Unit 8 - Mid-Unit Review

Calc. AB/BC

Warmup: Questions from Lesson 8.6?

If you haven’t finished the CA worksheet from yesterday take it out and continue to work on it during this time.

Name: ______________________________ Date:_______________ Period: _____ **Review**

Mid-Unit 8 Review – Applications of Integration

Lessons 8.1 through 8.6

Reviews do NOT cover all material from the lessons but will hopefully remind you of key points. To be prepared, you must study all packets from Unit 8.

Average Rate of Change	Mean Value Theorem	Average Value of a Function
$\frac{f(b)-f(a)}{b-a}$	$f'(c)=\frac{f(b)-f(a)}{b-a}$	$\frac{1}{b-a}\int_a^b f(x)\,dx$

Find the average value of each function on the given interval.

1. $f(x) = x^3$ on $[0, 2]$

2. $f(x) = \frac{1}{x}$ on $[1, e]$

$$\int \text{rate of change} =$$

$\int \text{velocity} =$	$\int \lvert\text{velocity}\rvert =$

3. A particle's velocity is given by $v(t) = 6t^2 - 18t + 12$, where t is measured in seconds, v is measured in feet per second, and $s(t)$ represents the particle's position.

(a) If $s(1) = 3$, what is the value of $s(2)$?

(b) What is the net change in distance over the first 3 seconds?

(c) What is the total distance traveled by the particle during the first 2 seconds? Show the set up AND your answer.

4. A particle moves along a coordinate line. Its acceleration function is $a(t) = 6t - 22$ for $t \geq 0$. If $v(0) = 24$ find the velocity at $t = 4$.

5. A particle's velocity is given by $v(t) = \cos t$, where t is measured in months, v is measured in kilometers per month, and $s(t)$ represents the particle's position.

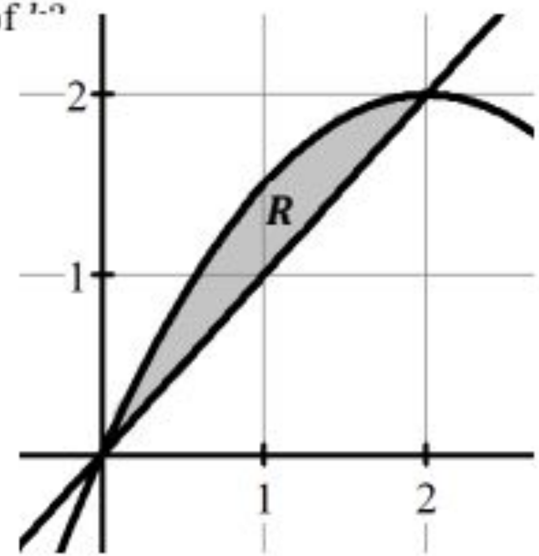
(a) If $s\left(\frac{\pi}{6}\right) = 10$, what is the value of $s\left(\frac{3\pi}{2}\right)$?

(b) What is the net change in distance over the first π months?

(c) What is the total distance traveled by the particle during the first π months? Show the set up AND your answer.

6. Find the area between the two curves $y = x^2 - 4$ and $y = 2 - x$.

7. **Calculator active.** Let R be the region bounded by the graphs $y = 2x - \frac{1}{2}x^2$ and $y = x$ as sown in the figure. If the line $x = k$ divides R into two regions of equal area, what is the value of



8. **Calculator active.** A 10,000-liter tank of water is filled to capacity. At time $t = 0$, water begins to drain out of the tank at a rate modeled by $r(t)$, measured in liters per hour, where r is given by the piecewise-defined function

$$r(t) = \frac{400t}{t+2} \text{ for } 0 \leq t \leq 6.$$

a. Find $\int_0^6 r(t)\,dt$

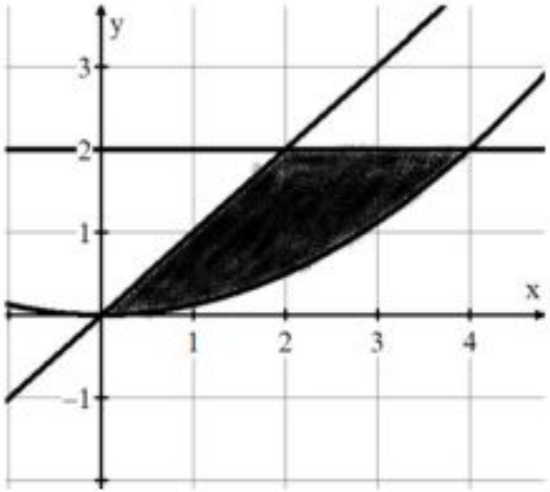
b. Explain the meaning of your answer to part *a* in the context of this problem.

c. Write, but do not solve, an equation involving an integral to find the time *A* when the amount of water in the tank is 8,000 liters.

