Unit 3 - End of Unit Review

AP Calc. AB/BC

Warmup: Questions from Lesson 3.6?

If you haven’t finished the CA worksheet from yesterday take it out and continue to work on it during this time.

Name: _____ Date:_____ Period: _____

Review

Unit 3 REVIEW – Composite, Implicit, and Inverse Functions

Reviews do NOT cover all material from the lessons but should remind you of key points. To be prepared, you should review all packets from Unit 3.

Find the derivative.

1. $h(x) = \cos^2(4x)$

2. $y = \ln\sqrt{x+3}$

3. $x^2 + 2y^5 = 10xy$

4. $y = \csc^{-1}(x^3)$

For each problem, let f and g be differentiable functions where $g(x) = f^{-1}(x)$ for all x.

5. $f(6) = -1$, $f(4) = -2$, $f'(6) = 3$, and $f'(4) = 7$. What is the value of $g'(-1)$?

6. Let f be the function defined by $f(x) = x^3 + 3x + 1$. Let $g(x) = f^{-1}(x)$, where $g(-3) = -1$. What is the value of $g'(-3)$?

Find $\frac{d^2y}{dx^2}$ based on the given information.

7. $y = x^5 - e^{4x}$

8. $y = y^2 + x$

9. Find the equation of the tangent line.
$x^2 + 7y^2 = 8y^3$ at $(-6, 2)$

10. If $x = y^2 - \cos x$ find $\frac{d^2y}{dx^2}$ at $(0, -1)$.

Practice - Test Prep.

Take the next 5-10 minutes to work together on the practice - test prep section of our notes.

We will go through it together on the board after the time is up!

Name: Solutions Date: Period: **Review**

Unit 3 REVIEW – Composite, Implicit, and Inverse Functions

Reviews do NOT cover all material from the lessons but should remind you of key points. To be prepared, you should review all packets from Unit 3.

Find the derivative.

1. $h(x) = \cos^2(4x)$

$h'(x) = 2\cos(4x) \cdot (-\sin(4x)) \cdot 4$

$h'(x) = -8\cos(4x)\sin(4x)$

2. $y = \ln\sqrt{x+3}$

$y' = \frac{1}{\sqrt{x+3}} \cdot \frac{1}{2\sqrt{x+3}}$

$y' = \frac{1}{2x+6}$

3. $x^2 + 2y^5 = 10xy$

$2x + 10y^4\frac{dy}{dx} = 10y + 10x\frac{dy}{dx}$

$\frac{dy}{dx}(10y^4 - 10x) = 10y - 2x$

$\frac{dy}{dx} = \frac{5y - x}{5y^4 - 5x}$

4. $y = \csc^{-1}(x^3)$

$\frac{dy}{dx} = -\frac{1}{|x^3|\sqrt{x^6 - 1}} \cdot (3x^2)$

$\frac{dy}{dx} = -\frac{3}{|x|\sqrt{x^6 - 1}}$

For each problem, let f and g be differentiable functions where $g(x) = f^{-1}(x)$ for all x.

5. $f(6) = -1$, $f(4) = -2$, $f'(6) = 3$, and $f'(4) = 7$. What is the value of $g'(-1)$?

$$g'(-1) = \frac{d}{dx} f^{-1}(-1) = \frac{1}{f'(f^{-1}(-1))}$$

$$= \frac{1}{f'(6)}$$

$$= \boxed{\frac{1}{3}}$$

6. Let f be the function defined by $f(x) = x^3 + 3x + 1$. Let $g(x) = f^{-1}(x)$, where $g(-3) = -1$. What is the value of $g'(-3)$?

$$g'(-3) = \frac{d}{dx} f^{-1}(-3) = \frac{1}{f'(f^{-1}(-3))}$$

$f'(x) = 3x^2 + 3$

$f'(-1) = 6$

$$= \frac{1}{f'(-1)}$$

$$= \boxed{\frac{1}{6}}$$

Find $\frac{d^2y}{dx^2}$ based on the given information.

7. $y = x^5 - e^{4x}$

$$\frac{dy}{dx} = 5x^4 - e^{4x} \cdot 4$$

$$\frac{d^2y}{dx^2} = 20x^3 - e^{4x} \cdot 4 \cdot 4$$

$$\boxed{\frac{d^2y}{dx^2} = 20x^3 - 16e^{4x}}$$

8. $y = y^2 + x$

$$\frac{dy}{dx} = 2y\frac{dy}{dx} + 1$$

$$\frac{dy}{dx}(1 - 2y) = 1$$

$$\frac{dy}{dx} = \frac{1}{(1-2y)} = (1-2y)^{-1}$$

$$\frac{d^2y}{dx^2} = -1(1-2y)^{-2} \cdot \left(-2\frac{dy}{dx}\right)$$

$$= -\frac{1}{(1-2y)^2} \cdot \left(\frac{-2}{(1-2y)}\right) = \boxed{\frac{2}{(1-2y)^3}}$$

