Unit 4 Progress Check: FRQ Part A

1. A GRAPHING CALCULATOR IS REQUIRED FOR THIS QUESTION.

You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your question, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results. Your work must be expressed in standard mathematical notation rather than calculator syntax.

Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

For time $0 \le t \le 10$, water is flowing into a small tub at a rate given by the function F defined by $F(t) = \arctan\left(\frac{\pi}{2} - \frac{t}{10}\right)$. For time $5 \le t \le 10$, water is leaking from the tub at a rate given by the function L defined by $L(t) = 0.03\left(20t - t^2 - 75\right)$. Both $F(t)$ and $L(t)$ are measured in cubic feet per minute, and t is measured in minutes. The volume of water in the tub, in cubic feet, at time t minutes is given by $W(t)$.

(a) At time $t = 3$, there are 2.5 cubic feet of water in the tub. Write an equation for the locally linear approximation of W at $t = 3$, and use it to approximate the volume of water in the tub at time $t = 3.5$.

(b) Find $W''(8)$. Using correct units, interpret the meaning of $W''(8)$ in the context of the problem.

(c) Is there a time t, for $5 < t < 10$, at which the rate of change of the volume of water in the tub changes from positive to negative? Give a reason for your answer.

(d) The tub is in the shape of a rectangular box that is 0.5 foot wide, 4 feet long, and 3 feet deep. What is the rate of change of the depth of the water in the tub at time $t = 6$?