Set up the integral(s) that give the area of the region bounded by the given equations. Show the equivalent set up with respect to x as well as with respect to y.

9. $y = x,\ \ y = \frac{x^2}{8},\ \ y = 2$

with respect to x



with respect to y

Notes Filled In:

AP Calc. AB/BC - Mid-Unit 8 Review - Filled In

Practice - Test Prep.

Take the next 5-10 minutes to work together on the practice - test prep section of our notes.

We will go through it together on the board after the time is up!

Name: ______________________________ Date:_______________

Corrective Assignment

Mid-Unit 8 CA – Applications of Integration

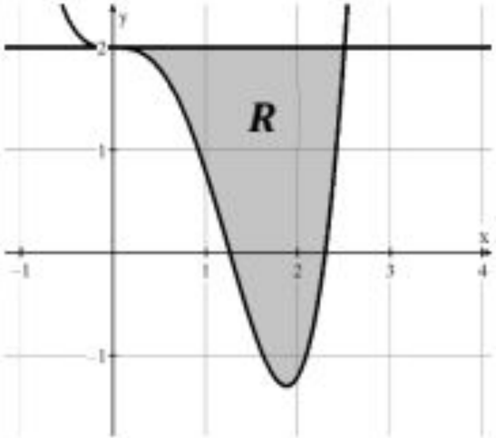
Find the average value of the function over the given interval.

1. $f(x) = \frac{10}{x^2};\ [1,5]$

2. **Calculator active.** $f(x) = e^{2x}\cos(x);\ [-1,4]$

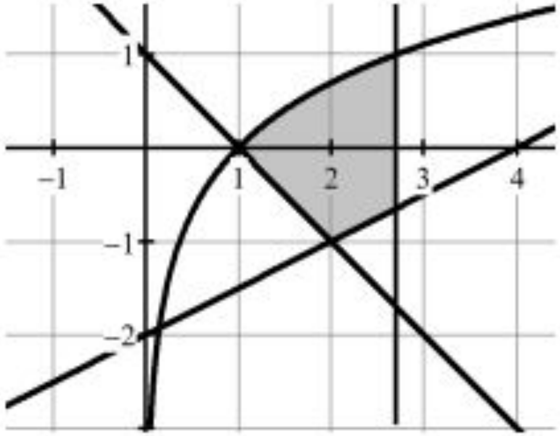
3. Find the area of the region bounded by the graphs $y = x^2, y = -x, x = 0$, and $x = 2$.

4. **Calculator active.** Let R be the region bounded by the graphs $y = 0.8x^4 - 2x^3 + 2$ and $y = 2$ as sown in the figure. If the line $x = k$ divides R into two regions of equal area, what is the value of k?

