Unit 8 - Review

Score: _______/ (AB)30-(BC)35 Name:____________________

+3pts

1. The depth of the ocean just off the coast changes according the tides. The rate at which it is changing can be modeled by $R(t) = 2.12\sin\left(\frac{\pi}{4}t\right)$, where $R(t)$ is feet per hour and t is hours after 9:00 a.m. If the depth of the ocean is 12 feet at this particular spot, how deep will it be at 11:00 a.m.?

$$12 + \int_0^2 R(t)\,dt = \boxed{14.699 \text{ ft}}$$

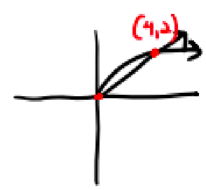
+3pts

2. Traffic flow measures the number of cars that pass through an intersection per minute. It can be modeled by the function $f(t) = 10 + 8\cos\left(\frac{t}{3}\right)$ for $0 \le t \le 15$ where $f(t)$ is measured in cars per minute and t is measured in minutes. Is the traffic flow increasing or decreasing at $t = 10$? Give a reason for your answer.

$f'(10) = 0.508$ $\boxed{\text{Increasing b/c } f'(10) > 0.}$

4. Let R be the region in the first quadrant bounded by the graphs of $y = \sqrt{x}$ and $y = \frac{x}{2}$.

$y^2 = x$ $2y = x$



+2pts

a. Find the area of R.

$$A = \int_0^4 \left(\sqrt{x} - \frac{x}{2}\right)dx = \boxed{1.333}$$

+2pts

b. Find the volume of the solid generated when R is rotated about the vertical line $x = -2$.

$$V = \pi\int_0^2 \left[(2y+2)^2 - (y^2+2)^2\right]dy$$

$$V \approx \boxed{30.159}$$

+2pts

c. The region R is the base of a solid. For this solid, the cross sections perpendicular to the y-axis are semicircles. Find the volume of this solid.

$$V = \int_0^2 \frac{\pi}{2}\left(\frac{2y - y^2}{2}\right)^2 dy \approx \boxed{0.4188}$$

$\boxed{15}$

6. The area of the region in the first quadrant bounded by the graph of $f(x) = \frac{\ln x}{x}$ and the lines $x = 1$ and $x = e$ is

+3pts

B

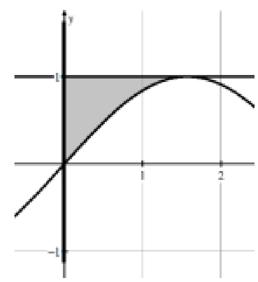
(A) $\frac{1}{3}$ (B) $\frac{1}{2}$ (C) 1 (D) e (E) e

7. Setup integral(s) with respect to y that represent the area bounded by $y = \sin(x)$, $y = 1$, and $x = 0$. Do NOT evaluate.

+2pts

$\sin^{-1}(y) = x$

$$\boxed{\int_0^1 \sin^{-1}(y)\, dy}$$

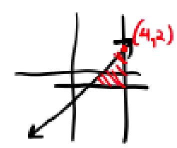


$x = y+2$

8. Revolve the region bounded by the graphs of $y = x - 2$, $x = 4$, and $y = -1$ about the line $x = 4$. Set up the integral for the volume of this solid, but do NOT evaluate.

+2pts

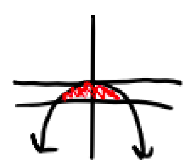
$$\pi \int_{-1}^{2} (4-(y+2))^2 \quad \text{or} \quad \pi \int_{-1}^{2} (y+2-4)^2 \rightarrow \boxed{\pi \int_{-1}^{2} (2-y)^2\, dy \quad \text{or} \quad \pi \int_{-1}^{2} (y-2)^2\, dy}$$



9. Revolve the region bounded by the graphs of $y = -x^2$ and $y = -1$ about the line $y = -1$. Set up the integral for the volume of this solid, but do NOT evaluate.

