Name: Solutions Date:______________ Period: _____ **Review**

End-of-Unit 6 Review – Integration and Accumulation of Change

Lessons 6.6 through 6.14

Reviews do NOT cover all material from the lessons but will hopefully remind you of key points. To be prepared, you must study all packets from Unit 6.

Find the value of the definite integral.

1. $\int_{-2}^{-1}\left(\frac{1}{x^2}+x^2-5x\right)dx$

$-\frac{1}{x}+\frac{x^3}{3}-\frac{5x^2}{2}\Big|_{-2}^{-1}$

$\left[-\frac{1}{(-1)}+\frac{-1}{3}-\frac{5}{2}\right]-\left[-\frac{1}{(-2)}-\frac{8}{3}-\frac{20}{2}\right]$

$\left[1-\frac{1}{3}-\frac{5}{2}\right]-\left[\frac{1}{2}-\frac{8}{3}-10\right]$

$11+\frac{7}{3}-\frac{6}{2}$

$8+\frac{7}{3}$

$\frac{24}{3}+\frac{7}{3}=\boxed{\frac{31}{3}}$

2. $\int_{-1}^{8}\left(x^{2/3}-x\right)dx$

$\frac{x^{5/3}}{5/3}-\frac{x^2}{2}\Big|_{-1}^{8}$

$\frac{3}{5}\left(\sqrt[3]{x}\right)^5-\frac{1}{2}x^2\Big|_{-1}^{8}$

$\left[\frac{3}{5}(2)^5-32\right]-\left[-\frac{3}{5}-\frac{1}{2}\right]$

$\left[\frac{96}{5}-32\right]-\left[-\frac{6}{10}-\frac{5}{10}\right]$

$\frac{192}{10}-\frac{320}{10}+\frac{11}{10}$

$\boxed{-\frac{117}{10}}$

3. $\int_0^{\pi}(x-\sin x)\,dx$

$\frac{x^2}{2}+\cos x\Big|_0^{\pi}$

$\left[\frac{\pi^2}{2}+(-1)\right]-\left[0+1\right]$

$\frac{\pi^2}{2}-1-1$

$\boxed{\frac{\pi^2}{2}-2}$

4. $\int_{-1}^{1}x\sqrt{1-x^2}\,dx$

$u=1-x^2$

$\frac{du}{-2x}=dx$

$\int_0^0 x\sqrt{u}\,\frac{du}{-2x}$

$-\frac{1}{2}\int_0^0\sqrt{u}\,du$

$\boxed{0}$

lower bound = upper bound

5. $\int_0^{\frac{\pi}{6}}\frac{\sin(2x)}{\cos^2(2x)}dx$

$u=\cos(2x)$

$du=-\sin(2x)\cdot 2dx$

$\frac{du}{-2\sin(2x)}=dx$

$\int_1^{\frac{1}{2}}\frac{\sin(2x)}{u^2}\left(\frac{du}{-2\sin(2x)}\right)$

$-\frac{1}{2}\int_1^{\frac{1}{2}}u^{-2}\,du$

$-\frac{1}{2}\left[\frac{u^{-1}}{-1}\right]\Big|_1^{\frac{1}{2}}$

$\frac{1}{2}\left[\frac{1}{\frac{1}{2}}-\frac{1}{1}\right]=\frac{1}{2}[1]=\boxed{\frac{1}{2}}$

6. $\int_e^{e^2}\frac{1}{x\ln x}dx$

$u=\ln x$

$du=\frac{1}{x}dx$

$x\,du=dx$

$\int_1^2\frac{1}{xu}\cdot(x\,du)$

$\int_1^2\frac{1}{u}\,du$

$\ln|u|\Big|_1^2$

$\ln 2-\ln 1$

$\boxed{\ln 2}$

7. If $\int_{-5}^{2}f(x)dx=-17$ and $\int_5^2 f(x)dx=4$, what is the value of $\int_{-5}^{5}f(x)dx$?

$\int_{-5}^{5}f(x)\,dx=\int_{-5}^{2}f(x)\,dx+\int_2^5 f(x)\,dx$

$=(-17)+(-4)$

(A) -21 (B) -13 (C) 0 (D) 13 (E) 21

Find the following indefinite integrals.

