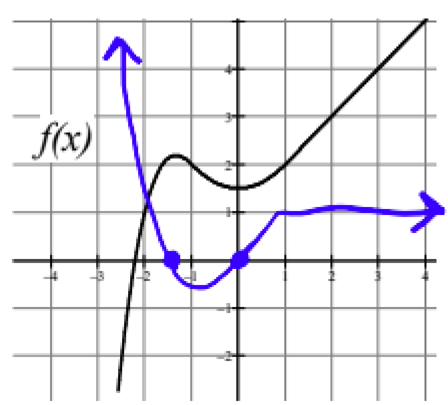
Sketch the derivative of the given function.

+1 pt zero at $x = -1.5$

+1 pt zero at $x = 0$

+1 pt constant for $x > 1$.

+1 pt general shape of f'



A particle moves along the x-axis with velocity given by $v(t) = \frac{10\sin(0.4t^2)}{t^2-t+3}$ for time $0 \le t \le 3.5$ where t is measured in minutes, and v is measured in feet per minute.

a. Find the acceleration of the particle at time $t = 2$.

$$a(2) \approx \underbrace{-1.2929}_{+2\text{ pts}}\ \underbrace{\text{ft/min}^2}_{+1\text{ pt}}$$

b. Is the particle moving to the left or right at $t = 2$? Justify your answer with specific values.

$v(2) \approx 1.999$ Right because $v(2) > 0$.

At time $t \ge 0$, the position of a particle moving along the x-axis is given by $x(t) = \frac{t^3}{3} + 2t + 2$. For what value of t in the interval $[0,3]$ will the instantaneous velocity of the particle equal the average velocity of the particle from time $t = 0$ to time $t = 3$?

$$\frac{x(3)-x(0)}{3-0} = 5 \qquad x'(t) = t^2+2$$

$$5 = t^2+2$$
$$3 = t^2$$

B

(A) 1 (B) $\sqrt{3}$ (C) $\sqrt{7}$ (D) 3 (E) 5

A rectangle is formed in Quadrant I with one side on the x-axis, another on the y-axis, and the corner opposite the origin on the graph $y = 9 - x^2$. Find the dimensions of the rectangle with the largest area.

$$A = x(9-x^2)$$
$$A = 9x - x^3$$
+2 pts → $9 - 3x^2 = 0$
$$x^2 = 3$$

$9 - (\sqrt{3})^2 = 6$ ← +1 pt

$$\boxed{\sqrt{3} \times 6}$$

+1 pt

The derivative of g is given by $g'(x) = x^3(4-x)(x-2)$. Find all relative extrema and justify your conclusions.

$x^3 = 0 \quad 4 - x = 0 \quad x - 2 = 0$

$x = 0 \quad x = 4 \quad x = 2$

x	$(-\infty, 0)$	0	$(0,2)$	2	$(2,4)$	4	$(4,\infty)$
g'	+	0	−	0	+	0	−

Rel max at $x=0$ and $x=4$ because g' changes sign from positive to negative.
Rel min at $x=2$ because g' changes sign from negative to positive.

A particle's position along the y-axis is measured by $y(t) = t^3 - 2t^2 - 4t$ where $t > 0$. Find the intervals where the particle is slowing down.

$y'(t) = 3t^2 - 4t - 4$

$(3t+2)(t-2)$

+1 pt → $t = -\frac{2}{3} \quad t = 2$

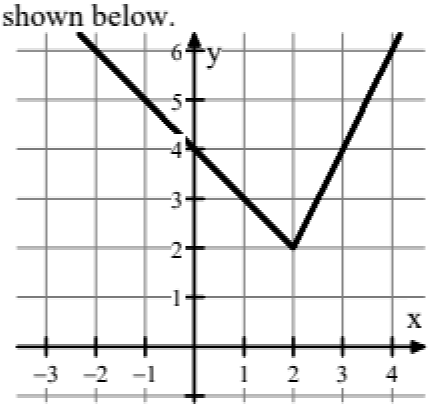
$y''(t) = 6t - 4 = 0$

+1 pt → $t = \frac{2}{3}$

t	$(0, \frac{2}{3})$	$\frac{2}{3}$	$(\frac{2}{3}, 2)$	2	$(2, \infty)$
y'	−	−	−	0	+
y''	−	0	+	+	+

$\boxed{(\frac{2}{3}, 2)}$ ← +2 pts

. The graph of the function h is shown below.



Graph of h

If f is the function given by $f(x) = h(h(x))$, what is the value of $f'(1)$?

$f'(1) = h'(h(1)) \cdot h'(1)$

$h'(3) \cdot (-1)$

$2 \cdot (-1)$

E

(A) 3 (B) −1 (C) 5 (D) −4 (E) −2

The third derivative of the function f is continuous on the interval $(1, 6)$. Values for f and its first three derivatives at $x = 5$ are given in the table below. What is $\lim_{x \to 5} \frac{f(x)}{(x-5)^2}$?

A

x	$f(x)$	$f'(x)$	$f''(x)$	$f'''(x)$
5	0	0	-1	6

$$\frac{0}{0} \rightarrow \lim_{x \to 5} \frac{f'(x)}{2(x-5)} = \frac{0}{0} \longrightarrow \lim_{x \to 5} \frac{f''(x)}{2} = \frac{-1}{2}$$

(A) $-\frac{1}{2}$ (B) $-\frac{1}{5}$ (C) -1 (D) 6 (E) The limit does not exist.