Unit 5 - End of Unit Review

Calc. AB/BC

Warmup: Questions from Lesson 5.12?

If you haven't finished the CA worksheet from yesterday take it out and continue to work on it during this time.

Unit 5 Review – Analytical Applications of Differentiation

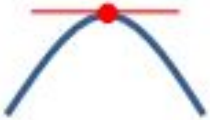
This review summarizes everything from Unit 5 along with examples but contains no problems to work through.

DEFINITIONS

Extrema: The maximum and minimum points. Extrema can be absolute or relative.

Critical Points: Where the first derivative is zero or DNE. These are possible maximum, minimum, or points of inflection!

$f'(x) = 0$

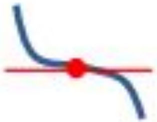


Horizontal Tangent Maximum

$f'(x) = 0$

Horizontal Tangent Minimum

$f'(x) = 0$



Horizontal Tangent Point of Inflection

$f'(x) = DNE$

Vertical Tangent Point of Inflection

$f'(x) = DNE$

Cusp (No Tangent) Maximum

Concavity: Where the function is "cupping" up or down

Points of Inflection: Where the second derivative is zero or DNE and changes sign!

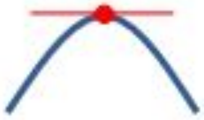
FIRST DERIVATIVE

The first derivative is the instantaneous rate of change, or the slope of the tangent line, and can determine if the function is increasing or decreasing at a given point.

$f'(x) > 0$

Function is increasing

$f'(x) = 0$



Function is not increasing or decreasing

$f'(x) < 0$

Function is decreasing

SECOND DERIVATIVE

The second derivative determines concavity.

$f''(x) > 0$

Concave Up

$f''(x) = 0$

Neither concave up or concave down

$f''(x) < 0$

Concave Down

FINDING EXTREMA

The First Derivative Test

<table>
<tr><th>STEPS</th><th>EXAMPLE
$f(x) = x^2 + 2x + 1$</th></tr>
<tr><td>1. Find the critical points.</td><td>$f'(x) = 2x + 2$
$0 = 2x + 2$
$x = -1$</td></tr>
<tr><td>2. Determine whether the function is increasing or decreasing on each side of every critical point.
A chart or number line helps!</td><td>
<table>
<tr><th>Interval</th><th>$(-\infty, -1)$</th><th>-1</th><th>$(-1, \infty)$</th></tr>
<tr><td>Test Value</td><td>-2</td><td>-1</td><td>2</td></tr>
<tr><td>$f'(x)$</td><td>$f'(-2) = -$
Negative</td><td>$f'(-1)$
$= 0$</td><td>$f'(2) = 6$
Positive</td></tr>
</table>
Function decreases to the left and increases to the right of $x = -1$ so it must be relative minimum point</td></tr>
</table>

The Second Derivative Test

STEPS	EXAMPLE $\boldsymbol{f(x) = x^2 + 2x + 1}$
1. Find the critical points.	$f'(x) = 2x + 2$ $0 = 2x + 2$ $x = -1$
2. Determine whether the function is concave up or concave down at every critical point using the second derivative.	$f''(-1) = 2$ Second derivative is positive at $x = -1$ Concave up $x = -1$ is a relative minimum point

Finding Absolute Extrema on an Interval (Candidates Test)

STEPS	EXAMPLE $f(x) = x^2 + 2x + 1$ on the interval $[-3, 0]$
1. Find the critical points. The critical points are candidates as well as the endpoints of the interval.	$f'(x) = 2x + 2$ $0 = 2x + 2$ $x = -1$
2. Check all candidates using the $f(x)$.	$f(-3) = 4$ absolute maximum $f(-1) = 0$ absolute minimum $f(0) = 1$

LINEAR MOTION

The chart matches up function vocabulary with linear motion vocabulary.

FUNCTION	LINEAR MOTION
Value of a function at x	Position at time t
First Derivative	Velocity
Second Derivative	Acceleration
$f'(x) > 0$ Increasing Function	Moving right or up
$f'(x) < 0$ Decreasing Function	Moving left or down
$f'(x) = 0$	Not moving
Absolute Max	Farthest right or up
Absolute Min	Farthest left or down
$f'(x)$ changes signs	Object changes direction
$f'(x)$ and $f''(x)$ have same sign	Speeding Up
$f'(x)$ and $f''(x)$ have different signs	Slowing Down

Example:

A particle moves along the x-axis with the position function $x(t) = t^4 - 4t^3 + 2$ where $t > 0$.

