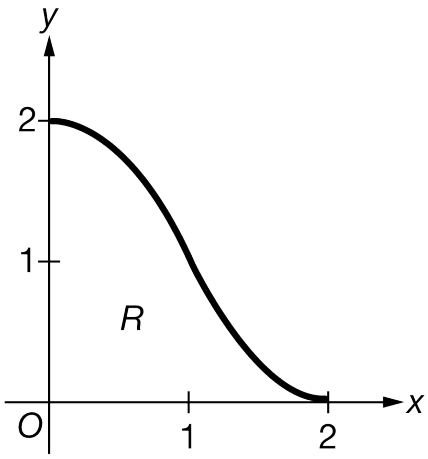
Unit 8 Progress Check: FRQ Part B

1. NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.



Let f be the function defined by $f(x) = \begin{cases} 2 - x^2 & \text{for } 0 \le x < 1 \\ (x-2)^2 & \text{for } 1 \le x \le 2 \end{cases}$. Let R be the region in the first quadrant bounded by the graph of f and the x- and y-axes, as shown in the figure above.

(a) Find the area of R.

(b) Region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.

(c) The portion of the region R for $1 \le x \le 2$ is revolved about the y-axis to form a solid. Find the volume of the solid.

Part A

At most 2 out of 3 points earned for a copy error in the integrand.

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Unit 8 Progress Check: FRQ Part B

0	1	2	3 ✓

The student response accurately includes all three of the criteria below.

- [ ] definite integrals
- [ ] antiderivatives
- [ ] answer

Solution:

$$\begin{aligned}\text{Area} &= \int_0^1 \left(2 - x^2\right)\, dx + \int_1^2 (x-2)^2\, dx \\ &= \left(2x - \tfrac{1}{3}x^3\right)\Big|_0^1 + \tfrac{1}{3}(x-2)^3\Big|_1^2 \\ &= \left(\tfrac{5}{3} - 0\right) + \left(0 - \left(-\tfrac{1}{3}\right)\right) = 2\end{aligned}$$

Part B

At most 1 out of 2 points earned if there is one error in the integrand OR 1 error with limits of integration.

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The student response accurately includes both of the criteria below.

- [ ] integrands
- [ ] expression for total volume

Solution:

$$\text{Volume} = \int_0^1 \left(2 - x^2\right)^2\, dx + \int_1^2 (x-2)^4\, dx$$

Part C

Third and fourth points may be earned if there is a maximum of one error in either the integrand OR one error in the limits and constant. Eligibility requires a difference of squares for the integrand.

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Unit 8 Progress Check: FRQ Part B

0	1	2	3	4 ✓

The student response accurately includes all four of the criteria below.

- [ ] integrand
- [ ] limits and π
- [ ] antiderivative
- [ ] answer

Solution:

$y = (x-2)^2 \;\Rightarrow\; x = \sqrt{y} + 2$

$$\text{Volume} = \pi \int_0^1 \left((\sqrt{y}+2)^2 - 1^2 \right) \, dy = \pi \int_0^1 (y + 4\sqrt{y} + 4 - 1) \, dy$$

$$= \pi \int_0^1 (y + 4\sqrt{y} + 3) \, dy = \pi \left(\tfrac{y^2}{2} + \tfrac{8}{3} y^{\frac{3}{2}} + 3y \right) \Big|_0^1$$

$$= \pi \left(\tfrac{1}{2} + \tfrac{8}{3} + 3 \right) = \tfrac{37\pi}{6}$$

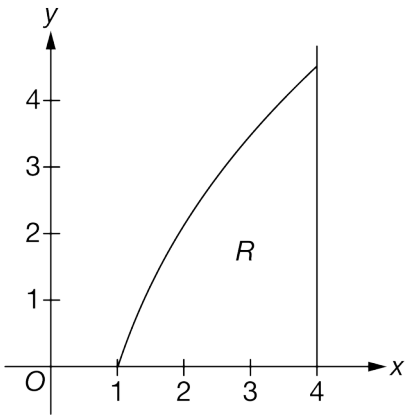
Unit 8 Progress Check: FRQ Part B

2. NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.



Let R be the region bounded by the graph of $y = \frac{3(x-1)}{\sqrt{x}}$, the x-axis, and the vertical line $x = 4$, as shown in the figure above.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is revolved about the x-axis.

Part A

The first point is for the integrand only. At most 3 out of 4 points earned for a copy error in the integrand. Subsequent points may be earned.

The second and third points are for antiderivatives. At most 1 out of 2 antiderivative points earned if only one antiderivative is correct OR there is at most one arithmetic sign error in the antiderivatives.

The fourth point is for the limits of integration and evaluation.

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Unit 8 Progress Check: FRQ Part B

0	1	2	3	4 ✓

The student response accurately includes all four of the criteria below.

- [ ] integrand
- [ ] antiderivatives (no criteria)
- [ ] answer

Solution:

$$\text{Area} = \int_1^4 \frac{3(x-1)}{\sqrt{x}}\,dx = \int_1^4 \left(3\sqrt{x} - \frac{3}{\sqrt{x}}\right)dx$$

$$= \left(3 \cdot \tfrac{2}{3}x^{\frac{3}{2}} - 3 \cdot 2x^{\frac{1}{2}}\right)\Big|_{x=1}^{x=4} = \left(2x^{\frac{3}{2}} - 6\sqrt{x}\right)\Big|_{x=1}^{x=4}$$

$$= (16 - 12) - (2 - 6) = 8$$

Part B

The first point is for the integrand only. At most 4 out of 5 points earned for a copy error in the integrand. Subsequent points may be earned.

The second point is for limits and π.

The third and fourth points are for antiderivatives. At most 1 out of 2 antiderivative points earned if only one antidifferentiation error OR there is at most one arithmetic sign error in the antiderivatives.

The fifth point is for the evaluation, which does not need to be simplified. An error with the factor of 9 applies to this point.

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3	4	5 ✓

The student response accurately includes all five of the criteria below.

- [ ] integrand
- [ ] limits and constant
- [ ] antiderivatives (no criteria)
- [ ] answer

Unit 8 Progress Check: FRQ Part B

Solution:

$$\text{Volume} = \pi \int_1^4 \left(\frac{3(x-1)}{\sqrt{x}} \right)^2 dx = 9\pi \int_1^4 \left(\frac{x^2 - 2x + 1}{x} \right) dx$$

$$= 9\pi \int_1^4 \left(x - 2 + \frac{1}{x} \right) dx = 9\pi \left(\tfrac{x^2}{2} - 2x + \ln\, x \right) \Big|_{x=1}^{x=4}$$

$$= 9\pi \left((8 - 8 + \ln\, 4) - \left(\tfrac{1}{2} - 2 + 0 \right) \right) = 9\pi \left(\ln\, 4 + \tfrac{3}{2} \right)$$