Unit 7 Progress Check: FRQ Part A

1. NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

A large vat is initially filled with a saltwater solution. A solution with a higher concentration of salt flows into the vat, and solution flows out of the vat at the same rate. The number of pounds of salt in the vat at time t minutes is modeled by the function A that satisfies the differential equation $\frac{dA}{dt} = 6 - 0.02A$. At time $t = 10$ minutes, the vat contains 50 pounds of salt.

(a) Write an equation for the line tangent to the graph of A at $t = 10$. Use the tangent line to approximate the number of pounds of salt in the vat at time $t = 12$ minutes.

(b) Show that $A(t) = 300 - 250e^{0.2-0.02t}$ satisfies the differential equation $\frac{dA}{dt} = 6 - 0.02A$ with initial condition $A(10) = 50$.

(c) The flow of solution into the vat is stopped, and the solution is drained. The depth of solution in the vat is modeled by the function h that satisfies the differential equation $\frac{dh}{dt} = -k\sqrt{h}$, where $h(t)$ is measured in meters, t is the number of minutes since draining began, and k is a constant. If the depth of the solution is 16 meters at time $t = 0$ minutes and 4 meters at time $t = 30$ minutes, what is $h(t)$ in terms of t ?

Part A

For the second point, it is incorrect to state $A(12) = 60$ rather than $A(12) \approx 60$.

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

0	1	2 ✓

The student response accurately includes both of the criteria below.

- ☐ tangent line equation
- ☐ approximation

Unit 7 Progress Check: FRQ Part A

Solution:

$A(10) = 50$

$\left.\frac{dA}{dt}\right|_{t=10} = 6 - 0.02 \cdot 50 = 5$

An equation for the line tangent to the graph of A at $t = 10$ is $y = 50 + 5(t - 10)$.

At time $t = 12$ minutes, the vat contains approximately $50 + 5(12 - 10) = 60$ pounds of salt. $A(12) \approx 60$.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The student response accurately includes all three of the criteria below.

- [ ] verification of initial condition
- [ ] $\frac{dA}{dt} = 5e^{0.2-0.02t}$
- [ ] verification that $\frac{dA}{dt} = 6 - 0.02A$

Solution:

$A(10) = 300 - 250e^{0.2-0.02 \cdot 10} = 300 - 250e^{0} = 50$

$\frac{dA}{dt} = \frac{d}{dt}\left(300 - 250e^{0.2-0.02t}\right)$
$= -250e^{0.2-0.02t}(-0.02) = 5e^{0.2-0.02t}$

$6 - 0.02A = 6 - 0.02\left(300 - 250e^{0.2-0.02t}\right)$
$= 6 - 6 + 5e^{0.2-0.02t} = 5e^{0.2-0.02t}$

Part C

Zero out of 4 points earned if no separation of variables.

At most 2 out of 4 points earned [1-1-0-0] if no constant of integration.

Both antiderivatives must be correct to earn the second point.

The fourth point requires an expression for $h(t)$. The domain of $h(t)$ is included with the solution; this is not a requirement to earn the fourth point.

Select a point value to view scoring criteria, solutions, and/or examples and to score the response

Unit 7 Progress Check: FRQ Part A

0	1	2	3	4 ✓

The student response accurately includes all four of the criteria below.

- ☐ separation of variables
- ☐ antiderivatives
- ☐ constant of integration and uses initial conditions
- ☐ $h(t)$

Solution:

$$\frac{dh}{dt} = -k\sqrt{h}$$
$$\frac{dh}{\sqrt{h}} = -k\ dt$$
$$\int \frac{1}{\sqrt{h}}\ dh = -\int k\ dt$$
$$2\sqrt{h} = -kt + C$$
$$2\sqrt{16} = 0 + C \ \Rightarrow\ C = 8$$
$$2\sqrt{4} = -30k + 8 \ \Rightarrow\ k = \tfrac{2}{15}$$
$$2\sqrt{h} = -\tfrac{2}{15}t + 8$$
$$h(t) = \left(-\tfrac{t}{15} + 4\right)^2$$

Note: This is valid for $0 \leq t \leq 60$.