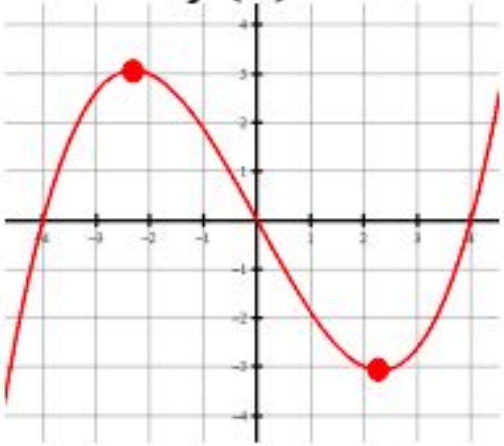
Interval	$(\mathbf{0, 2})$	**2**	$(\mathbf{2, 3})$	**3**	$(\mathbf{3, \infty})$
$\boldsymbol{x'(t)}$ **velocity**	$x'(t) > 0$ increasing right	$x'(t) > 0$ Increasing right	$x'(t) > 0$ increasing right	$x'(t) = 0$ Not moving	$x'(t) < 0$ decreasing left
$\boldsymbol{x''(x)}$ **acceleration**	$x''(t) > 0$ Concave up	$x''(t) = 0$	$x''(t) < 0$ Concave down	$x''(t) < 0$ Concave down	$x''(t) < 0$ Concave down
Conclusion	Speeding Up	Moving Right	Slowing Down	Not Moving	Speeding Up

FUNCTION	**LINEAR MOTION**
$t = 3$ is maximum	$t = 3$ has no velocity Changing direction
Increasing (0,3)	Moving right (0,3)
Decreasing $(3, \infty)$	Moving left $(3, \infty)$

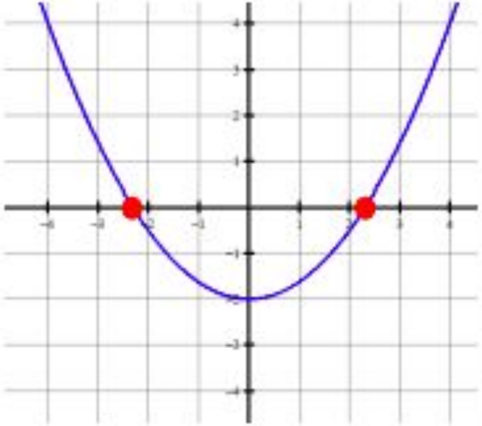
GRAPHICAL ANALYSIS

Connecting $f(x)$ to $f'(x)$

$f(x)$



$f'(x)$



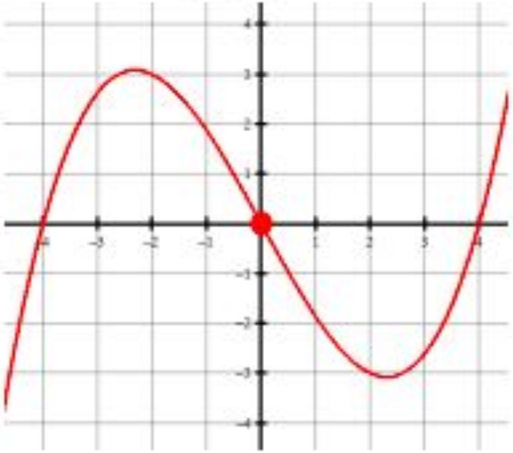
$f(x)$ max at $x = -2.2$ and min $x = 2.2$
so $f'(x) = 0$ at these points

$f(x)$ is increasing on
$(-\infty, -2.2)(2.2, \infty)$
so $f'(x) > 0$ on these intervals

