

Unit 4 Progress Check: FRQ Part A

You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your question, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results. Your work must be expressed in standard mathematical notation rather than calculator syntax.

Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which f(x) is a real number.

For time $0 \le t \le 10$, water is flowing into a small tub at a rate given by the function F defined by $F(t) = \arctan\left(\frac{\pi}{2} - \frac{t}{10}\right)$. For time $5 \le t \le 10$, water is leaking from the tub at a rate given by the function L defined by $L(t) = 0.03 \left(20t - t^2 - 75\right)$. Both F(t) and L(t) are measured in cubic feet per minute, and t is measured in minutes. The volume of water in the tub, in cubic feet, at time t minutes is given by W(t).

- (a) At time t=3, there are 2.5 cubic feet of water in the tub. Write an equation for the locally linear approximation of W at t=3, and use it to approximate the volume of water in the tub at time t=3.5.
- (b) Find W''(8). Using correct units, interpret the meaning of W''(8) in the context of the problem.
- (c) Is there a time t, for 5 < t < 10, at which the rate of change of the volume of water in the tub changes from positive to negative? Give a reason for your answer.
- (d) The tub is in the shape of a rectangular box that is 0.5 foot wide, 4 feet long, and 3 feet deep. What is the rate of change of the depth of the water in the tub at time t = 6?

Part A

The second point may be earned if response contains an incorrect value for slope based on one computational error.

The third point does not require a simplified answer. Substitution of function values is required. Units are not required.

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

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0	1	2	3



Unit 4 Progress Check: FRQ Part A

The student response accurately includes all three of the criteria below.

- slope equation f
- equation for tangent line approximation

Solution:

$$W(3) = 2.5$$

$$W'(3) = F(3) = 0.904089$$

$$y = W(3) + W'(3)(t-3) = 2.5 + 0.904(t-3)$$

$$W(3.5) \approx W(3) + W'(3)(3.5 - 3) = 2.952$$
 cubic feet

Part B

The second point requires a time reference, rate, and units. The point may be earned with an incorrect approximation based on one computational error and provided the behavior described is consistent with the sign of the approximation, the numerical value, and the time reference.

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0 1 2

The student response accurately includes both of the criteria below.

- W''(8)
- interpretation with units

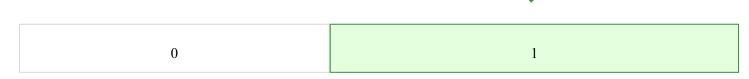
Solution:

$$W''(8) = F'(8) - L'(8) = -0.183$$
 (or -0.182)

At time t = 8 minutes, the rate at which the rate of change of the volume of water is changing by -0.183 (or -0.182) cubic feet per minute per minute.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.



Unit 4 Progress Check: FRQ Part A

The student response accurately includes a correct answer with reason.

Solution:

$$W'(t) = F(t) - L(t) = 0$$
 for $5 < t < 10$.

$$W'(t) = 0 \implies t = 8.149287$$

Yes, there is such a time t, because W'(t) > 0 for 5 < t < 8.149 and W'(t) < 0 for 8.149 < t < 10.

Part D

The first point is earned with $\frac{dV}{dt} = 2\frac{dh}{dt}$.

The third point may be earned with a maximum of one error in the evaluation of $\frac{dV}{dt}$.

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

/

3

0

1

2

The student response accurately includes all three of the criteria below.

- relationship between $\frac{dV}{dt}$ and $\frac{dh}{dt}$
- $\frac{dV}{dt}\Big|_{t=6}$
- answer

The volume of water in the tub is V = (0.5)(4)h = 2h, where h is the depth of water, in feet.

$$rac{dV}{dt}=2rac{dh}{dt}$$

At time
$$t=6, rac{dV}{dt}\left|_{t=6}=W'(t)=F(6)-L(6)=0.500581.$$

Therefore,
$$\left. rac{dh}{dt} \, \right|_{t=6} = rac{1}{2} \, rac{dV}{dt} = 0.250$$