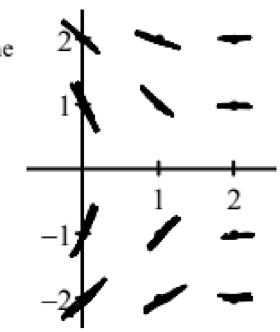
Unit 7 - Review

Score: _______/ (AB)38-(BC)40 Name:_____________________

1. Consider the differential equation $\frac{dy}{dx} = \frac{x-2}{y}$.

+2 pts

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.



(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy-plane for which $y \neq 0$. Describe all points in the xy-plane, $y \neq 0$, for which $\frac{dy}{dx} = 1$.

+2

$$\frac{x-2}{y} = 1$$
$$x - 2 = y$$

$\boxed{\text{All points that fall on the line } y = x - 2}$

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = -3$.

+4 pts

$$y\,dy = (x-2)\,dx$$
$$\frac{y^2}{\cancel{2}} = \frac{x^2}{\cancel{2}} - 2x + C_1$$
$$y^2 = x^2 - 4x + C_2$$

$9 = C_2$ +1 pt

+1 pt

$$\boxed{y = -\sqrt{x^2 - 4x + 9}}$$

2. A population y grows according to the equation $\frac{dy}{dt} = ky$, where k is a constant and t is measured in years. If the population doubles every 21 years, then what is the value of k?

+2 pts

$$2 = e^{21k}$$
$$k = \frac{\ln 2}{21} \simeq \boxed{0.033}$$

3. Mr. Brust is running down his street. His position is given by the function $p(t)$, where t is measured in minutes since the start of his run. His velocity is inversely proportional to the natural logarithm of the time since the start of his run. What is a differential equation that represents this relationship? Use k as a constant.

+2 pts

$$\boxed{\frac{dp}{dt} = \frac{k}{\ln(t)}}$$

4. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = 3y - x$ with initial condition $f(1) = 3$. What is the approximation for $f(0)$ obtained using Euler's method with 2 steps of equal length, starting at $x = 1$?

$\Delta x = \frac{0-1}{2} = -0.5$

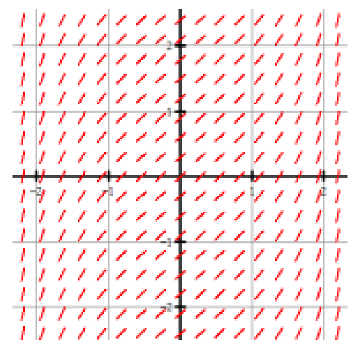
$y = y_1 + y' \cdot \Delta x$

x	y	y'	new y
1	3	$3(3)-1 = 8$	$3+8(-0.5) = -1$
0.5	-1	$3(-1)-0.5 = -3.5$	$-1-3.5(-0.5) = 0.75$
0	0.75		

+3

$\boxed{f(0) \approx 0.75}$

5. Which of the following could be a path through the slope field created by the differential equation $\frac{dy}{dx} = \sec^2(0.5x)$?



(A) $y = \tan x$

(B) $y = 2\tan x + 1$

+3

(C) $y = \frac{1}{2}\tan(x) + 4$

D

(D) $y = 2\tan(0.5x) - 1$

(E) $y = \frac{1}{2}\tan(0.5x) + 2$

6. During a certain epidemic, the number of people that are infected at any time increases at a rate proportional to the number of people that are infected at that time. If 1,300 people are infected when the epidemic is first discovered, and 2,000 people are infected 6 days later, how many people are infected 10 days after the epidemic is first discovered? Round down to the nearest whole number.