$f(x)$ is decreasing on $(-2.2, 2.2)$
so $f'(x) < 0$ on this interval

Connecting $f(x)$ to $f''(x)$

$f(x)$

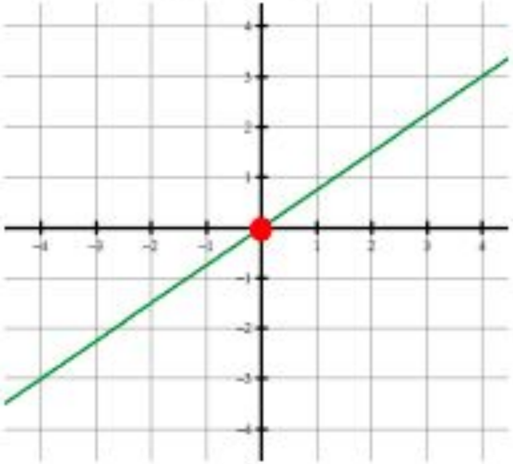


$f(x)$ is concave down on $(-\infty, 0)$
so $f''(x) < 0$ on this interval

$f(x)$ is concave up on $(0, \infty)$
so $f''(x) > 0$ on this interval

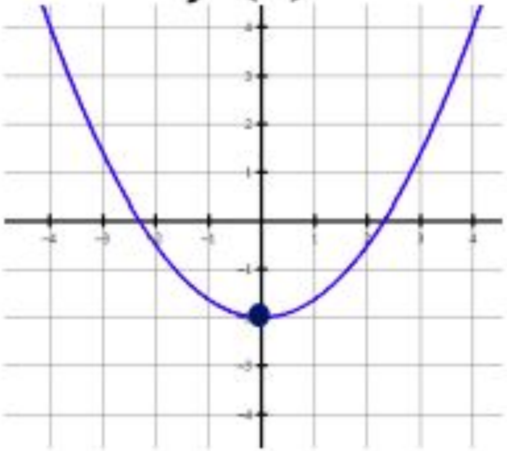
$x = 0$ is a point of inflection on $f(x)$
so $f''(x)$ changes sign at $x = 0$.

$f''(x)$



Connecting $f'(x)$ to $f''(x)$

$f'(x)$

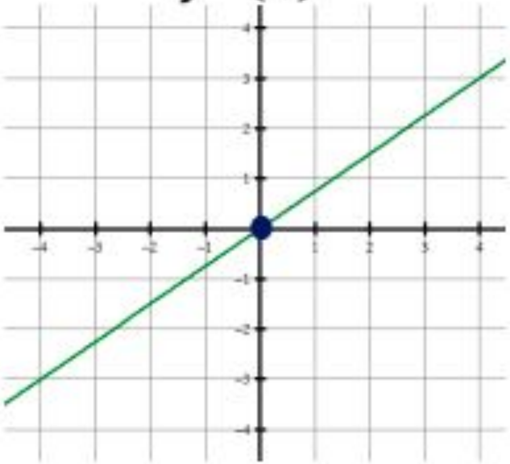


$f'(x)$ min $x = 0$
so $f''(x) = 0$ at $x = 0$

$f'(x)$ is decreasing on $(-\infty, 0)$
so $f''(x) < 0$ on this interval

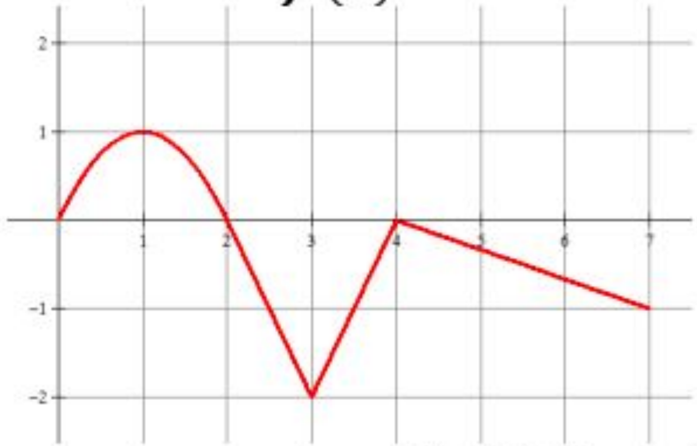
$f'(x)$ is increasing on $(0, \infty)$
so $f''(x) > 0$ on this interval

