Score: $\frac{\quad}{60}$ = ______% Calculus – Unit 1

Name: ______________________________ Date:______________ Period: _____

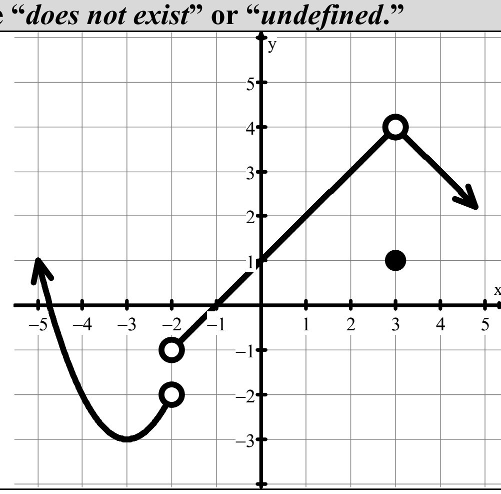
**Test #1**

# Mid-Unit 1 Test – Limits and Continuity

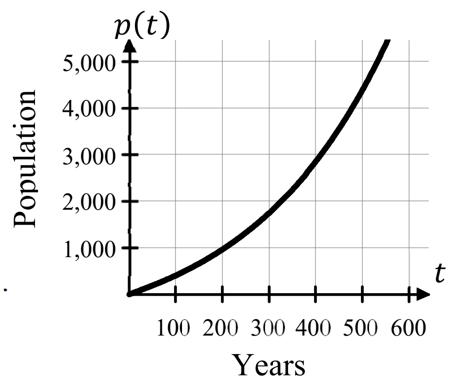
NO CALCULATOR!

**Give the value of each statement. If the value does not exist, write "*does not exist*" or "*undefined*."**

1. $\lim_{x \to 2^-} g(x) =$
2. $g(-2) =$
3. $\lim_{x \to -2} g(x) =$
4. $\lim_{x \to 3} g(x) =$
5. $\lim_{x \to -2^-} g(x) =$
6. $\lim_{x \to -2^+} g(x) =$
7. $g(3) =$
8. $\lim_{x \to 0} g(x) =$




A city's population can be modeled by the function $p$, where $p(t)$ gives the population and $t$ gives the number of years since 1500 for $0 \le t \le 500$. The graph of the function $p$ is shown to the right.




9. Draw a tangent line at $t = 100$.

10. Give a rough estimate of the instantaneous rate of change at $t = 100$.

11. Give an example of how to calculate a rate of change that would give a close estimate to the rate of change in the year 1800.

**Sketch a graph of a function $f$ that satisfies all of the following conditions.**

12.

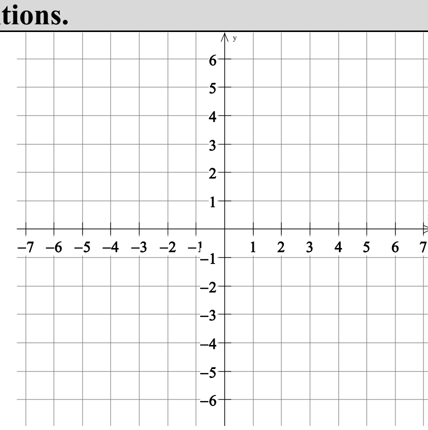
a. $f(2) = 3$

b. $\lim_{x \to 2} f(x) = -2$

c. $\lim_{x \to -4^+} f(x) = -2$

d. $f(-4) = 1$

e. $\lim_{x \to -4^-} f(x) > f(-4)$



Mr. Sullivan is losing hair due to stress caused by students who are behind pace. His hair loss can be modeled by $h$, where $h(t)$ is the number of hairs lost through month $t$ for $0 \leq t \leq 48$.

13. What does $h(12)$ represent?

14. What does $\frac{h(12)-h(6)}{12-6}$ represent?

15. What does $\frac{h(12)-h(11.999)}{12-11.999}$ represent?

16. According to the table, what is value of $\lim_{x \to 1} f(x)$?

| $x$ | 0.8 | 0.99 | 1.01 | 1.1 |
|---|---|---|---|---|
| $f(x)$ | $-0.5$ | $-0.001$ | 0.001 | 0.5 |

**Evaluate the limit.**

17. $\lim_{x \to 1} \sqrt{7x+42}$

18. $\lim_{x \to -1} \frac{x^2-1}{x+1}$

19. $\lim_{x \to 3} \frac{x^2-3x}{x-3}$

20. $\lim_{x \to 0} \frac{\sin(7x)}{11x}$

21. $\lim_{x \to 0} \frac{\sqrt{x+2}-\sqrt{2}}{x}$

22. $\lim_{x \to 3^+} \frac{x-3}{|x-3|}$

23. $\lim_{x \to 0} \frac{x}{\frac{1}{x+6}-\frac{1}{6}}$

24. If $g(x) = \begin{cases} \sqrt{4-x}, & x < -3 \\ x^2 - 2, & -3 \le x < 2 \\ |x-4|, & x \ge 2 \end{cases}$

find the following:

a. $\lim_{x \to 2^-} g(x) =$

b. $\lim_{x \to 2^+} g(x) =$

c. $\lim_{x \to -3^+} g(x) =$

d. $\lim_{x \to 2} g(x) =$

e. $g(2) =$

f. $\lim_{x \to -3^-} g(x) =$

g. $g(-3) =$

h. $\lim_{x \to -3} g(x) =$

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25. The function $f$ is continuous and increasing for $x \ge 0$. The table gives values of $f$ at selected values of $x$.

