Name: Solutions Date:_______________ Period: _____

Review

Unit 3 REVIEW – Composite, Implicit, and Inverse Functions

Reviews do NOT cover all material from the lessons but should remind you of key points. To be prepared, you should review all packets from Unit 3.

Find the derivative.

1. $h(x) = \cos^2(4x)$

$$h'(x) = 2\cos(4x) \cdot (-\sin(4x)) \cdot 4$$

$$\boxed{h'(x) = -8\cos(4x)\sin(4x)}$$

2. $y = \ln\sqrt{x+3}$

$$y' = \frac{1}{\sqrt{x+3}} \cdot \frac{1}{2\sqrt{x+3}}$$

$$\boxed{y' = \frac{1}{2x+6}}$$

3. $x^2 + 2y^5 = 10xy$

$$2x + 10y^4\frac{dy}{dx} = 10y + 10x\frac{dy}{dx}$$

$$\frac{dy}{dx}(10y^4 - 10x) = 10y - 2x$$

$$\boxed{\frac{dy}{dx} = \frac{5y-x}{5y^4-5x}}$$

4. $y = \csc^{-1}(x^3)$

$$\frac{dy}{dx} = -\frac{1}{|x^3|\sqrt{x^6-1}} \cdot (3x^2)$$

$$\boxed{\frac{dy}{dx} = -\frac{3}{|x|\sqrt{x^6-1}}}$$

For each problem, let f and g be differentiable functions where $g(x) = f^{-1}(x)$ for all x.

5. $f(6) = -1$, $f(4) = -2$, $f'(6) = 3$, and $f'(4) = 7$. What is the value of $g'(-1)$?

$$g'(-1) = \frac{d}{dx}f^{-1}(-1) = \frac{1}{f'(f^{-1}(-1))}$$

$$= \frac{1}{f'(6)}$$

$$= \boxed{1/3}$$

6. Let f be the function defined by $f(x) = x^3 + 3x + 1$. Let $g(x) = f^{-1}(x)$, where $g(-3) = -1$. What is the value of $g'(-3)$?

$$g'(-3) = \frac{d}{dx}f^{-1}(-3) = \frac{1}{f'(f^{-1}(-3))}$$

$f'(x) = 3x^2 + 3$

$f'(-1) = 6$

$$= \frac{1}{f'(-1)}$$

$$= \boxed{1/6}$$

Find $\frac{d^2y}{dx^2}$ based on the given information.

7. $y = x^5 - e^{4x}$

$$\frac{dy}{dx} = 5x^4 - e^{4x} \cdot 4$$

$$\frac{d^2y}{dx^2} = 20x^3 - e^{4x} \cdot 4 \cdot 4$$

$$\boxed{\frac{d^2y}{dx^2} = 20x^3 - 16e^{4x}}$$

8. $y = y^2 + x$

$$\frac{dy}{dx} = 2y\frac{dy}{dx} + 1$$

$$\frac{dy}{dx}(1-2y) = 1$$

$$\frac{dy}{dx} = \frac{1}{(1-2y)} = (1-2y)^{-1}$$

$$\frac{d^2y}{dx^2} = -1(1-2y)^{-2} \cdot \left(-2\frac{dy}{dx}\right)$$

$$= -\frac{1}{(1-2y)^2} \cdot \left(\frac{-2}{(1-2y)}\right) = \boxed{\frac{2}{(1-2y)^3}}$$

9. Find the equation of the tangent line.
$x^2 + 7y^2 = 8y^3$ at $(-6, 2)$

$$2x + 14y\frac{dy}{dx} = 24y^2\frac{dy}{dx}$$

$$2(-6) + 14(2)\frac{dy}{dx} = 24(4)\frac{dy}{dx}$$

$$-12 + 28\frac{dy}{dx} = 96\frac{dy}{dx}$$

$$-12 = 68\frac{dy}{dx}$$

$$-\frac{3}{17} = \frac{dy}{dx}$$

$$\boxed{y - 2 = -\frac{3}{17}(x+6)}$$

10. If $x = y^2 - \cos x$ find $\frac{d^2y}{dx^2}$ at $(0, -1)$.

$$1 = 2y\frac{dy}{dx} + \sin x$$

$$\frac{1-\sin x}{2y} = \frac{dy}{dx} \qquad \left.\frac{dy}{dx}\right|_{(0,-1)} = \frac{1-\sin(0)}{2(-1)} = \frac{1}{-2}$$

$$\frac{d^2y}{dx^2} = \frac{-\cos x(2y) - (1-\sin x)\left(2\frac{dy}{dx}\right)}{4y^2} \qquad \frac{dy}{dx} = -\frac{1}{2}$$

$$\left.\frac{d^2y}{dx^2}\right|_{(0,-1)} = \frac{-(1)(-2) - (1-0)(-1)}{4(-1)^2} = \frac{2+1}{4} = \boxed{\frac{3}{4}}$$