# Unit 1 - End of Unit Review

AP Calc. AB/BC

# Warmup: Questions from CA1 - Lesson 1.16?

If you haven’t finished the CA worksheet from yesterday take it out and continue to work on it during this time.

NO CALCULATOR!

**Give the value of each statement. If the value does not exist, write "*does not exist*" or "*undefined*."**

1. $\lim_{x \to 2^-} g(x) =$

2. $g(-2) =$

3. $\lim_{x \to -2} g(x) =$

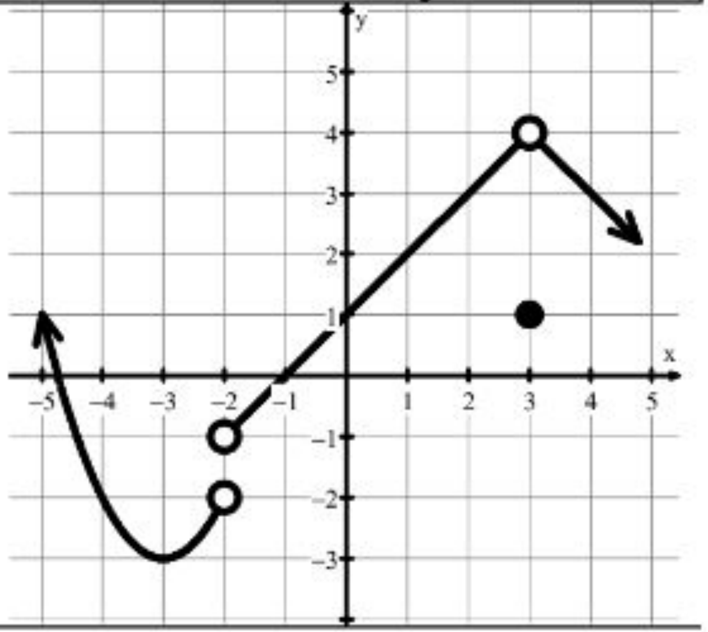
4. $\lim_{x \to 3} g(x) =$

5. $\lim_{x \to -2^-} g(x) =$

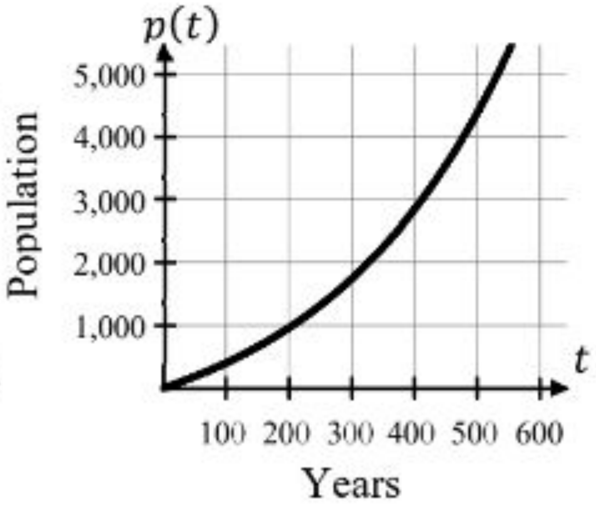
6. $\lim_{x \to -2^+} g(x) =$

7. $g(3) =$

8. $\lim_{x \to 0} g(x) =$



A city's population can be modeled by the function $p$, where $p(t)$ gives the population and $t$ gives the number of years since 1500 for $0 \leq t \leq 500$. The graph of the function $p$ is shown to the right.




9. Draw a tangent line at $t = 100$.

10. Give a rough estimate of the instantaneous rate of change at $t = 100$.

11. Give an example of how to calculate a rate of change that would give a close estimate to the rate of change in the year 1800.

**Sketch a graph of a function $f$ that satisfies all of the following conditions.**

12.

