Unit 1 - Mid-Unit Review

AP Calc. AB

Name: _____ Date:_____ Period: _____

Review

Mid-Unit 1 Review – Limits

Lessons 1.1 through 1.9

Reviews do NOT cover all material from the lessons but will hopefully remind you of key points. To be prepared, you must study all packets from Unit 1.

A salesman tracks the number of cars he sells through the model c, where $c(m)$ is number of cars sold and m is the month for $0 \leq m \leq 24$.

1. What does $c(10)$ represent?

2. What does $\frac{c(16)-c(8)}{16-8}$ represent?

3. What does $\frac{c(7)-c(6.999)}{7-6.999}$ represent?

Evaluate the limit.

4. $\lim_{x \to 0} \frac{\sqrt{x+19}-\sqrt{19}}{x}$

5. $\lim_{x \to -3} \frac{x+3}{x^2+2x-3}$

6. $\lim_{x \to 0} \frac{\sin^2(3x)}{\sin^2(5x)}$

7. $\lim_{x \to 2^-} \frac{|x-2|}{x-2}$

8. $\lim_{x \to 10} \frac{x^2-5x-50}{x-10}$

9. $\lim_{x \to 0} \frac{\frac{1}{x+1}-1}{x}$

10. If $f(x) = \begin{cases} \sin x, & x < -\pi \\ \tan x & -\pi < x < \frac{\pi}{4} \\ \cos x, & x \geq \frac{\pi}{4} \end{cases}$, find the following:

a. $\lim_{x \to -\pi^-} f(x) =$

b. $\lim_{x \to -\pi} f(x) =$

c. $\lim_{x \to \frac{\pi}{4}} f(x) =$

d. $f\left(\frac{\pi}{4}\right) =$

Give the value of each statement. If the value does not exist, write "*does not exist*" or "*undefined.*"

11. $\lim_{x \to 3} f(x) =$

12. $\lim_{x \to 1} f(x) =$

13. $f(3) =$

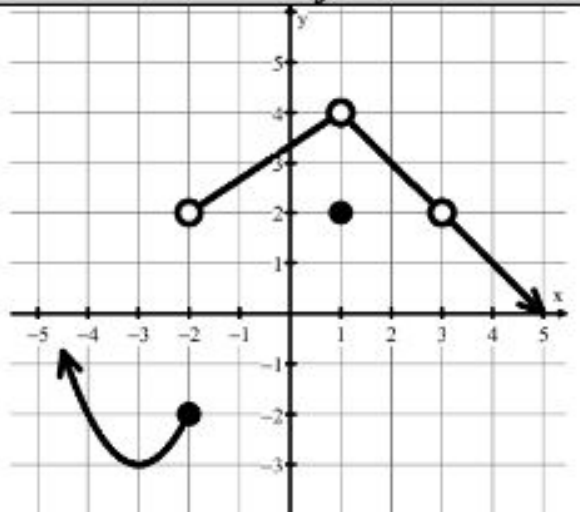
14. $f(-2) =$

15. $\lim_{x \to 2} f(x) =$

16. $\lim_{x \to -2^+} f(x) =$

17. $f(1) =$

18. $\lim_{x \to -2^-} f(x) =$



19. Let g and h be the functions defined by $g(x) = -\frac{1}{4}x^2 - \frac{1}{2}x - \frac{9}{4}$ and $h(x) = \sin\left(\frac{\pi}{2}x\right) - 1$. If f is a function that satisfies $g(x) \leq f(x) \leq h(x)$ for all x, what is $\lim_{x \to -1} f(x)$?

CALCULATOR ALLOWED:

20. If $f(x) = \frac{x^2+10x+21}{x+3}$, create your own table of values to help you evaluate $\lim_{x \to -3} f(x)$.

$\lim_{x \to -3} f(x) =$

x						
$f(x)$						

Solutions

Unit 1 - Mid-Unit Review

Independent Work Time

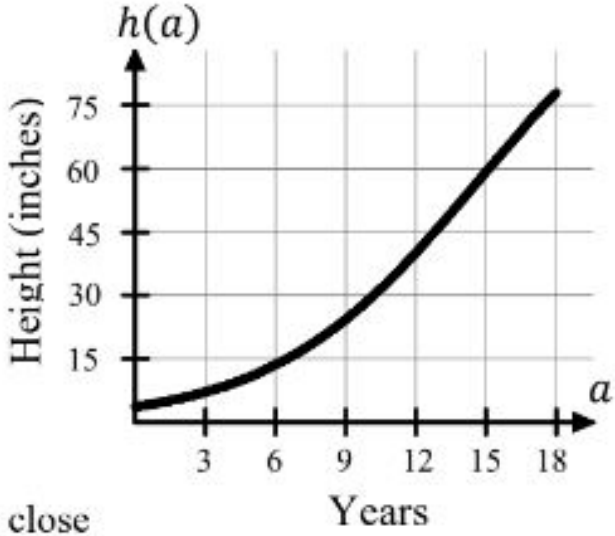
The following CA will cover questions from sections 1.1-1.9 we can take some time to go through any of these questions during the next lesson.