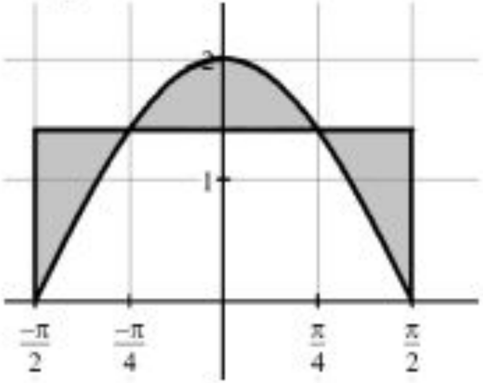


5. Set up the integral(s), with respect to x, that represent the area of the bounded region. Do not solve.

$$y = \ln x,\ y = 1 - x,\ y = \frac{1}{2}x - 2, \text{ and } x = e$$

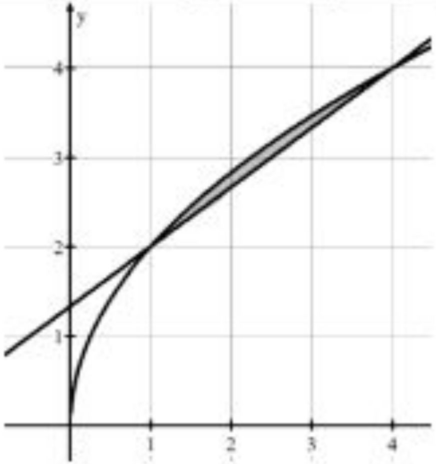


6. The figure shows the graph of $y = 2\cos(x)$, and the line $y = \sqrt{2}$, for $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$. Write a set of integrals that represents the sum of all the areas of the shaded regions. Use exact values for your boundaries, not rounded decimals.

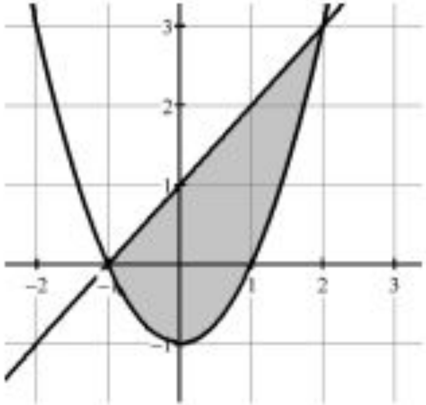


Set up the integral(s), with respect to y, that represent the area of the shaded region.

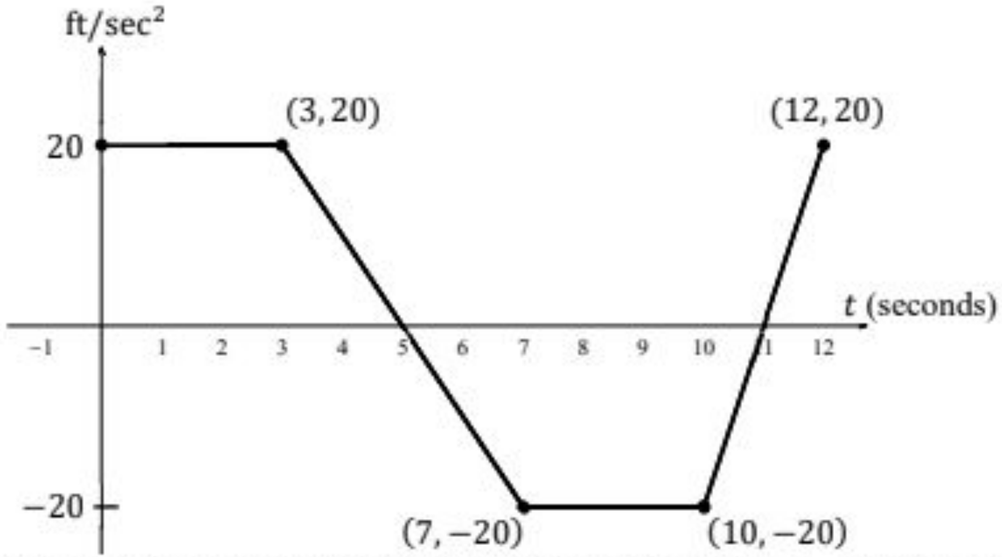
7. $x = \frac{y^2}{4},\ x = \frac{3}{2}y - 2$



8. $y = x^2 - 1,\ y = x + 1$

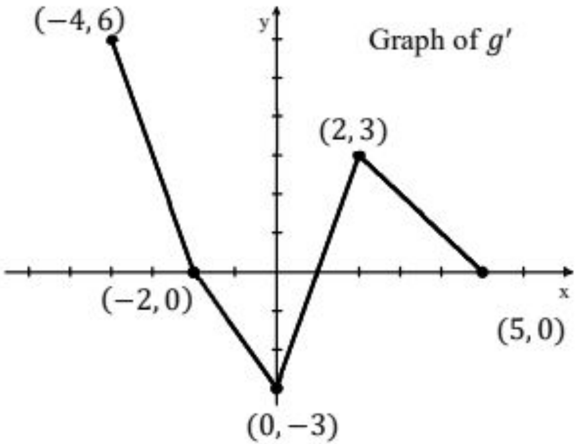


9.