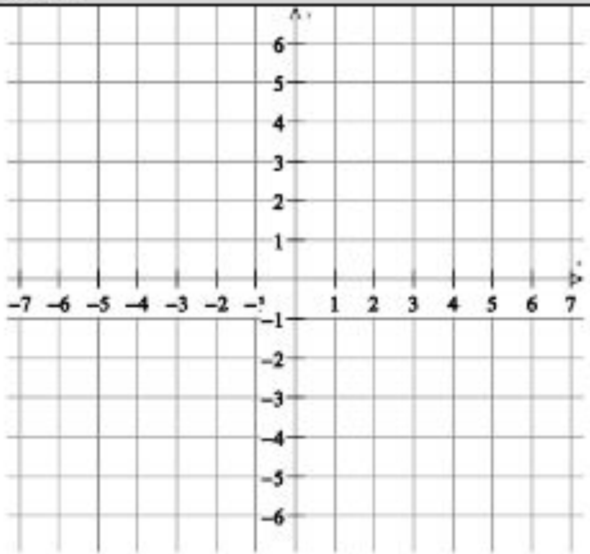
a. $f(2) = 3$

b. $\lim_{x \to 2} f(x) = -2$

c. $\lim_{x \to -4^+} f(x) = -2$

d. $f(-4) = 1$

e. $\lim_{x \to -4^-} f(x) > f(-4)$



Mr. Sullivan is losing hair due to stress caused by students who are behind pace. His hair loss can be modeled by $h$, where $h(t)$ is the number of hairs lost through month $t$ for $0 \leq t \leq 48$.

13. What does $h(12)$ represent?

14. What does $\frac{h(12)-h(6)}{12-6}$ represent?

15. What does $\frac{h(12)-h(11.999)}{12-11.999}$ represent?

16. According to the table, what is value of $\lim_{x \to 1} f(x)$?

| $x$ | 0.8 | 0.99 | 1.01 | 1.1 |
|---|---|---|---|---|
| $f(x)$ | −0.5 | −0.001 | 0.001 | 0.5 |

**Evaluate the limit.**

17. $\lim_{x \to 1} \sqrt{7x + 42}$

18. $\lim_{x \to -1} \frac{x^2-1}{x+1}$

19. $\lim_{x \to 3} \frac{x^2-3x}{x-3}$

20. $\lim_{x \to 0} \frac{\sin(7x)}{11x}$

21. $\lim_{x \to 0} \frac{\sqrt{x+2}-\sqrt{2}}{x}$

22. $\lim_{x \to 3^+} \frac{x-3}{|x-3|}$

23. $\lim_{x \to 0} \frac{x}{\frac{1}{x+6}-\frac{1}{6}}$

24. If $g(x) = \begin{cases} \sqrt{4-x}, & x < -3 \\ x^2 - 2, & -3 \le x < 2 \\ |x-4|, & x \ge 2 \end{cases}$

find the following:

a. $\lim_{x \to 2^-} g(x) =$

b. $\lim_{x \to 2^+} g(x) =$

c. $\lim_{x \to -3^+} g(x) =$

d. $\lim_{x \to 2} g(x) =$

e. $g(2) =$

f. $\lim_{x \to -3^-} g(x) =$

g. $g(-3) =$

h. $\lim_{x \to -3} g(x) =$

25. The function $f$ is continuous and increasing for $x \geq 0$. The table gives values of $f$ at selected values of $x$.

| $x$ | 8.87 | 8.999 | 9.001 | 9.01 |
|---|---|---|---|---|
| $f(x)$ | 3.86 | 3.999 | 4.001 | 4.7 |

Approximate the value of $\lim_{x \to 9} \sqrt{f(x)}$.

26. Let $f$ be a function where $\lim_{x \to 4} f(x) = \frac{1}{2}$. Which of the following could represent the function $f$?

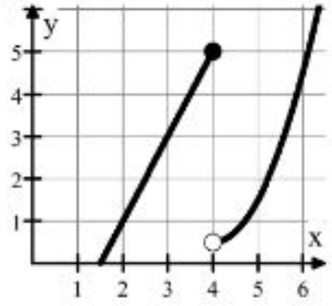
I.

$$f(x) = \begin{cases} \frac{x-4}{x^2-6x+8}, & x \neq 4 \\ 4, & x = 4 \end{cases}$$

II.

| $x$ | 3.8 | 3.9 | 3.999 | 4 | 4.001 | 4.1 | 4.2 |
|---|---|---|---|---|---|---|---|
| $f(x)$ | 0.47 | 0.49 | 0.499 | 2 | 0.5001 | 0.51 | 0.53 |

III.




(A) I only (B) II only (C) III only (D) I and II only (E) none

27. If $f$ is a piecewise linear function such that $\lim_{x \to 8} f(x)$ does not exist, which of the following could be representative of the function $f$?

