Name: _______________________________ Date:_______________ **Corrective Assignment**

Mid-Unit 8 CA – Applications of Integration

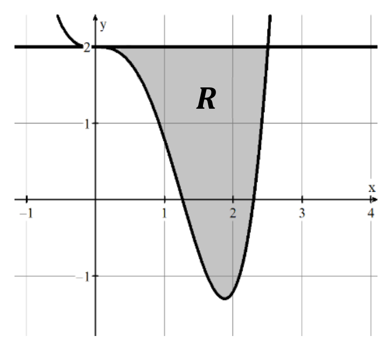
Find the average value of the function over the given interval.

1. $f(x) = \frac{10}{x^2}$; $[1, 5]$

2. **Calculator active.** $f(x) = e^{2x} \cos(x)$; $[-1, 4]$

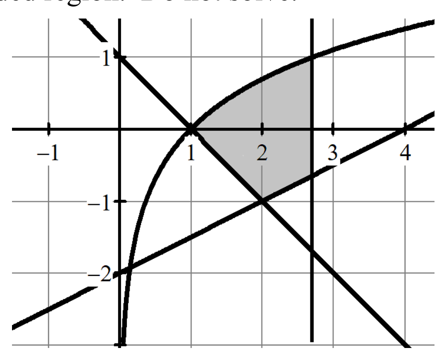
3. Find the area of the region bounded by the graphs $y = x^2$, $y = -x$, $x = 0$, and $x = 2$.

4. **Calculator active.** Let R be the region bounded by the graphs $y = 0.8x^4 - 2x^3 + 2$ and $y = 2$ as sown in the figure. If the line $x = k$ divides R into two regions of equal area, what is the value of k?

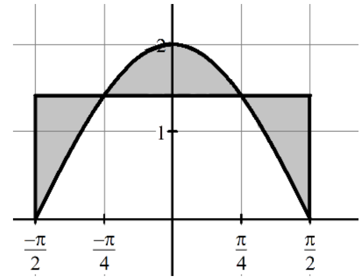


5. Set up the integral(s), with respect to x, that represent the area of the bounded region. Do not solve.

$$y = \ln x,\ y = 1 - x,\ y = \frac{1}{2}x - 2, \text{ and } x = e$$

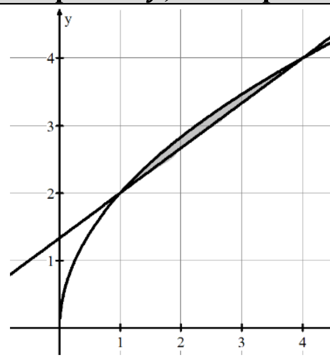


6. The figure shows the graph of $y = 2\cos(x)$, and the line $y = \sqrt{2}$, for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$. Write a set of integrals that represents the sum of all the areas of the shaded regions. Use exact values for your boundaries, not rounded decimals.

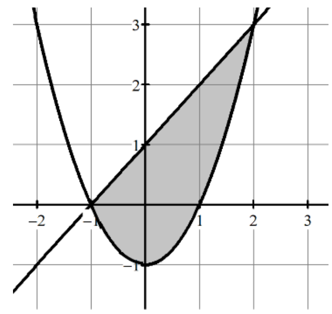


Set up the integral(s), with respect to y, that represent the area of the shaded region.

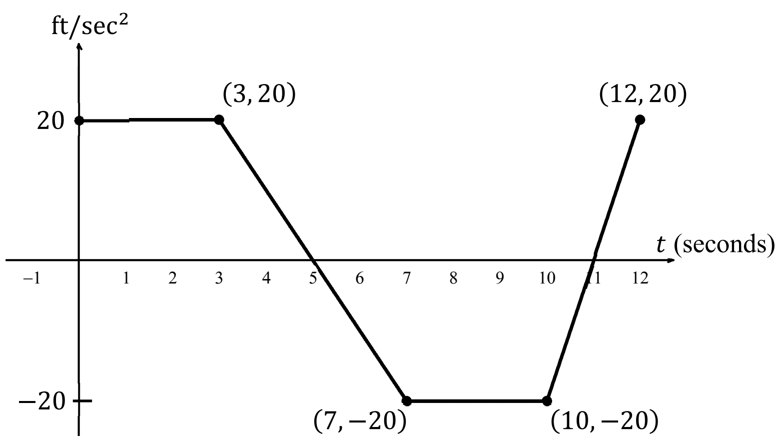
7. $x = \frac{y^2}{4}$, $x = \frac{3}{2}y - 2$



8. $y = x^2 - 1$, $y = x + 1$

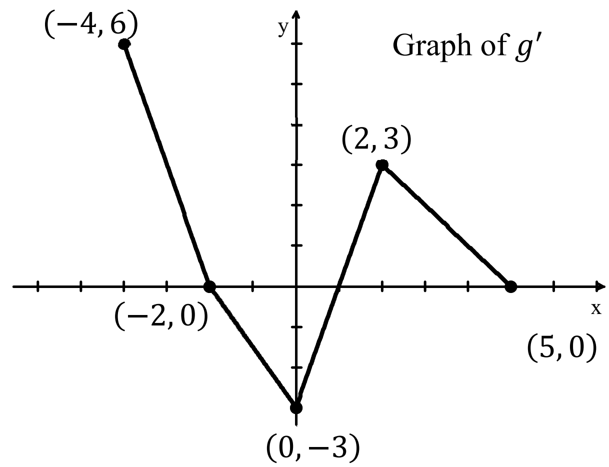


9.



