Unit 7 - End of Unit Review

Calc. AB/BC

Warmup: Questions from Lesson 7.9?

If you haven't finished the CA worksheet from yesterday take it out and continue to work on it during this time.

Name: ______________________________ Date:______________ Period: _____

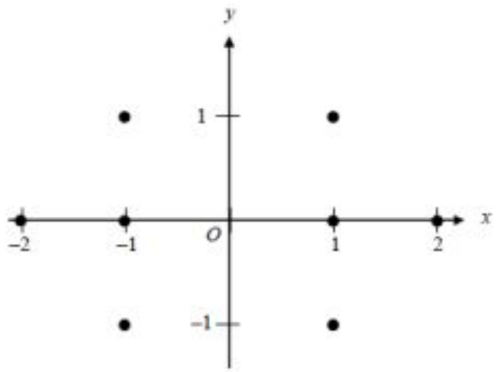
Review

Unit 7 Review – Differential Equations

Reviews do NOT cover all material from the lessons but will hopefully remind you of key points. To be prepared, you must study all packets from Unit 7.

1. Consider the differential equation $\frac{dy}{dx} = \frac{y^2}{x}$, where $x \neq 0$.

   a. On the axes provided, sketch a slope field for the given differential equation at the eight points indicated.



   b. Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(1) = -1$.

   c. Write an equation for the tangent line to the curve $y = f(x)$ through the point $(1, -1)$. Then use your tangent line equation to estimate the value of $f(1.2)$.

2. The rate of change of the volume, $V(t)$, of water in a swimming pool is directly proportional to the cube root of the volume. If $V = 27 \text{ ft}^3$ when $\frac{dV}{dt} = 5$, what is a differential equation that models this situation?

Find the general solution of the differential equation.

4. $\frac{dy}{dx} = \frac{2x}{y}$

5. $\frac{dy}{dx} = x(y + 4)$

For each differential equation, find the particular solution that passes through the given point.

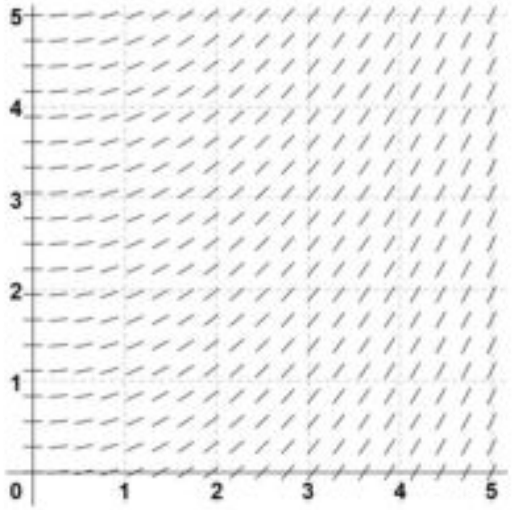
6. $\frac{dy}{dx} = \frac{18}{6x+3} + \frac{4}{x^3}$; $\left(-\frac{1}{3}, -15\right)$

7. $\frac{dy}{dx} = 2y$ and $y = -0.2$ when $x = 0$

8. A population y grows according to the equation $\frac{dy}{dt} = ky$, where k is a constant and t is measured in years. If the population doubles every 12 years, then what is the value of k?

9. The number of people in a store is modeled by a function F that satisfies the logistic differential equation $\frac{dF}{dt} = \frac{1}{500}F(100 - F)$, where t is in hours and $F(0) = 10$. What is the greatest rate of change, in people per hour, of the number of people in the store?

10. Explain why the following slope field cannot represent the differential equation $\frac{dy}{dt} = 0.4y$



11. Given that $y = f(t)$ is a solution to the logistic differential equation $\frac{dy}{dt} = \frac{y}{5} - \frac{y^2}{1500}$, where t is time in years. What is $\lim_{t \to \infty} f(t)$?

12. For what value of k, if any, will $y = k\cos(2x) + 3\sin(4x)$ be a solution to the differential equation $y'' + 16y = -6\cos(2x)$?

13. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = 3x + y$ with initial condition $f(0) = 1$. What is the approximation for $f(0.5)$ obtained using Euler's method with 2 steps of equal length, starting at $x = 0$?

Notes Filled In:

AP Calc. AB/BC - Unit 7 Review - Filled In

Practice - Test Prep.

Take the next 5-10 minutes to work together on the practice - test prep section of our notes.

We will go through it together on the board after the time is up!

Name: ______________________________ Date:_______________

Corrective Assignment

Unit 7 CA – Differential Equations (BC)

1. The rate at which a project $p(x)$ is completed is proportional to the square root of the number of employees x working on the project, where p is measured as a percent of the project that has been completed. If 5 people can complete the project at a rate of 3% per day, what is a differential equation that models this situation?

Find the general solution of the differential equation.

2. $\frac{dy}{dx} = (x + 7)y$

3. $\frac{dy}{dx} = -xy^3$

4. A population y grows according to the equation $\frac{dy}{dt} = ky$, where k is a constant and t is measured in years. If the population doubles every 14 years, then what is the value of k?

5. A dose of 500 milligrams of a drug is administered to a patient. The amount of the drug, in milligrams, in the person's bloodstream at time t, in hours, is given by $A(t)$. The rate at which the drug leaves the bloodstream can be modeled by the differential equation $\frac{dA}{dt} = -0.8A$. Write an expression for $A(t)$.