+2pts

$$\pi \int_{-1}^{1} [(-1)-(-x^2)]^2 dx \quad \text{or} \quad \pi \int_{-1}^{1} [(-x^2)+1]^2 dx \rightarrow \boxed{\pi \int_{-1}^{1} [-1+x^2]^2 dx \quad \text{or} \quad \pi \int_{-1}^{1} [-x^2+1]^2 dx}$$



10. If the region enclosed by the y-axis, the curve $y = 4\sqrt{x}$, and the line $y = 8$ is revolved about the x-axis, the volume of the solid generated is

+3pts

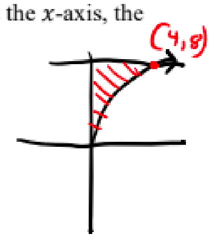
$\pi \int_0^4 \left[(8)^2 - (4\sqrt{x})^2\right] dx$

$\pi \int_0^4 [64 - 16x]\, dx$

$\pi \left[64x - 8x^2\right] \Big|_0^4$

$\pi \left[(256 - 128) - (0)\right]$

B



(A) $\frac{32\pi}{3}$ (B) 128π (C) $\frac{128}{3}$ (D) 128 (E) $\frac{128\pi}{3}$

13. The average value of the function $f(x) = (x-1)^2$ on the interval from $x = 1$ to $x = 5$ is

+3pts

$\frac{1}{5-1} \int_1^5 (x-1)^2\, dx$

$\frac{1}{4} \frac{(x-1)^3}{3} \Big|_1^5$

$\frac{1}{12}\left[(5-1)^3 - (1-1)^3\right]$

$\frac{1}{12}(4^3 - 0)$

$\frac{64}{12} = \frac{32}{6}$

(A) $-\frac{16}{3}$ (B) $\frac{16}{3}$ (C) $\frac{64}{3}$ (D) $\frac{66}{3}$ (E) $\frac{256}{3}$

3. Find the distance traveled (to three decimal places) from $t = 1$ to $t = 5$ seconds, for a particle whose velocity is given by $v(t) = t + \ln t$.

+3pts

C

$\int_1^5 |v(t)|\, dt$

(A) 6.000 (B) 1.609 (C) 16.047 (D) 0.800 (E) 148.413

11. Which of the following integrals gives the length of the curve $y = \sin x^2$ from $x = 0$ to $x = \frac{\pi}{5}$?

A. $\int_0^{\frac{\pi}{5}} \sqrt{1 + 2x \sin x^2}\, dx$

B. $\int_0^{\frac{\pi}{5}} \sqrt{1 + \sin^2(x^2)}\, dx$

C. $\int_0^{\frac{\pi}{5}} \sqrt{1 + 2x \cos x^2}\, dx$

D. $\int_0^{\frac{\pi}{5}} \sqrt{1 + 2x \cos^2(x^2)}\, dx$

E. $\int_0^{\frac{\pi}{5}} \sqrt{1 + 4x^2 \cos^2(x^2)}\, dx$

+3 pts

$y' = \cos x^2 \cdot 2x$

$[y']^2 = 4x^2 \cos^2 x^2$

$\int_a^b \sqrt{1 + 4x^2 \cos^2 x^2}$

E

5. The table below gives the values of f', the derivative of f. If $f(4.2) = 3$, what is the approximation of $f(4.4)$ obtained by using Euler's method with 2 steps of equal size?

x	4.1	4.2	4.3	4.4
$f'(x)$	-0.2	-0.27	-0.32	-0.41

new y = old y + $y'(\Delta x)$

$\Delta x = \frac{4.4 - 4.2}{2} = 0.1$

x	old y	y'	new y
4.2	3	-0.27	$3 + -0.27(0.1) = 2.973$
4.3	2.973	-0.32	$2.973 + -0.32(0.1) = 2.941$
4.4	2.941		

$\boxed{f(4.4) \approx 2.941}$

+2 pts