| $x$ | 8.87 | 8.999 | 9.001 | 9.01 |
|---|---|---|---|---|
| $f(x)$ | 3.86 | 3.999 | 4.001 | 4.7 |

Approximate the value of $\lim_{x \to 9} \sqrt{f(x)}$.

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26. Let $f$ be a function where $\lim_{x \to 4} f(x) = \frac{1}{2}$. Which of the following could represent the function $f$?

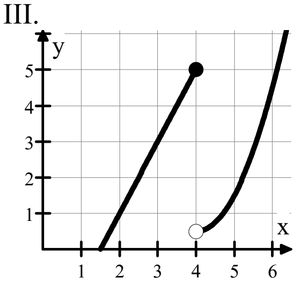
I.

$f(x) = \begin{cases} \frac{x-4}{x^2-6x+8}, & x \ne 4 \\ 4, & x = 4 \end{cases}$

II.

| $x$ | 3.8 | 3.9 | 3.999 | 4 | 4.001 | 4.1 | 4.2 |
|---|---|---|---|---|---|---|---|
| $f(x)$ | 0.47 | 0.49 | 0.499 | 2 | 0.5001 | 0.51 | 0.53 |

III.




(A) I only (B) II only (C) III only (D) I and II only (E) none

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27. If $f$ is a piecewise linear function such that $\lim_{x \to 8} f(x)$ does not exist, which of the following could be representative of the function $f$?

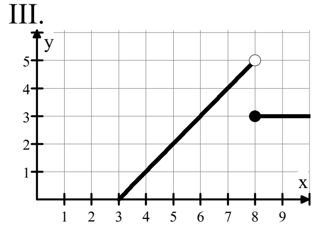
I.

$f(x) = \begin{cases} \frac{1}{4}x - 7, & x < 8 \\ 11 - 2x, & x > 8 \end{cases}$

II.

| $x$ | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|
| $f(x)$ | $-5$ | $-3$ | $-1$ | 6 | $\frac{8}{5}$ | $\frac{11}{5}$ | $\frac{14}{5}$ |

III.




(A) I only (B) II only (C) III only (D) II and III only (E) none

28. Let $f$ and $g$ be the functions defined by $f(x) = \frac{\sin x}{3x}$ and $g(x) = x^3 \cos\left(\frac{1}{x^5}\right)$ for $x \neq 0$. The following inequalities are true for $x \neq 0$. State whether each inequality can be used with the squeeze theorem to find the limit of the function as $x$ approaches 0?

I. $\frac{1}{3} \leq f(x) \leq \frac{1}{2}$

II. $-x^3 \leq g(x) \leq x^3$

III. $-\frac{1}{x^5} \leq g(x) \leq \frac{1}{x^5}$

(A) I only (B) II only (C) III only (D) I and II only (E) II and III only

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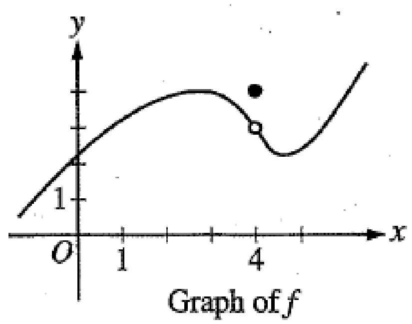
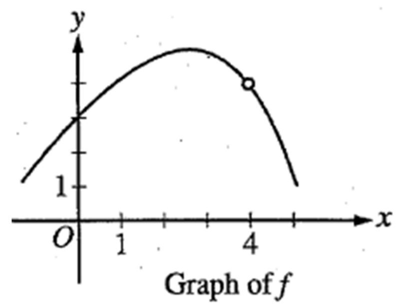
29. For which of the following does $\lim_{x \to 4} f(x)$ exist?

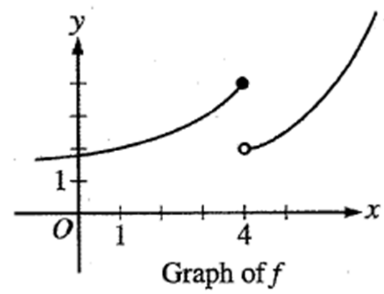
I.




II.




III.




(A) I only (B) II only (C) III only (D) I and II only (E) I and III only

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30. If $a \neq 0$, then $\lim_{x \to a} \frac{x^2 - a^2}{x^4 - a^4}$ is

(A) $\frac{1}{a^2}$ (B) $\frac{1}{2a^2}$ (C) $\frac{1}{6a^2}$ (D) 0 (E) nonexistent