A car is traveling on a straight road with velocity 80 ft/sec at time $t = 0$. For $0 \leq t < 12$ seconds, the car's acceleration $a(t)$, in ft/sec^2, is the piecewise linear function defined by the graph above. On the time interval $0 \leq t < 12$, what is the car's absolute maximum velocity, in ft/sec, and at what timed does it occur? Justify your answer.

10. Let g be a continuous function with $g(-2) = 4$. The graph of the piecewise-linear function $g'(x)$, the derivative of g, is shown for $-4 \le x \le 5$.



a. Find the x-coordinate of all points of inflections of the graph $y = g(x)$ for $-4 \le x \le 5$.

b. Find the absolute minimum value of g on the interval $-4 \le x \le 5$. Justify your answer.

c. Find the average rate of change of $g'(x)$ on the interval $-4 \le x \le 5$.

d. Find the average rate of change of $g(x)$ on the interval $-4 \le x \le 5$.

11. When a grocery store opens, it has 80 pounds of apples on a table for customers to purchase. Customers remove apples from the table at a rate modeled by $f(t) = 8 + (0.7t)\cos\left(\frac{t^3}{50}\right)$ for $0 < t \leq 10$ where $f(t)$ is measured in pounds per hour and t is the number of hours after the store opened. What amount of apples are there 4 hours after the store opens?

12. At $t = 0$ a particle starts at rest and moves along a line in such a way that at time t its acceleration is $18t^2$ feet per second per second. Through how many feet does the particle move during the first 2 seconds?

ANSWERS to Mid-Unit 8 Corrective Assignment

1. 2

2. -246.1198

3. $\int_0^2 (x^2+x)\,dx = \frac{14}{3}$

4. $\int_0^k -0.8x^4 + 2x^3\,dx = \int_k^{2.5} -0.8x^4 + 2x^3\,dx$
   $k \approx 1.715$

5. $\int_1^2 (\ln x - 1 + x)\,dx + \int_2^e \left(\ln x + 2 - \frac{1}{2}x\right)dx$

6. $\int_{-\frac{\pi}{2}}^{-\frac{\pi}{4}} (\sqrt{2} - 2\cos x)\,dx + \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (2\cos x - \sqrt{2})\,dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\sqrt{2} - 2\cos x)\,dx$

7. $\int_2^4 \left(\frac{3}{2}y - 2 - \frac{y^2}{4}\right)dy$

8. $\int_{-1}^0 \left(2\sqrt{y+1}\right)dy + \int_0^3 \left(\sqrt{y+1} - y + 1\right)dy$

9. The absolute maximum must occur at $t = 5$ or at an endpoint.
   $v(5) = 80 + \int_0^5 a(t)\,dt = 160$ ft/sec.
   $\int_5^{12} a(t)\,dt < 0$, so $v(12) < v(5)$
   Therefore, the absolute maximum velocity occurs at $t = 5$.

10a. g' changes from decreasing to increasing at $x = 0$.
   g' changes from increasing to decreasing at $x = 2$.

10b. The only sign change of g' from negative to positive in the interval is at $x = 1$.
   $g(-4) = 4 + \int_{-2}^{-4} g'(x)\,dx = 4 + (-6) = -2$
   $g(1) = 4 + \int_{-2}^{1} g'(x)\,dx = 4 + (-3) + \left(-\frac{3}{2}\right) = -\frac{1}{2}$
   $g(5) = -\frac{1}{2} + \int_1^5 g'(x)\,dx = -\frac{1}{2} + \frac{1}{2} + \frac{9}{2} = \frac{9}{2}$
   The minimum value of g for $-4 \le x \le 5$ is -2.

10c. $\frac{g'(5)-g'(-4)}{5--4} = \frac{0-6}{9} = -\frac{2}{3}$

10d. $\frac{g(5)-g(-4)}{5-(-4)} = \frac{\left[\frac{11}{2}\right]-[-2]}{9} = \frac{\frac{11}{2}+\frac{4}{2}}{9} = \frac{15}{18} = \frac{5}{6}$

11. 43.461 pounds of apples

12. 24