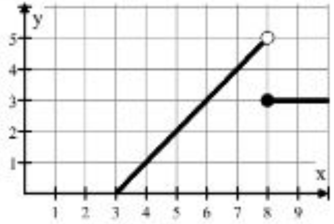
I.

$$f(x) = \begin{cases} \frac{1}{4}x - 7, & x < 8 \\ 11 - 2x, & x > 8 \end{cases}$$

II.

| $x$ | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|
| $f(x)$ | $-5$ | $-3$ | $-1$ | 6 | $\frac{8}{5}$ | $\frac{11}{5}$ | $\frac{14}{5}$ |

III.




(A) I only (B) II only (C) III only (D) II and III only (E) none

28. Let $f$ and $g$ be the functions defined by $f(x) = \frac{\sin x}{3x}$ and $g(x) = x^3 \cos\left(\frac{1}{x^5}\right)$ for $x \neq 0$. The following inequalities are true for $x \neq 0$. State whether each inequality can be used with the squeeze theorem to find the limit of the function as $x$ approaches 0?

I. $\frac{1}{3} \leq f(x) \leq \frac{1}{2}$

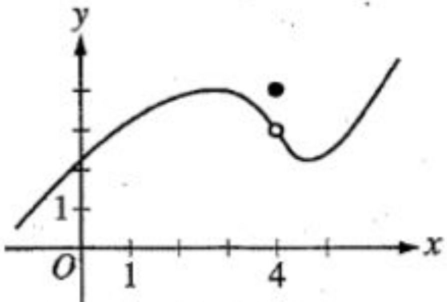
II. $-x^3 \leq g(x) \leq x^3$

III. $-\frac{1}{x^5} \leq g(x) \leq \frac{1}{x^5}$

(A) I only (B) II only (C) III only (D) I and II only (E) II and III only

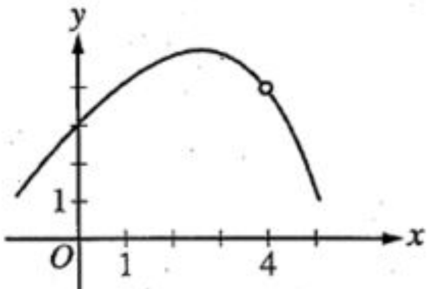
29. For which of the following does $\lim_{x \to 4} f(x)$ exist?

I.



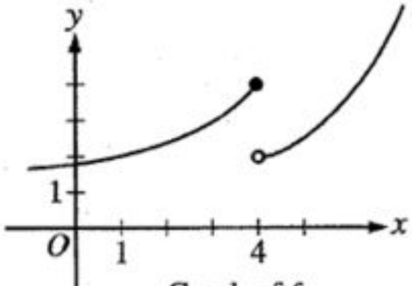

Graph of $f$

II.




Graph of $f$

III.




Graph of $f$

(A) I only (B) II only (C) III only (D) I and II only (E) I and III only

30. If $a \neq 0$, then $\lim_{x \to a} \frac{x^2 - a^2}{x^4 - a^4}$ is

(A) $\frac{1}{a^2}$ (B) $\frac{1}{2a^2}$ (C) $\frac{1}{6a^2}$ (D) 0 (E) nonexistent

# Notes Filled In:

AP Calc. AB/BC - End of Unit Review (1) - Filled In

# End of Unit 1 Review– Limits and Continuity

## Lessons 1.10 through 1.16.

Reviews do NOT cover all material from the lessons but will hopefully remind you of key points. To be prepared, you should review all packets from Unit 1 (including the Mid-Unit Review).

1. If $f(x) = \frac{x+3}{x^2-2x-15}$, identify the type of each discontinuity and where it is located.

**State whether the function is continuous at the given $x$ values. Justify your answers!**

2. $f(x) = \begin{cases} \cos(3x), & x < 0 \\ \tan x, & 0 \le x < \frac{\pi}{4} \\ \sin(2x), & x \ge \frac{\pi}{4} \end{cases}$

Continuous at $x = 0$?