8. $\int \left(\frac{x^2-x+5}{x}\right) dx$

$\int \left(x-1+\frac{5}{x}\right) dx$

$\boxed{\frac{x^2}{2}-x+5\ln x + C}$

9. $\int \sec x \tan x \, dx$

$\boxed{\sec x + C}$

10. $\int \frac{2x}{3} \ln 4x \, dx$

Int. by Parts

$f = \ln(4x) \qquad g' = \frac{2x}{3}$

$f' = \frac{1}{x} \qquad g = \frac{2}{3}\frac{x^2}{2}$

$\frac{x^2}{3}\ln(4x) - \int \frac{1}{3} x \, dx$

$\boxed{\frac{x^2}{3}\ln(4x) - \frac{1}{6}x^2 + C}$

11. $\int \sqrt{x}\left(x-\frac{4}{x}\right) dx$

$\int x^{3/2} - 4x^{-1/2} \, dx$

$\frac{x^{5/2}}{5/2} - \frac{4x^{1/2}}{1/2} + C$

$\boxed{\frac{2}{5}x^{5/2} - 8\sqrt{x} + C}$

12. $\int \frac{50x^3-55x^2-26x+33}{10x-7} dx$

$$\begin{array}{r|l} & 5x^2-2x-4+\frac{5}{10x-7} \\ 10x-7 & 50x^3-55x^2-26x+33 \\ & -(50x^3-35x^2) \\ & -20x^2-26x+33 \\ & -(-20x^2+14x) \\ & -40x+33 \\ & -(-40x+28) \\ & 5 \end{array}$$

$\int 5x^2-2x-4+\frac{5}{10x-7} dx$

$u = 10x-7$

$\frac{du}{10} = dx$

$\frac{5x^3}{3} - \frac{2x^2}{2} - 4x + \frac{5}{10}\ln|10x-7| + C$

$\boxed{\frac{5}{3}x^3 - x^2 - 4x + \frac{1}{2}\ln|10x-7| + C}$

13. $\int_0^6 \frac{1}{\sqrt{6-x}} dx$

Improper Integral

$\lim_{t\to 6^-} \int_0^t \frac{1}{\sqrt{6-x}} dx$

$u = 6-x$

$\frac{du}{-1} = dx$

$\lim_{t\to 6^-} \int_6^{6-t} u^{-1/2} \frac{du}{-1}$

$\lim_{t\to 6^-} -2u^{1/2} \Big|_6^{6-t}$

$\lim_{t\to 6^-} \left[-2\sqrt{6-t}\right] - \left[-2\sqrt{6}\right]$

$0 + 2\sqrt{6}$

$\boxed{2\sqrt{6}}$

14. $\int (e^x + 2^x)\,dx$

$$\boxed{e^x + \frac{1}{\ln 2} 2^x + C}$$

15. $\int \left(\frac{1}{x} + \frac{1}{x^3}\right) dx$

$$\ln|x| + \frac{x^{-2}}{-2} + C$$

$$\boxed{\ln|x| - \frac{1}{2x^2} + C}$$

16. $\int x^2 e^x\,dx$

Integration by parts

f		g'
x^2	$+$	e^x
$2x$	$-$	e^x
2	$+$	e^x
0		e^x

$$\boxed{x^2e^x - 2xe^x + 2e^x + C}$$

17. $\int \frac{1}{x^2+6x+8}\,dx$

Linear Partial Fr.

$$\int \frac{1}{(x+4)(x+2)}\,dx \qquad \frac{1}{(x+4)(x+2)} = \frac{A}{x+4} + \frac{B}{x+2}$$

$$1 = A(x+2) + B(x+4)$$

Let $x=-4$

$1 = -2A$

$-\frac{1}{2} = A$

Let $x=-2$

$1 = 2B$

$\frac{1}{2} = B$

$$\int \frac{-\frac{1}{2}}{x+4} + \frac{\frac{1}{2}}{x+2}\,dx$$

$$-\frac{1}{2}\ln|x+4| + \frac{1}{2}\ln|x+2| + C$$

$$\boxed{\frac{1}{2}\ln\left|\frac{x+2}{x+4}\right| + C}$$

18. $\int_0^\infty \frac{1}{9+x^2}\,dx$

$u = \frac{x}{3}$

$3du = dx$

$$\lim_{t\to\infty} \int_0^t \frac{\frac{1}{9}}{1+\left(\frac{x}{3}\right)^2}\,dx$$

$$\lim_{t\to\infty} \frac{1}{9}\int_0^{t/3} \frac{1}{1+u^2}\,(3du)$$

$$\lim_{t\to\infty} \frac{1}{3}\left[\tan^{-1} u\right]_0^{t/3}$$

$$\frac{1}{3}\left[\tan^{-1}(\infty) - \tan^{-1}(0)\right]$$

$$\frac{1}{3}\left[\frac{\pi}{2} - 0\right]$$

$$\boxed{\frac{\pi}{6}}$$

19. $\int \frac{1}{x^2+2x+2}\,dx$

Complete the square

$$(x^2+2x+1)+2-1$$

$$\int \frac{1}{(x+1)^2+1}\,dx$$

$$\boxed{\tan^{-1}(x+1) + C}$$

20. **Calculator active problem.** If $f'(x) = \sin(e^x)$ and $f(0) = 5.7$, then $f(2) =$

$$\boxed{5.7 + \int_0^2 \sin(e^x)\,dx \approx 6.2509}$$