Ans. Key - Unit 6 - Review

Score: _______/ 35 Name:____________________

If $\int_4^{-10} g(x)\,dx = -3$ and $\int_4^{6} g(x)\,dx = 5$, find $\int_{-10}^{6} g(x)\,dx =$ $3+5=\boxed{8}$

$\int_{-10}^{4} g(x) = 3$

x	0	5
$f(x)$	3	-2
$g(x)$	0	2
$g'(x)$	1	-3

Let f be the function given by $f(x) = \int_{-1}^{x} g(t)\,dt$ where g is a differentiable function. The table above gives selected values of f, g, and g'. If h is the function given by $h(x) = x^2 - e^x + 1$ for which of the following values of x is $h(x) = f'(5)$?

$2x - e^x = g(5)$

$2x - e^x = 2$

B

(A) -2.032 (B) -1.147 (C) 0 (D) 1.873 (E) 2.158

Suppose $g(x)$ is a continuous function. A table of selected values of $g(x)$ is shown below.

x	0	3	6	9	12	15	18
$g(x)$	-4	-2	3	4	9	5	1

The approximate value of $\int_0^{18} g(x)dx$ using a midpoint Riemann sum with three subintervals of equal length is

$6(-2) + 6(4) + 6(5)$

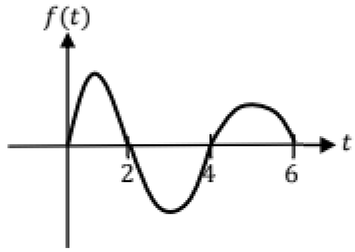
$-12 + 24 + 30$

B

(A) 48 (B) 42 (C) 39 (D) 24 (E) 21

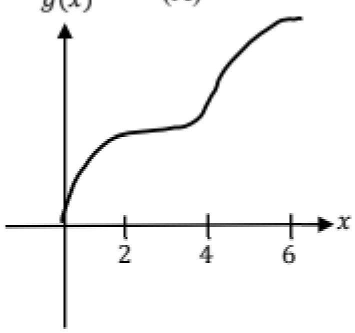
Let $g(x) = \int_0^x f(t)\,dt$, where $f(t)$ has the graph shown below.

B

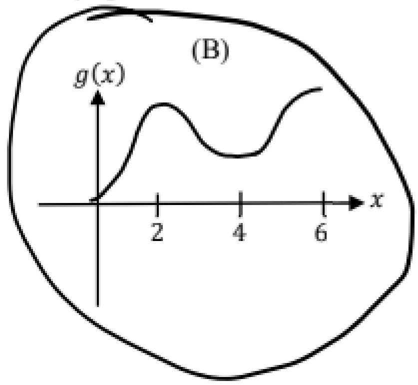


Which of the following could be the graph of g?

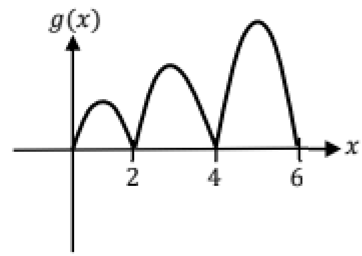
(A)



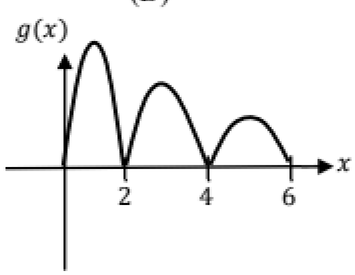
(B)



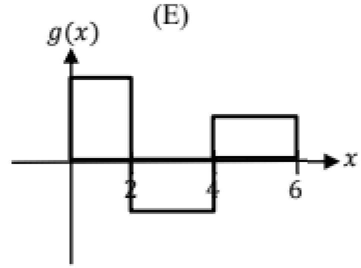
(C)



(D)



(E)



Find the following indefinite integrals.

