# Lesson 1.3 - Estimating Limit Values from Graphs

AP Calc. AB/BC

## Warmup: Questions from CA1 - Lesson 1.2?

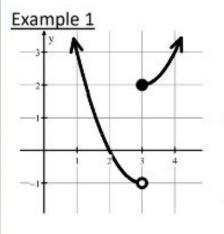
If you haven't finished the CA worksheet from yesterday take it out and continue to work on it during this time.

## 1.3 Finding Limits from Graphs

# Notes

#### What is a one-sided limit?

A *one-sided limit* is the \_\_\_\_\_ a function approaches as you approach a given from either the \_\_\_\_ or \_\_ side.



The limit of f as w approaches

3 from the right side is 2.

The limit of 
$$f$$
 as  $x$  approaches 3 from the left side is  $-1$ .

The limit of 
$$f$$
 as  $x$  approaches

$$\lim_{x \to \infty} f(x) =$$

 $\lim f(x) =$ 

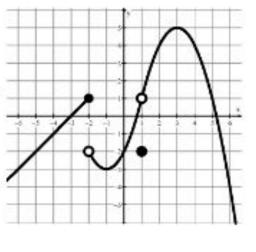
If the two sides are different?  $\lim_{x \to a} f(x) = 0$ 

#### Example 2

a. 
$$\lim_{x \to -2^-} f(x) =$$

b. 
$$\lim_{x \to -2^+} f(x) =$$

$$c. \lim_{x \to -2} f(x) =$$



e. 
$$\lim_{x\to 0} f(x) =$$

$$f. \lim_{x \to 3^{-}} f(x) =$$

$$\mathrm{d.}\lim_{x\to 1}f(x)=$$

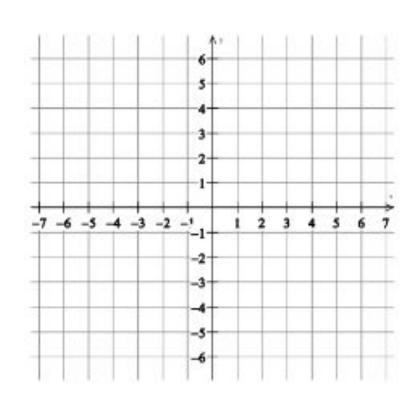
i. 
$$f(-2) =$$

g. 
$$\lim_{x \to -1} f(x) =$$

#### Example 3

Sketch a graph of a function g that satisfies all of the following conditions.

- a. g(3) = -1
- $b. \lim_{x\to 3}g(x)=4$
- $c. \quad \lim_{x \to -2^+} g(x) = 1$
- d. g is increasing on -2 < x < 3
- e.  $\lim_{x \to -2^{-}} g(x) > \lim_{x \to -2^{+}} g(x)$



## Notes Filled In:

AP Calc. AB/BC - Lesson 1.3 - Filled In

#### Practice - Test Prep.

Take the next 5-10 minutes to work together on the practice - test prep section of our notes.

We will go through it together on the board after the time is up!

For 1-3, give the value of each statement. If the value does not exist, write "does not exist" or "undefined."

a. 
$$\lim_{x \to -1^{-}} f(x) = 3$$
 b.  $f(1) = 1$  c.  $\lim_{x \to 0} f(x) = 1$ 

b. 
$$f(1) =$$

c. 
$$\lim_{x\to 0} f(x) = \bigcirc$$

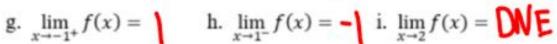
d. 
$$\lim_{x \to 2^+} f(x) =$$

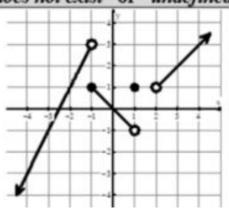
$$f(-1) =$$

d. 
$$\lim_{x \to 2^+} f(x) =$$
 e.  $f(-1) =$  f.  $f(2) =$ 

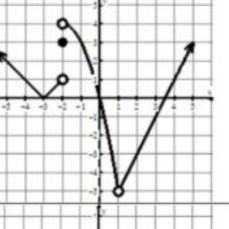
$$g. \lim_{x \to -1^+} f(x) =$$

$$\lim_{x\to 1^-} f(x) = -1$$
 i.

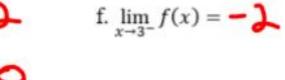


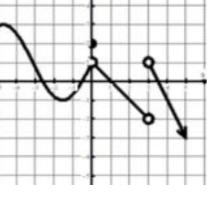


- a.  $\lim_{x \to -3} f(x) = 0$  b. f(1) = DNE c.  $\lim_{x \to 1} f(x) = -5$
- d.  $\lim_{x \to -2^+} f(x) = 4$  e. f(3) = -1 f.  $\lim_{x \to -2^-} f(x) = 1$
- g.  $\lim_{x\to -2} f(x) = DNE$  h. f(-2) = 3 i. f(4) = 1



- a.  $\lim_{x \to 3^+} f(x) =$  b. f(3) = DNE c.  $\lim_{x \to 0} f(x) =$
- d.  $\lim_{x \to 3} f(x) = \text{ of } e. f(0) = \lambda$
- g.  $\lim_{x\to 0^+} f(x) =$  h. f(1) =





#### 4. Sketch a graph of a function f that satisfies all of the following conditions.

a. 
$$f(-2) = 5$$

b. 
$$\lim_{x \to -2} f(x) = 1$$

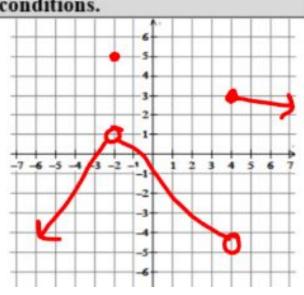
$$c. \quad \lim_{x \to 4^+} f(x) = 3$$

d. f is increasing on x < -2

e. 
$$\lim_{x \to 4^-} f(x) < \lim_{x \to 4^+} f(x)$$

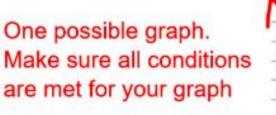
One possible graph.