Name: _____ Date:_____

Corrective Assignment

Mid-Unit 1 Corrective Assignment – Limits and Continuity

A child's height can be modeled by the function h, where $h(a)$ gives the height in inches and a gives the child's age in years for $0 \leq a \leq 18$. The graph of the function h is shown to the right.



1. Draw a tangent line at $a = 9$.

2. Give a rough estimate of the instantaneous rate of change at $a = 9$.

3. Give an example of how to calculate a rate of change that would give a close estimate to the rate of change for $a = 17$.

A house loan is being paid off each month. The interest being paid each month m can be modeled by I, where $I(m)$ is the interest payment and m is the month for $0 \leq m \leq 360$.

4. What does $I(3)$ represent?

5. What does $\frac{I(3)-I(1)}{3-1}$ represent?

6. What does $\frac{I(4)-I(3.999)}{4-3.999}$ represent?

Give the value of each statement. If the value does not exist, write "*does not exist*" or "*undefined*."

7.

a. $\lim_{x\to 3^-} f(x) =$

b. $f(-1) =$

c. $\lim_{x\to -3} f(x) =$

d. $\lim_{x\to -1} f(x) =$

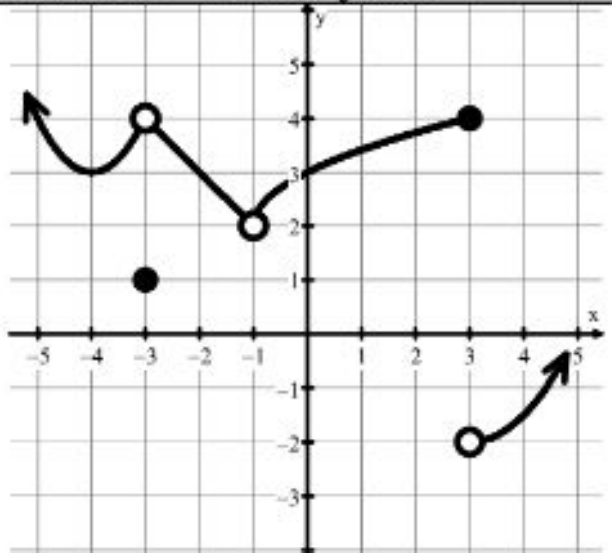
e. $f(-3) =$

f. $\lim_{x\to 3^+} f(x) =$

g. $f(3) =$

h. $\lim_{x\to 0} f(x) =$

i. $f(-4) =$



Sketch a graph of a function h that satisfies all of the following conditions.

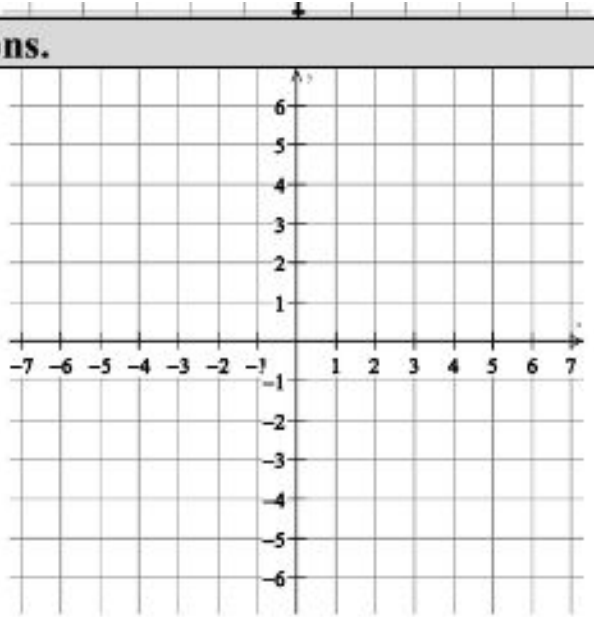
8.

a. $\lim_{x \to 3} h(x) = h(-2) = 1$

b. h is constant on $-2 < x < 3$ and decreasing everywhere else.

c. $h(3)$ is undefined.

d. $\lim_{x \to -2^-} h(x) < \lim_{x \to -2^+} h(x)$



9. According to the table, what is value of $\lim_{x \to -3} f(x)$?

x	-3.4	-3.001	-2.999	-2.7
$f(x)$	7.7	7.999	8.001	8.18

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10. If $f(x) = \begin{cases} x^2 - 1, & x \leq 1 \\ \ln x & 1 < x \leq e \\ \sqrt{x}, & x > e \end{cases}$, find the following:

a. $\lim_{x \to e^-} f(x) =$

b. $\lim_{x \to 1} f(x) =$

c. $\lim_{x \to e} f(x) =$

d. $f(e) =$

Evaluate the limit.