Continuous at $x = \frac{\pi}{4}$?

| **Find the domain of each function.** | |
|---|---|
| 3. $h(t) = \frac{\sqrt{t+3}}{t-5}$ | 4. $f(x) = \ln\left(\frac{2}{x-1}\right)$ |

5. If the function $f$ is continuous for all real numbers and if $f(x) = \frac{x^2+6x+8}{x+2}$ when $x \neq -2$, then $f(-2) =$

6. Let $f$ be the function defined by $f(x) = \begin{cases} \frac{x^2+8x+12}{x+6}, & x \neq -6 \\ b, & x = -6 \end{cases}$. For what value of $b$ is $f$ continuous at $x = -6$?

| Evaluate the limit. | | |
|---|---|---|
| 7. $\lim_{x\to\infty} \sin\left(\frac{x+3\pi x^2}{2x^2}\right)$ | 8. $\lim_{x\to -5^-} \frac{-3}{25-x^2}$ | 9. $\lim_{x\to\infty} \frac{\sin x}{x}$ |
| 10. $\lim_{x\to\infty} \frac{4x^5-2x^2+3}{3x^2+2x^5-x^4}$ | 11. $\lim_{x\to -1} \frac{x^2+1}{x+1}$ | 12. $\lim_{x\to\infty} x^5 3^{-x}$ |

13. Identify all horizontal asymptotes of $f(x) = \frac{\sqrt{16\quad{}^{6}+x^{3}+5x}}{5x^{3}-8x}$.

# Notes Filled In:

AP Calc. AB/BC - End of Unit Review (2) - Filled In

# Independent Work Time

Please work on the CA worksheet during this time.

We will go through any questions from the CA worksheet tomorrow during the Warmup.

If you finish the CA worksheet early then you can go to AP Classroom and work on homework, step-by-step, watch a daily video, or utilize a different resource from GC

# End of Unit 1 Corrective Assignment – Limits and Continuity

**Give the value of each statement. If the value does not exist, write "*does not exist*" or "*undefined*."**

1.

a. $\lim_{x \to 3^-} f(x) =$

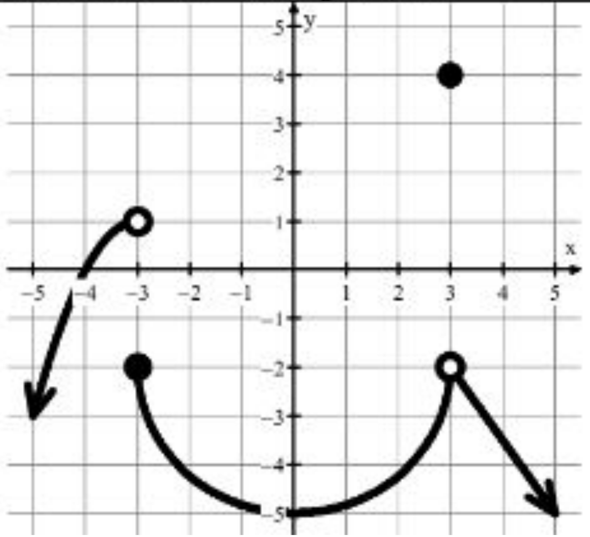
b. $f(-3) =$

c. $\lim_{x \to -3^-} f(x) =$

d. $\lim_{x \to -3^+} f(x) =$

e. $f(3) =$

f. $\lim_{x \to 3} f(x) =$



| Evaluate the limit. | | |
|---|---|---|
| 2. $\lim_{x\to 1}\frac{\sqrt{x+15}-4}{x-1}$ | 3. $\lim_{x\to -7^-}\frac{x}{x+7}$ | 4. $\lim_{x\to\infty}\sin\left(\frac{\frac{\sqrt{2}}{2}x+\pi x^2}{4x^2-x^3+2}\right)$ |
| 5. $\lim_{x\to 3}\frac{x-1}{x^2-6x+9}$ | 6. $\lim_{x\to 3}\frac{x^2-3x}{x^2-9}$ | 7. $\lim_{x\to 0}\frac{x}{\frac{1}{x+7}-\frac{1}{7}}$ |
| 8. $\lim_{x\to\infty}\frac{5x^4-3x^3-1}{x^3-2x^4}$ | 9. $\lim_{x\to 0}\frac{\sin^2(2x)}{2x^2}$ | 10. $\lim_{x\to -7^-}\frac{\lvert x+7\rvert}{x+7}$ |

11. If $f(x) = \begin{cases} \sin(2x), & x < \frac{\pi}{4} \\ \sin x, & \frac{\pi}{4} \le x \le \pi \\ \cos\left(\frac{x}{2}\right), & x > \pi \end{cases}$

find the following:

a. $\lim_{x \to \frac{\pi}{4}^+} f(x) =$

b. $\lim_{x \to \frac{\pi}{4}^-} f(x) =$

c. $\lim_{x \to \pi^-} f(x) =$

d. $\lim_{x \to \pi^+} f(x) =$

e. $f\left(\frac{\pi}{4}\right) =$

f. $f(\pi) =$

g. $\lim_{x \to \frac{\pi}{4}} f(x) =$

h. $\lim_{x \to \pi} f(x) =$

| **Identify any horizontal asymptote(s) of the following functions** | | |
|---|---|---|
| 12. $f(x) = 4^x$ | 13. $f(x) = \frac{(x+4)(x+1)}{(2x-1)^2}$ | 14. $f(x) = \frac{\sqrt{9x^4+x^3-4}}{x^2+2x-1}$ |