$f''(x)$



Using $f'(x)$ to draw conclusions about $f(x)$

$f'(x)$



Find Extrema of $f(x)$

$x = 2$ and 4 are critical points because
$f'(x) = 0$

$x = 2$ is a maximum because
$f'(x)$ is positive on left, negative on right

$x = 4$ is NOT an extrema because
$f'(x)$ is negative on left, negative on right

Find Points of Inflection of $f(x)$

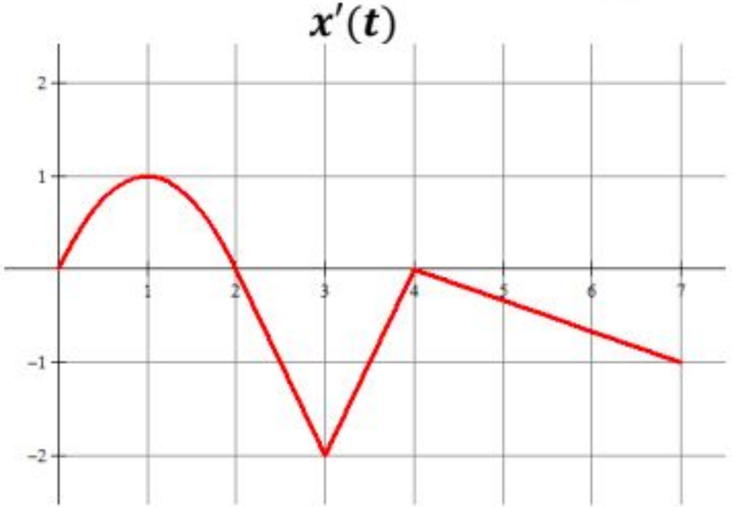
$x = 1, 3,$ and 4 are possible points of inflection because
$f''(x) = 0$ or DNE

$x = 1$ is a point of inflection because
$f''(x)$ changes sign from positive to negative at $x = 1$.

$x = 3$ is a point of inflection because
$f''(x)$ changes sign from negative to positive at $x = 3$.

$x = 4$ is a point of inflection because
$f''(x)$ changes sign from positive to negative at $x = 4$.

Now interpret the same graph as linear motion if the graph represents velocity of a particle moving along x-axis.



Moving Right or Left?

Particle moves right on (0,2).

$t = 2$ particle changes direction.

Particle moves left on (2,4)(4,7).

$t = 4$ particle has no velocity.

The maximum speed happens at $t = 3$.

Speeding up or Slowing down?

Particle speeds up on (0,1)
because $f'(x)$ has the same sign as $f''(x)$

Particle slows down on (1,2)
because $f'(x)$ has a different sign from $f''(x)$

Particle speeds up on (4,7)
because $f'(x)$ has the same sign as $f''(x)$

Particle speeds up on (2,3)
because $f'(x)$ has the same sign as $f''(x)$

Particle slows down on (3,4)
because $f'(x)$ has a different sign from $f''(x)$

Practice - Test Prep.

Take the next 5-10 minutes to work together on the practice - test prep section of our notes.

We will go through it together on the board after the time is up!

Name: ______________________________ Date:_______________ **Corrective Assignment**

End of Unit 5 CA – Analytical Applications of Differentiation

1. **Calculator active problem.** The first derivative of the function f is given by

$$f'(x) = -2 + x + 3e^{-\cos(4x)}$$

How many points of inflection does the graph of f have on the interval $0 < x < \pi$?

2. **Calculator active problem.** The rate of money in a particular mutual fund is represented by $m(t) = \sin\left(\frac{e}{3}\right)^t$ thousand dollars per year where t is measured in years. Is the amount of money from this mutual fund increasing or decreasing at time $t = 4$ years? Justify your answer.

3. A particle is traveling along the y-axis and its position from the origin can be modeled by

$$y(t) = 6t - 2t^3 + 10$$

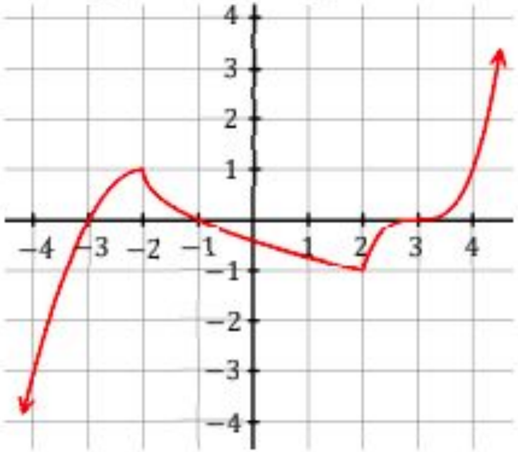
where y is meters and t is minutes.

a. On the interval $0 \leq t \leq 2$, when is the particle farthest above the origin.

b. On the interval $0 \leq t \leq 2$, what is the particle's maximum speed?