$\int 10x\sqrt[3]{5x^2-4}\,dx$

$u = 5x^2 - 4$

$\frac{du}{10x} = dx$

$\int \sqrt[3]{u}\,du$

$\frac{3}{4}u^{4/3} + C$

$\boxed{\frac{3}{4}(5x^2-4)^{4/3} + C}$

$\int xe^{x^2}dx$

$u = x^2$

$\frac{du}{2x} = dx$

$\frac{1}{2}\int e^u\,du$

$\boxed{\frac{1}{2}e^{x^2} + C}$

. $\int \sin x \, e^{\cos x} \, dx$

$u = \cos x$

$\frac{du}{-\sin x} = dx$

$-\int e^{u} \, du$

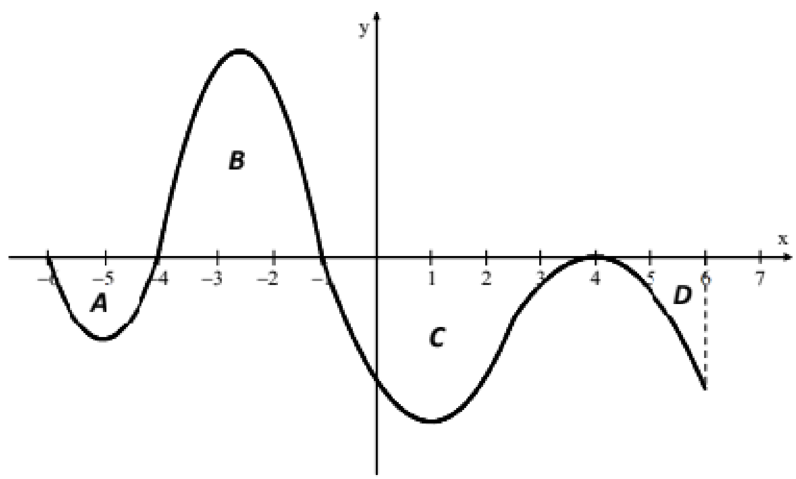
$\boxed{-e^{\cos x} + C}$

12. $\int \frac{1}{\sqrt{-x^2-8x-15}} \, dx$

$-(x^2+8x+16)-15+16$

$\int \frac{1}{\sqrt{1-(x+4)^2}} \, dx$

$\boxed{\sin^{-1}(x+4) + C}$



The figure above shows the graph of the continuous function f. The regions A, B, C, and D have areas 4, 13, 16, and 3, respectively. For $-6 \le x \le 6$, the function g is defined by $g(x) = 4 + \int_{-1}^{x} f(t)\,dt$.

(a) Is there a value x, for $-1 \le x \le 4$, such that $g(x) = 0$? Justify your answer.

Yes. Region C will cause you to subtract 16. On the interval $-1 \le x \le 4$, $g(x)$ will be 0.

(b) Find the absolute minimum value of g on the interval $-6 \le x \le 6$.

$g(-6) = 4 - 13 + 4 = -5$

$g(-4) = 4 - 13 = -9$

$g(-1) = 4 + 0 = 4$

$g(4) = 4 - 16 = -12$

$g(6) = 4 - 16 - 3 = -15$

$\boxed{-15}$

(c) Find the value of $\int_1^{-1} f(5-x)\,dx$

$u = 5 - x$

$-du = dx$

$\int_4^6 f(u)\,(-du)$

$-\int_4^6 f(u)\,du = -(-3) = \boxed{3}$

Find the value of the definite integral.

15. $\int_0^{\frac{\pi}{6}} \sin(3x)\cos(3x)\,dx$

$u = \sin(3x)$

$\frac{du}{3\cos(3x)} = dx$

$\frac{1}{3}\int_0^1 u\,du$

$\frac{1}{3}\frac{u^2}{2}\Big|_0^1$

$\frac{1}{6}[1 - 0]$

$\boxed{1/6}$

. $\int_0^4 \frac{2}{\sqrt{2x+1}}\,dx$

$u = 2x + 1$

$\frac{du}{2} = dx$

$\int_1^9 u^{-\frac{1}{2}}\,du$

$2u^{\frac{1}{2}}\Big|_1^9$

$2[3 - 1] = \boxed{4}$

17. $\int_0^{\ln 3} e^x(4 - e^x)dx$

$u = 4 - e^x$

$\frac{du}{-e^x} = dx$

$-\int_3^1 u\,du$

$-\frac{u^2}{2}\Big|_3^1$

$-\left[\frac{1}{2} - \frac{9}{2}\right] = \boxed{4}$

9. A curve given by the equation $x^3 + xy = 8$ has slope given by $\frac{dy}{dx} = \frac{-3x^2 - y}{x}$. The value of $\frac{d^2y}{dx^2}$ at the point where $x = 2$ is

C

$2^3 + 2y = 8$
$2y = 0$
$y = 0$

$\frac{dy}{dx}_{(2,0)} = \frac{-3(4) - 0}{2} = -6$

$\frac{d^2y}{dx^2} = \frac{\left(-6x - \frac{dy}{dx}\right)(x) - (-3x^2 - y)(1)}{x^2}$

$\frac{d^2y}{dx^2}_{(2,0)} = \frac{(-12 - -6)(2) - (-12 - 0)}{4} = \frac{-12 + 12}{4}$

(A) -6 (B) -3 (C) 0 (D) 4 (E) undefined