15. Let $g$ and $h$ be the functions defined by $g(x) = -x^2 + 2x + 3$ and $h(x) = 2x - 1$. If $f$ is a function that satisfies $g(x) \leq f(x) \leq h(x)$ for all $x$, what is $\lim_{x \to -2} f(x)$?

In a certain country, the number of deaths in a year can be modeled by $d$, where $d(t)$ is the number of deaths and $t$ is the year since 1975 for $0 \leq t \leq 30$.

16. What does $d(25)$ represent?

17. What does $\frac{d(30)-d(20)}{30-20}$ represent?

18. What does $\frac{d(15)-d(14.999)}{15-14.999}$ represent?

**For each function identify the type of each discontinuity and where it is located.**

19. $g(x) = \begin{cases} \ln(ex), & x < 1 \\ 2, & x = 1 \\ x - 1, & 1 < x \leq 2 \\ x^2 - 3 & x > 2 \end{cases}$

20. $f(x) = \frac{x}{x^2-4x}$

21. $f(x) = \frac{x^2+9x+14}{x+7}$

**State whether the function is continuous at the given $x$ values. Justify your answers!**

22. $f(x) = \begin{cases} \frac{1}{x^2+5}, & x \leq -2 \\ 3^x, & -2 < x < 1 \\ \cos(3\pi x), & x \geq 1 \end{cases}$

a. Continuous at $x = -2$?

b. Continuous at $x = 1$?

**Find the domain of each function.**

23. $w(t) = \frac{t-7}{\sqrt{t+49}}$

24. $f(x) = \ln(6x - 5)$

25. If the function $f$ is continuous for all real numbers and if $f(x) = \frac{x^2-2x-63}{x-9}$ when $x \neq 9$, then $f(9) =$

26. Let $f$ be the function defined by $f(x) = \begin{cases} \frac{3\sin(5x)}{2x}, & x \neq 0 \\ a, & x = 0 \end{cases}$. For what value of $a$ is $f$ continuous at $x = 0$?

27. According to the table, what is value of $\lim_{x \to 2} f(x)$?

| $x$ | 1.8 | 1.99 | 2.01 | 2.3 |
|---|---|---|---|---|
| $f(x)$ | $-5.6$ | $-5.501$ | $-5.499$ | $-5.3$ |

## Answers

| | | | | | |
|---|---|---|---|---|---|
| 1a. $-2$ | 1b. $-2$ | 1c. $1$ | 1d. $-2$ | 1e. $4$ | 1f. $-2$ |
| 2. $\frac{1}{8}$ | 3. $\infty$ | 4. $0$ | 5. $\infty$ | 6. $\frac{1}{2}$ | 7. $-49$ |
| 8. $-\frac{5}{2}$ | 9. $2$ | 10. $-1$ | 11a. $\frac{\sqrt{2}}{2}$ | 11b. $1$ | 11c. $0$ |
| 11d. $0$ | 11e. $\frac{\sqrt{2}}{2}$ | 11f. $0$ | 11g. DNE | 11h. $0$ | 12. $y = 0$ |
| 13. $y = \frac{1}{4}$ | 14. $y = 3$ | 15. $-5$ | 16. The number of deaths in the year 2000. | 17. The average number of deaths per year from 1995 to 2005. | |
| 18. The number of deaths per year in 1990. | 19. Jump at $x = 1$ | 20. Hole at $x = 0$ V.A. at $x = 4$ | 21. Hole at $x = -7$ | | |
| 22a. Yes. $f(-2) = \frac{1}{9}$ and $\lim_{x\to -2} f(x) = f(-2)$ | 22b. No. $\lim_{x\to 1^-} f(x) \neq \lim_{x\to 1^+} f(x)$ | 23. $t > -49$ | 24. $x > \frac{5}{6}$ | | |
| 25. $16$ | 26. $\frac{15}{2}$ | 27. $-5.5$ | | | |