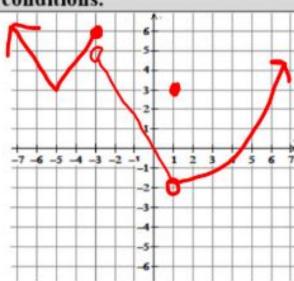
Make sure all conditions are met for your graph



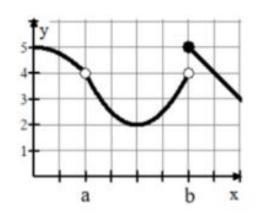
#### 5. Sketch a graph of a function g that satisfies all of the following conditions.

- a. g(1) = 3
- b.  $\lim_{x \to 1} g(x) = -2$
- c.  $\lim_{x \to -3^+} g(x) = 5$
- d. g is increasing only on -5 < x < -3 and x > 1
- e.  $\lim_{x \to -3^{-}} g(x) > \lim_{x \to -3^{+}} g(x)$

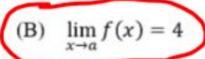




6. The graph of the function f is shown. Which of the following statements about f is true?



- (A)  $\lim_{x \to a} f(x) = \lim_{x \to b} f(x)$
- (C)  $\lim_{x \to b} f(x) = 4$
- (E)  $\lim_{x\to a} f(x)$  does not exist.



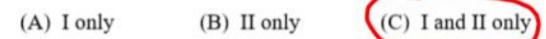
 $(D) \quad \lim_{x \to b} f(x) = 5$ 

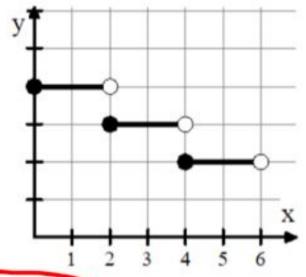
7. The figure below shows the graph of a function f with domain  $0 \le x < 6$ . Which of the following statements are true?

I.  $\lim_{x\to 4^-} f(x)$  exists.

II.  $\lim_{x\to 4^+} f(x)$  exists.

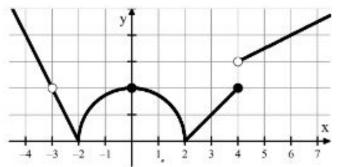
III.  $\lim_{x\to 4} f(x)$  exists.





(D) I and III only (E) I, II, and III

8. The graph of a function f is shown below. For which of the following values of c does  $\lim_{x\to c} f(x) = 2$ ?



(A) 0 only

(B) 0 and 4 only

(D) -3 and 4 only

(E) -3, 0, and 4

(C) -3 and 0 only

## Independent Work Time

Please work on the CA worksheet during this time.

We will go through any questions from the CA worksheet tomorrow during the Warmup.

If you finish the CA worksheet early then you can go to AP Classroom and work on homework, step-by-step, watch a daily video, or utilize a different resource from GC

#### 1.3 Finding Limits from Graphs

Calculus

g.  $\lim_{x \to 1^{+}} f(x) =$ 

Name:

**CA #1** 

For 1-2, give the value of each statement. If the value does not exist, write "does not exist" or "undefined."

- 1.
  - a.  $\lim_{x \to 2} f(x) =$ 
    - b. f(-3) = c.  $\lim_{x \to -3^{-}} f(x) =$

  - $d. \lim_{x \to 2^+} f(x) =$ e. f(2) =
    - $f. \lim_{x \to 2^{-}} f(x) =$
  - $g. \lim_{x \to -3^+} f(x) =$ i.  $\lim_{x \to -3} f(x) =$ h. f(4) =
- - a.  $\lim_{x \to 1} f(x) =$  b. f(-2) = c.  $\lim_{x \to -2^+} f(x) =$

  - $f. \lim_{x \to 1^{-}} f(x) =$  $d. \lim_{x \to 2} f(x) =$ e. f(-4) =

    - h. f(-5) = i. f(1) =

-5 -4 -3 -2 -1

### 3. Sketch a graph of a function f that satisfies all of the following conditions.

a. 
$$f(3) = 4$$

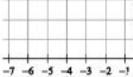
b. 
$$\lim_{x \to 3^{-}} f(x) = 2$$

c. 
$$\lim_{x \to 3^+} f(x) = -4$$

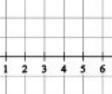
d. 
$$f(-2)$$
 is undefined.

e. 
$$\lim_{x \to -2^{-}} f(x) > \lim_{x \to -2^{+}} f(x)$$









Answers to 1.3 CA #1

1a. DNE	b2	c. 4	d2	e. 3	f. 2	g3	h. DNE	i. DNE
<b>2</b> a. 2	b1	c1	d. 1	e2	f. 2	g. 2	h. DNE	i. 4

#### 3. One possible graph:

Double check that each condition is satisfied with your graph and it passes the vertical line test.