4. A rectangle is formed with the base on the x-axis and the top corners on the function $y = 36 - x^2$. What length and width should the rectangle have so that its area is a maximum?

5. The graph shows the derivative of f, f'. Identify the intervals when f is increasing and decreasing. Include a justification statement.



Increasing:

Decreasing:

6. For the table below, selected values of x and $f(x)$ are given. Assume that $f'(x)$ and $f''(x)$ do not change signs.

x	$f(x)$
0	-10
1	-8
2	-5
3	-1

   a. Is $f(x)$ increasing or decreasing?

   b. Is $f(x)$ concave up or concave down?

7. Given the function $g(x) = -x^4 + 2x^2 - 1$, find the interval(s) when g is **concave up** and **decreasing** at the same time.

8. The Mean Value Theorem can be applied to which of the following function on the closed interval $[0, 5]$?

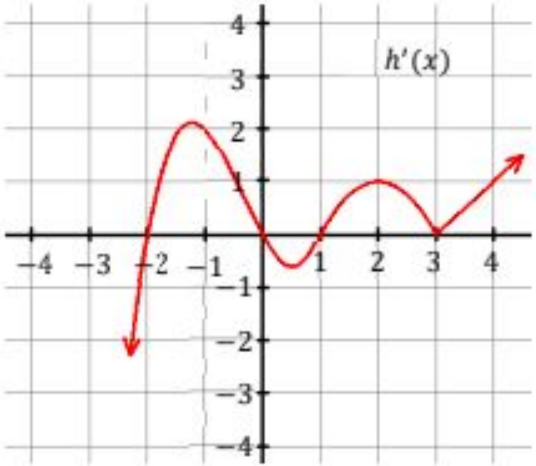
(A) $f(x) = \frac{x-3}{x+3}$

(B) $f(x) = (x-1)^{\frac{2}{3}}$

(C) $f(x) = \frac{x+3}{x-3}$

(D) $f(x) = |x-4|$

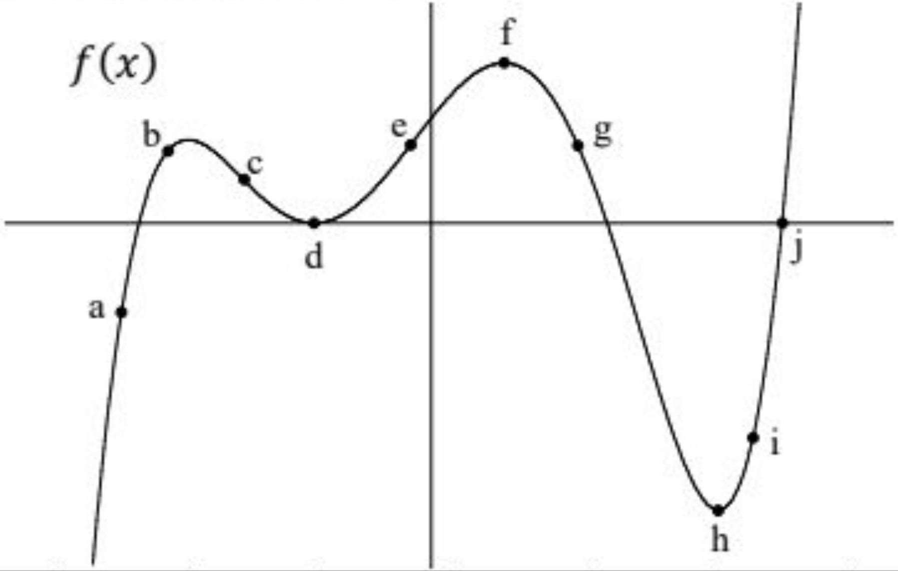
9. To the right is the graph of $h'(x)$. Identify all extrema of $h(x)$. No justification necessary on this problem.



10. The derivative of g is given by $g'(x) = (5 - x)x^{-3}$ for $x > 0$. Find all relative extrema and justify your conclusions.