9. Find the equation of the tangent line.
$x^2 + 7y^2 = 8y^3$ at $(-6, 2)$

$$2x + 14y\frac{dy}{dx} = 24y^2\frac{dy}{dx}$$

$$2(-6) + 14(2)\frac{dy}{dx} = 24(4)\frac{dy}{dx}$$

$$-12 + 28\frac{dy}{dx} = 96\frac{dy}{dx}$$

$$-12 = 68\frac{dy}{dx}$$

$$-\frac{3}{17} = \frac{dy}{dx}$$

$$\boxed{y - 2 = -\frac{3}{17}(x + 6)}$$

10. If $x = y^2 - \cos x$ find $\frac{d^2y}{dx^2}$ at $(0, -1)$.

$$1 = 2y\frac{dy}{dx} + \sin x$$

$$\frac{1 - \sin x}{2y} = \frac{dy}{dx} \qquad \left.\frac{dy}{dx}\right|_{(0,-1)} = \frac{1 - \sin(0)}{2(-1)} = \frac{1}{-2}$$

$$\frac{d^2y}{dx^2} = \frac{-\cos x\,(2y) - (1 - \sin x)\left(2\frac{dy}{dx}\right)}{4y^2} \qquad \frac{dy}{dx} = -\frac{1}{2}$$

$$\left.\frac{d^2y}{dx^2}\right|_{(0,-1)} = \frac{-(1)(-2) - (1 - 0)(-1)}{4(-1)^2} = \frac{2 + 1}{4} = \boxed{\frac{3}{4}}$$

Independent Work Time

Please work on the CA worksheet during this time.

We will go through any questions from the CA worksheet tomorrow during the Warmup.

If you finish the CA worksheet early then you can go to AP Classroom and work on homework, step-by-step, watch a daily video, or utilize a different resource from GC

Name: _____ Date:_____

Corrective Assignment

Unit 3 CA – Composite, Implicit, and Inverse Functions

Find $\frac{dy}{dx}$.

1. $y = \frac{e^{\tan 3x}}{3}$	2. $y = \ln(\sin 5x)$	3. $y = x\ln(4x)$
4. $e^{y^2} = x^5 + 10$	5. $y = \cos^{-1}(7x^3)$	6. $2x^3 - xy = \ln(y)$

Find the equation of the tangent line at the given point.

7. $4x^3 = -5xy + 4y$ at $(1, -4)$

8. $y = \arccos(5x)$ at $x = -\frac{\sqrt{3}}{10}$

9. $h(x) = (2x - 1)^3(x + 2)$ at $x = 1$.

10. Find the equation of any horizontal tangent lines for the graph of $(y^3 + 1)^2 = x^2 + 4x + 4$.

11. Slope of the tangent line of $g(x) = 4\sin^3 x$ at $x = \frac{\pi}{4}$.

12. Let f and g be differentiable functions where $g(x) = f^{-1}(x)$ for all x. $f(6) = 8$, $f(8) = 2$, $f'(6) = -3$, and $f'(8) = 4$. What is the value of $g'(8)$?

Find $\frac{d^2y}{dx^2}$ based on the given information.

13. $y = e^{x^4}$

14. $5y^2 + 3 = x^2$

Evaluate the 2nd derivative at the given point.

15. If $f(x) = x^3 + \frac{5}{x}$, find $f''(-1)$.

16. If $x^2 + y^2 = 13$, find $\frac{d^2y}{dx^2}$ at $(2, 3)$.

The table below gives values of the differentiable functions g and h, as well as their derivatives, g' and h', at selected values of x.

x	$g(x)$	$g'(x)$	$h(x)$	$h'(x)$
-1	0	4	3	6
0	9	2	0	-4
3	-1	-2	9	4
9	3	1	16	9

17. If $f(x) = \frac{g(x)}{\sqrt{h(x)}}$, find $f'(3)$.

18. Find $\frac{d}{dx}h^{-1}(9)$.

19. Find the equation of the tangent line to $g^{-1}(x)$ at $x = 3$.

Unit 3 Corrective Assignment – Answers

1. $e^{\tan(3x)}\sec^2(3x)$	2. $5\cot(5x)$	3. $\ln(4x)+1$	4. $\frac{5x^4}{2ye^{y^2}}$
5. $-\frac{21x^2}{\sqrt{1-49x^6}}$	6. $\frac{6x^2-y}{\frac{1}{y}+x}$	7. $y+4=8(x-1)$	8. $y-\frac{5\pi}{6}=-10\left(x+\frac{\sqrt{3}}{10}\right)$
9. $y-3=19(x-1)$	10. $y=-1$	11. $3\sqrt{2}$	12. $-\frac{1}{3}$
13. $12x^2e^{x^4}+16x^6e^{x^4}$			
14. $\frac{5y-\frac{x^2}{y}}{25y^2}$	15. -16	16. $-\frac{13}{27}$	17. $-\frac{16}{27}$
18. $\frac{1}{4}$	19. $y=x+6$		