A car is traveling on a straight road with velocity 80 ft/sec at time $t = 0$. For $0 \le t < 12$ seconds, the car's acceleration $a(t)$, in ft/sec^2, is the piecewise linear function defined by the graph above. On the time interval $0 \le t < 12$, what is the car's absolute maximum velocity, in ft/sec, and at what timed does it occur? Justify your answer.

10. Let g be a continuous function with $g(-2) = 4$. The graph of the piecewise-linear function $g'(x)$, the derivative of g, is shown for $-4 \leq x \leq 5$.



a. Find the x-coordinate of all points of inflections of the graph $y = g(x)$ for $-4 \leq x \leq 5$.

b. Find the absolute minimum value of g on the interval $-4 \leq x \leq 5$. Justify your answer.

c. Find the average rate of change of $g'(x)$ on the interval $-4 \leq x \leq 5$.

d. Find the average rate of change of $g(x)$ on the interval $-4 \leq x \leq 5$.

11. When a grocery store opens, it has 80 pounds of apples on a table for customers to purchase. Customers remove apples from the table at a rate modeled by $f(t) = 8 + (0.7t)\cos\left(\frac{t^3}{50}\right)$ for $0 < t \leq 10$ where $f(t)$ is measured in pounds per hour and t is the number of hours after the store opened. What amount of apples are there 4 hours after the store opens?

12. At $t = 0$ a particle starts at rest and moves along a line in such a way that at time t its acceleration is $18t^2$ feet per second per second. Through how many feet does the particle move during the first 2 seconds?

ANSWERS to Mid-Unit 8 Corrective Assignment

1. 2

2. -246.1198

3. $\int_0^2 (x^2 + x)\,dx = \frac{14}{3}$

4. $\int_0^k -0.8x^4 + 2x^3\,dx = \int_k^{2.5} -0.8x^4 + 2x^3\,dx$
   $k \approx 1.715$

5. $\int_1^2 (\ln x - 1 + x)\,dx + \int_2^e \left(\ln x + 2 - \frac{1}{2}x\right)dx$

6. $\int_{-\frac{\pi}{2}}^{-\frac{\pi}{4}} \left(\sqrt{2} - 2\cos x\right)dx + \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \left(2\cos x - \sqrt{2}\right)dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left(\sqrt{2} - 2\cos x\right)dx$

7. $\int_2^4 \left(\frac{3}{2}y - 2 - \frac{y^2}{4}\right)dy$

8. $\int_{-1}^0 \left(2\sqrt{y+1}\right)dy + \int_0^3 \left(\sqrt{y+1} - y + 1\right)dy$

9. The absolute maximum must occur at $t = 5$ or at an endpoint.
   $v(5) = 80 + \int_0^5 a(t)\,dt = 160$ ft/sec.
   $\int_5^{12} a(t)\,dt < 0$, so $v(12) < v(5)$
   Therefore, the absolute maximum velocity occurs at $t = 5$.

10a. g' changes from decreasing to increasing at $x = 0$.
   g' changes from increasing to decreasing at $x = 2$.

10b. The only sign change of g' from negative to positive in the interval is at $x = 1$.
   $g(-4) = 4 + \int_{-2}^{-4} g'(x)\,dx = 4 + (-6) = -2$
   $g(1) = 4 + \int_{-2}^{1} g'(x)\,dx = 4 + (-3) + \left(-\frac{3}{2}\right) = -\frac{1}{2}$
   $g(5) = -\frac{1}{2} + \int_1^5 g'(x)\,dx = -\frac{1}{2} + \frac{1}{2} + \frac{9}{2} = \frac{9}{2}$
   The minimum value of g for $-4 \le x \le 5$ is -2.

10c. $\frac{g'(5) - g'(-4)}{5—4} = \frac{0-6}{9} = -\frac{2}{3}$

10d. $\frac{g(5) - g(-4)}{5-(-4)} = \frac{\left[\frac{11}{2}\right] - [-2]}{9} = \frac{\frac{11}{2} + \frac{4}{2}}{9} = \frac{15}{18} = \frac{5}{6}$

11. 43.461 pounds of apples

12. 24