$\frac{dp}{dt} = kp$

$p = 1300e^{kt}$

$2000 = 1300e^{k(6)}$

$\ln\left(\frac{20}{13}\right) = 6k$

$k \approx 0.0717971$

$P(10) = \boxed{2665 \text{ people}}$

+3

a

7. For what value of k, if any, is $y = e^{4x} - ke^{-2x}$ a solution to the differential equation $\frac{y''}{2} - 2y' = 12e^{-2x}$?

$$y' = 4e^{4x} + 2ke^{-2x}$$

$$y'' = 16e^{4x} - 4ke^{-2x}$$

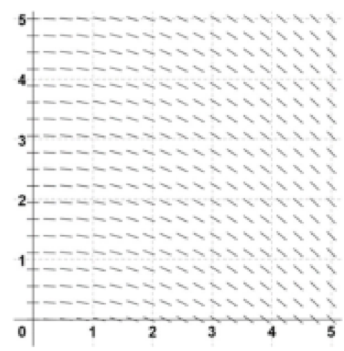
$$8e^{4x} - 2ke^{-2x} - 2\left[4e^{4x} + 2ke^{-2x}\right] = 12e^{-2x}$$

$$-6ke^{-2x} = 12e^{-2x}$$

$$\boxed{k = -2}$$

+2 pts

8. Explain why the following slope field cannot represent the differential equation $\frac{dy}{dt} = -0.2y$



+2 pts

Possible answer: When $y = 0, \frac{dy}{dt} = 0$. However, in the slope field, the slopes of the line segments for $y = 0$ are nonzero.

$\frac{dy}{dx} = x^2 e^{x^3}$

$$y = \int \tfrac{1}{3} e^u \, du \qquad u = x^3 \qquad \frac{du}{3x^2} = dx$$

$$\boxed{y = \tfrac{1}{3}e^{x^3} + C}$$

$\frac{dy}{dx} = 2xy^2$

$$y^{-2}\,dy = 2x\,dx$$

$$-\frac{1}{y} = x^2 + C$$

$$\boxed{y = -\frac{1}{x^2 + C}}$$

$\frac{dy}{dx} = 12\cos(4x) - e^{2x};\ (0, 1)$

$y = 3\sin(4x) - \frac{1}{2}e^{2x} + C$

$1 = 3\sin(0) - \frac{1}{2}e^{0} + C$

$1 = -\frac{1}{2} + C$

$\frac{3}{2} = C$

$\boxed{y = 3\sin(4x) - \frac{1}{2}e^{2x} + \frac{3}{2}}$

$\frac{d^2y}{dx^2} = 9x^2 - 12x$ and $y'(2) = 4$ and $y(1) = \frac{19}{4}$

$\frac{dy}{dx} = 3x^3 - 6x^2 + C_1$

$4 = 3(8) - 24 + C_1$

$4 = C_1$

$\frac{dy}{dx} = 3x^3 - 6x^2 + 4$

$y = \frac{3}{4}x^4 - 2x^3 + 4x + C_2$

$\frac{19}{4} = \frac{3}{4} - 2 + 4 + C_2$

$\frac{16}{4} = 2 + C_2$

$2 = C_2$

$\boxed{y = \frac{3}{4}x^4 - 2x^3 + 4x + 2}$

20. The position of a particle moving along the x-axis is given by $x(t) = e^{2t} - e^{t}$. When the particle is at rest, the acceleration of the particle is

+3 pts

A

$0 = 2e^{2t} - e^{t}$

$0 = e^{t}(2e^{t} - 1)$

$\frac{1}{2} = e^{t}$

$\ln(\frac{1}{2}) = t$

$x''(t) = 4e^{2t} - e^{t}$

$x''(\ln(\frac{1}{2})) = 4e^{2\ln\frac{1}{2}} - e^{\ln(\frac{1}{2})}$

$= 4e^{\ln\frac{1}{4}} - \frac{1}{2}$

$= 1 - \frac{1}{2}$

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\ln\frac{1}{2}$ (D) 2 (E) 4

19. A population changes at a rate modeled by the logistic differential equation $\frac{dy}{dt} = 4000y - \frac{1}{2}y^2$, where t is measured in years. What are all values of y for which the population is increasing at a $\underbrace{\text{decreasing rate}}_{y''<0}$?

$y'' = 4000 - y$

$\frac{dy}{dt} = \frac{1}{2}y(8000 - y)$

$L = 8000$

+3 pts

$0 = 4000 - y$

$y = 4000$

y	0		4000		8000
y''		pos	0	neg	

$\boxed{4000 < y < 8000}$