11. $\lim_{x \to 1} \frac{x^2-2x-15}{x+3}$

12. $\lim_{x \to 2} \frac{x^2+2x-8}{x-2}$

13. $\lim_{x \to -2} \frac{x+2}{x^2-4}$

14. $\lim_{x \to 0} \frac{1-\cos(2x)}{5x}$

15. $\lim_{x \to 2} \frac{\sqrt{x+2}-2}{x-2}$

16. $\lim_{x \to 0} \dfrac{\frac{1}{x+8} - \frac{1}{8}}{x}$

17. $\lim_{x \to -4^+} \dfrac{|x+4|}{x+4}$

18. Let f be a function where $\lim_{x\to 5} f(x) = 6$. Which of the following could represent the function f?

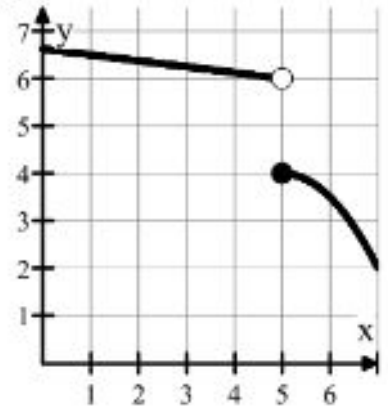
I.

$$f(x) = \begin{cases} \frac{x^2-4x-5}{x-5}, & x \neq 5 \\ 1, & x = 5 \end{cases}$$

II.

x	4.8	4.9	4.999	5	5.001	5.1	5.2
$f(x)$	6.2	6.1	6.001	-2	5.999	5.9	5.8

III.



(A) I only (B) II only (C) III only (D) I and II only (E) I and III only

19. If f is a piecewise linear function such that $\lim_{x \to 9} f(x)$ does not exist, which of the following could be representative of the function f?

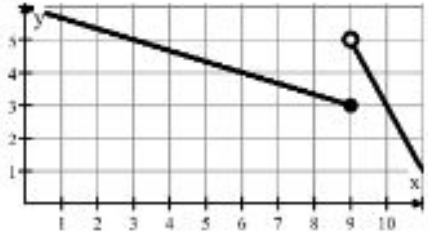
I.

$$f(x) = \begin{cases} 2x - 1, & x < 9 \\ \frac{1}{3}x + 14, & x > 9 \end{cases}$$

II.

x	6	7	8	9	10	11	12
$f(x)$	3	$\frac{10}{3}$	$\frac{11}{3}$	-3	8	12	16

III.



(A) I only (B) II only (C) III only (D) I and III only (E) none

20. Let f and g be the functions defined by $f(x) = \frac{8-8\cos x}{x^2}$ and $g(x) = x^2 \cos\left(\frac{1}{x}\right)$ for $x \neq 0$. The following inequalities are true for $x \neq 0$. State whether each inequality can be used with the squeeze theorem to find the limit of the function as x approaches 0?

I. $4 - x^2 \leq f(x) \leq 4$ II. $-x^2 - 1 \leq g(x) \leq 1 + x^2$ III. $-x^2 \leq g(x) \leq x^2$

(A) I only (B) II only (C) III only (D) I and II only (E) I and III only

21. The function f is continuous and increasing for $x \geq 0$. The table gives values of f at selected values of x.

x	6.5	6.999	7.001	7.5
$f(x)$	-6.5	-6.001	-5.999	-5.5

Approximate the value of $\lim_{x \to 7} 5e^{f(x)}$.

ANSWERS to Mid-Unit 1 Corrective Assignment

1. See graph. Straight line at $a = 9$.

2. ≈ 5 inches per year

3. $\frac{a(17)-a(16.999)}{17-16.999}$

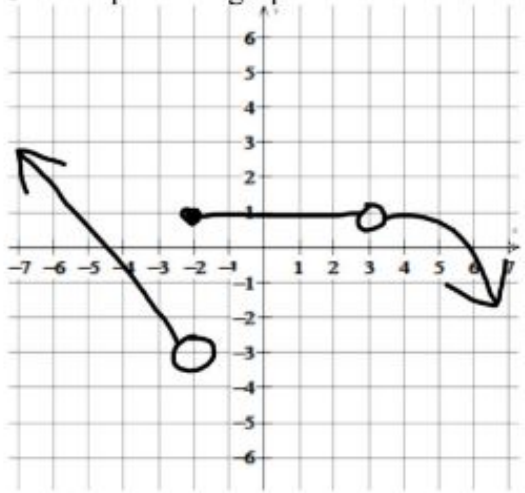
4. The interest paid on the 3rd month.

5. The average rate of change of interest being paid between the 1st and 3rd months.

6. An estimate of the rate at which the interest payment is changing on the 4th month.

7a. 4
7d. 2
7g. 4
7d. DNE
7e. 1
7h. 3
7c. 4
7f. -2
7i. 3

8. One possible graph:



9. 8

10a. 1
10b. 0
10c. DNE
10d. 1

11. -4

12. 6

13. $-\frac{1}{4}$

14. 0

15. $\frac{1}{4}$

16. $-\frac{1}{64}$

17. 1

18. D

19. C

20. E

21. $5e^{-6} = \frac{5}{e^6}$