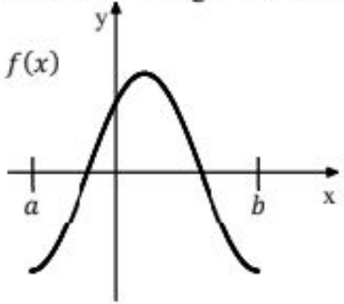
11. Consider the function f defined by $f(x) = e^x \sin x$ with domain $[0, 2\pi]$. Find the absolute maximum and minimum values of $f(x)$.

12. Using the figure below, complete the chart by indicating whether each value is positive (+), negative (-), or zero (0) at the indicated points. For these problems, if the point appears to be a max or min, assume it is. If it appears to be a point of inflection, assume it is.

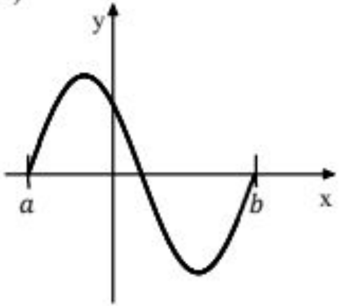


x	a	b	c	d	e	f	g	h	i	j
$f(x)$										
$f'(x)$										
$f''(x)$										

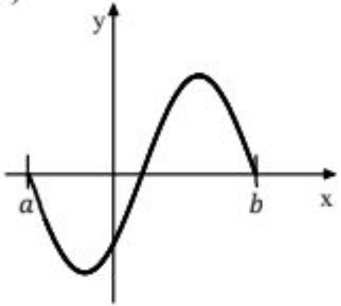
13. The graph of f is shown below. Which of the following could be the graph of the derivative of f?



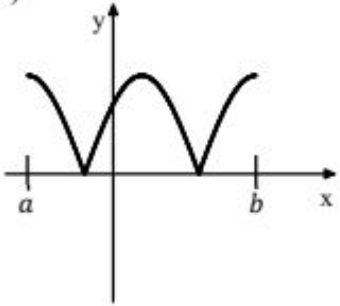
(A)



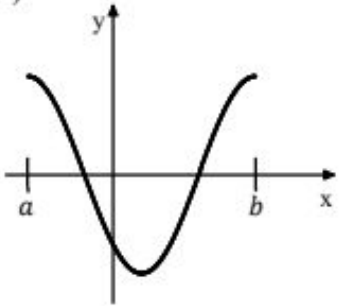
(B)



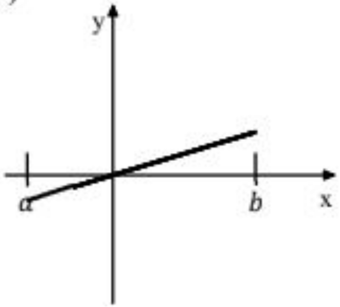
(C)



(D)



(E)



Answers

1. 4

2. Increasing because the rate $m(4)$ is positive. $m(4) \approx 0.3838$

3a. $y(0) = 10$
$\boldsymbol{y(1) = 14}$
$y(2) = 6$
At $t = 1$ minutes

3b. $y'(0) = 6$
$y'(2) = -18$
18 meters / minute

4. $2\sqrt{12} \times 24$

5. Increasing on the interval $(-3,-1)$ and $(3,\infty)$.
Decreasing on the interval $(-\infty,-3)$ and $(-1,3)$.

6a. Increasing

6b. Concave up

7. $\left(-\sqrt{\frac{1}{3}}, 0\right)$

8. A

9.
Min at $x = -2$ and $x = 1$.

Max at $x = 0$.

10. Relative maximum at $x = 5$ because g' changes sign from positive to negative.

11.
$g(0) = 0$
$g\left(\frac{3\pi}{4}\right) = e^{\frac{3\pi}{4}}\left(\frac{\sqrt{2}}{2}\right)$ **ABS MAX**
$g\left(\frac{7\pi}{4}\right) = -e^{\frac{7\pi}{4}}\left(\frac{\sqrt{2}}{2}\right)$ **ABS MIN**
$g(2\pi) = 0$

12.

x	a	b	c	d	e	f	g	h	i	j
$f(x)$	−	+	+	0	+	+	+	−	−	0
$f'(x)$	+	+	−	0	+	0	−	0	+	+
$f''(x)$	−	−	0	+	0	−	−	+	+	+

13. A