6. Consider the differential equation $\frac{dy}{dx} = (1 - 2x)y$. If $y = 10$ when $x = 1$, find an equation for y.

(A) $y = e^{x-x^2}$

(B) $y = 10 + e^{x-x^2}$

(C) $y = e^{x-x^2+10}$

(D) $y = 10e^{x-x^2}$

(E) $y = x - x^2 + 10$

7. The solution to the differential equation $\frac{dy}{dx} = \frac{x}{\cos y}$ with the initial condition $y(1) = 0$ is

(A) $y = \sin^{-1}\left(\frac{x^2-1}{2}\right)$

(B) $y = \sin^{-1}\left(\frac{x^2}{2}\right)$

(C) $y = \cos^{-1}(x^2 - 1)$

(D) $y = \ln[\cos(x - 1)]$

(E) $y = \ln(\sin x)$

8. If $\frac{dy}{dx} = \frac{3x^2+2}{y}$ and $y = 4$ when $x = 2$, then when $x = 3$, $y =$

(A) 18

(B) $\pm\sqrt{66}$

(C) 58

(D) $\pm\sqrt{74}$

(E) $\pm\sqrt{58}$

For each differential equation, find the particular solution that passes through the given point.

9. $\frac{dy}{dx} = 9e^{3x} - \cos x$; $(0, 2)$

10. $\frac{dy}{dx} = 4y$ and $y = 8$ when $x = 0$

11. $\frac{d^2y}{dx^2} = \cos(2x) + 1$ and $y'(\pi) = 0$ and $y(0) = 1$

12. For what value of k, if any, is $y = e^{3x} + ke^{-4x}$ a solution to the differential equation $y'' - 3y' = 7e^{-4x}$?

13. The table below gives the values of f', the derivative of f. If $f(1.3) = 1.7$, what is the approximation to $f(2.2)$ obtained by using Euler's method with 3 steps of equal size?

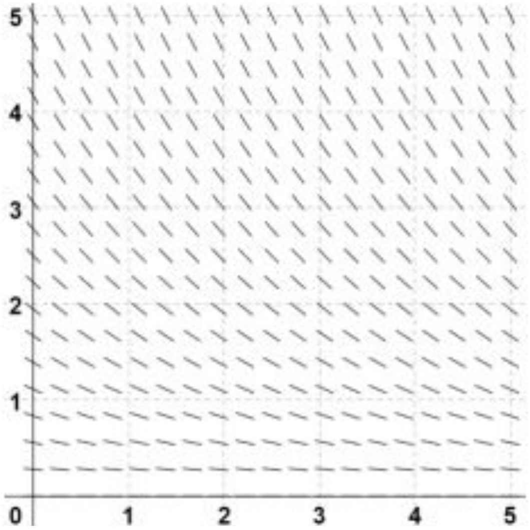
x	1.3	1.6	1.9	2.2
$f'(x)$	0.1	0.3	0.6	1.1

14. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = 2y - x$ with initial condition $f(1) = 3$. What is the approximation for $f(2)$ obtained using Euler's method with 2 steps of equal length, starting at $x = 1$?

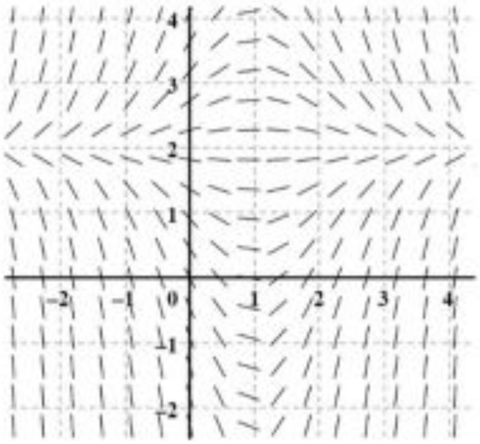
15. A populations rate of growth is modeled by the logistic differential equation $\frac{dP}{dt} = \frac{1}{1000}P(500 - P)$, where t is in weeks and $P(0) = 10$. What is the greatest rate of change for this population?

16. Using the logistic differential equation $\frac{dP}{dt} = 0.2P - 0.001P^2$, identify the carrying capacity.

17. Explain why the following slope field cannot represent the differential equation $\frac{dy}{dt} = 0.4y$



18.



(A) $\frac{dy}{dx} = y - 2x$

(B) $\frac{dy}{dx} = 1 + x + y$

(C) $\frac{dy}{dx} = (1 - x)(y - 2)$

(D) $\frac{dy}{dx} = xy^2$

(E) $\frac{dy}{dx} = (x - 1)y^2$

Answers to Unit 7 Corrective Assignment (BC)

1. $\frac{dp}{dx} = 1.3416\sqrt{x}$	2. $y = Ce^{\frac{1}{2}x^2+7x}$	3. $y = \pm\sqrt{\frac{1}{x^2+C}}$	4. $k \approx 0.0495$	5. $A(t) = 500e^{-0.8t}$
6. D	7. A	8. E	9. $y = 3e^{3x} - \sin x - 1$	10. $y = 8e^{4x}$
11. $y = -\frac{1}{4}\cos(2x) + \frac{1}{2}x^2 - \pi x + \frac{5}{4}$	12. $k = \frac{1}{4}$	13. $f(2.2) = 2$	14. $f(2.0) \approx 10.25$	
15. 62.5/week	16. 200	17. $\frac{dy}{dx} > 0$ when $y > 0$, but the slope field shows line segments with negative slope.	18. C	