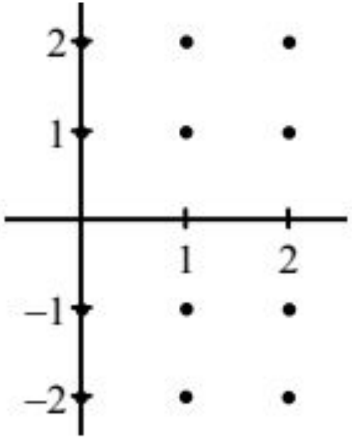
# Semester 2 - Unit 7 Review

AP Calc. AB/BC

1. Consider the differential equation $\frac{dy}{dx} = \frac{x-2}{y}$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.



(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the $xy$-plane for which $y \neq 0$. Describe all points in the $xy$-plane, $y \neq 0$, for which $\frac{dy}{dx} = 1$.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = -3$.

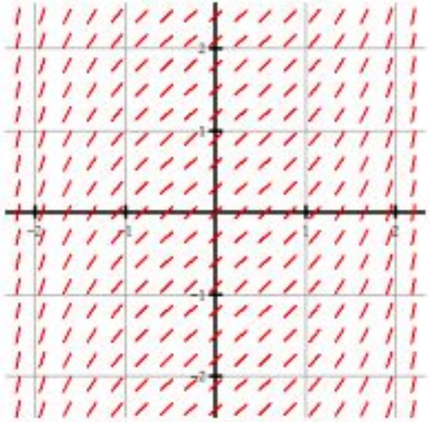
2. A population $y$ grows according to the equation $\frac{dy}{dt} = ky$, where $k$ is a constant and $t$ is measured in years. If the population doubles every 21 years, then what is the value of $k$?

3. Mr. Brust is running down his street. His position is given by the function $p(t)$, where $t$ is measured in minutes since the start of his run. His velocity is inversely proportional to the natural logarithm of the time since the start of his run. What is a differential equation that represents this relationship? Use $k$ as a constant.

4. Let $f$ be the function that is defined for all real numbers $x$ and has the following properties.
   - $f''(x) = 18x - 4$
   - $f'(1) = 5$
   - $f(-4) = 3$

   Find each $x$ such that the line tangent to the graph of $f$ at $(x, f(x))$ is horizontal.

5. Which of the following could be a path through the slope field created by the differential equation $\frac{dy}{dx} = \sec^2(0.5x)$?



(A) $y = \tan x$

(B) $y = 2\tan x + 1$

(C) $y = \frac{1}{2}\tan(x) + 4$

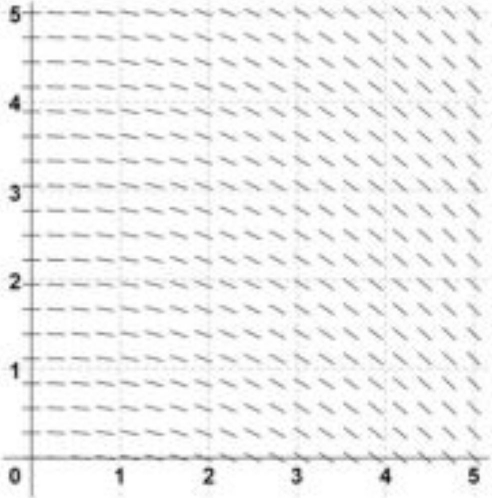
(D) $y = 2\tan(0.5x) - 1$

(E) $y = \frac{1}{2}\tan(0.5x) + 2$

6. During a certain epidemic, the number of people that are infected at any time increases at a rate proportional to the number of people that are infected at that time. If 1,300 people are infected when the epidemic is first discovered, and 2,000 people are infected 6 days later, how many people are infected 10 days after the epidemic is first discovered? Round down to the nearest whole number.

7. For what value of $k$, if any, is $y = e^{4x} - ke^{-2x}$ a solution to the differential equation $\frac{y''}{2} - 2y' = 12e^{-2x}$?

8. Explain why the following slope field cannot represent the differential equation $\frac{dy}{dt} = -0.2y$



**Find the general solution of each differential equation.**

$\frac{dy}{dx} = x^2 e^{x^3}$

$\frac{dy}{dx} = 2xy^2$

**For each differential equation, find the particular solution that passes through the given point.**

$\frac{dy}{dx} = 12\cos(4x) - e^{2x};\ \ (0, 1)$

$\frac{d^2y}{dx^2} = 9x^2 - 12x$ and $y'(2) = 4$ and $y(1) = \frac{19}{4}$

19. The position of a particle moving along the $x$-axis is given by $x(t) = e^{2t} - e^{t}$. When the particle is at rest, the acceleration of the particle is

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\ln\frac{1}{2}$ (D) 2 (E) 4

. A population changes at a rate modeled by the logistic differential equation $\frac{dy}{dt} = 4000y - \frac{1}{2}y^2$, where $t$ is measured in years. What are all values of $y$ for which the population is increasing at a decreasing rate?

# Answer Key:

[Semester 2 - Unit 7 Review - Answer Key]