

Name: \_\_\_\_\_ Date: \_\_\_\_\_

**Unit 7 CA – Differential Equations (BC)**

1. The rate at which a project  $p(x)$  is completed is proportional to the square root of the number of employees  $x$  working on the project, where  $p$  is measured as a percent of the project that has been completed. If 5 people can complete the project at a rate of 3% per day, what is a differential equation that models this situation?

**Find the general solution of the differential equation.**

2.  $\frac{dy}{dx} = (x + 7)y$

3.  $\frac{dy}{dx} = -xy^3$

4. A population  $y$  grows according to the equation  $\frac{dy}{dt} = ky$ , where  $k$  is a constant and  $t$  is measured in years. If the population doubles every 14 years, then what is the value of  $k$ ?
5. A dose of 500 milligrams of a drug is administered to a patient. The amount of the drug, in milligrams, in the person's bloodstream at time  $t$ , in hours, is given by  $A(t)$ . The rate at which the drug leaves the bloodstream can be modeled by the differential equation  $\frac{dA}{dt} = -0.8A$ . Write an expression for  $A(t)$ .

6. Consider the differential equation  $\frac{dy}{dx} = (1 - 2x)y$ . If  $y = 10$  when  $x = 1$ , find an equation for  $y$ .

(A)  $y = e^{x-x^2}$

(B)  $y = 10 + e^{x-x^2}$

(C)  $y = e^{x-x^2+10}$

(D)  $y = 10e^{x-x^2}$

(E)  $y = x - x^2 + 10$

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7. The solution to the differential equation  $\frac{dy}{dx} = \frac{x}{\cos y}$  with the initial condition  $y(1) = 0$  is

(A)  $y = \sin^{-1}\left(\frac{x^2-1}{2}\right)$

(B)  $y = \sin^{-1}\left(\frac{x^2}{2}\right)$

(C)  $y = \cos^{-1}(x^2 - 1)$

(D)  $y = \ln[\cos(x - 1)]$

(E)  $y = \ln(\sin x)$

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8. If  $\frac{dy}{dx} = \frac{3x^2+2}{y}$  and  $y = 4$  when  $x = 2$ , then when  $x = 3$ ,  $y =$

(A) 18

(B)  $\pm\sqrt{66}$

(C) 58

(D)  $\pm\sqrt{74}$

(E)  $\pm\sqrt{58}$

**For each differential equation, find the particular solution that passes through the given point.**

9.  $\frac{dy}{dx} = 9e^{3x} - \cos x$ ;  $(0, 2)$

10.  $\frac{dy}{dx} = 4y$  and  $y = 8$  when  $x = 0$

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11.  $\frac{d^2y}{dx^2} = \cos(2x) + 1$  and  $y'(\pi) = 0$  and  $y(0) = 1$

12. For what value of  $k$ , if any, is  $y = e^{3x} + ke^{-4x}$  a solution to the differential equation  $y'' - 3y' = 7e^{-4x}$ ?

13. The table below gives the values of  $f'$ , the derivative of  $f$ . If  $f(1.3) = 1.7$ , what is the approximation to  $f(2.2)$  obtained by using Euler's method with 3 steps of equal size?

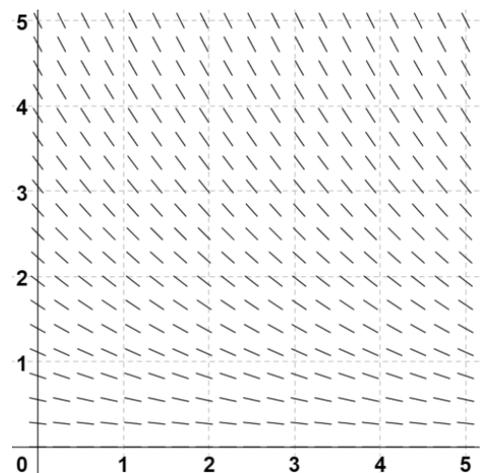
$x$	1.3	1.6	1.9	2.2
$f'(x)$	0.1	0.3	0.6	1.1

14. Let  $y = f(x)$  be the solution to the differential equation  $\frac{dy}{dx} = 2y - x$  with initial condition  $f(1) = 3$ . What is the approximation for  $f(2)$  obtained using Euler's method with 2 steps of equal length, starting at  $x = 1$ ?

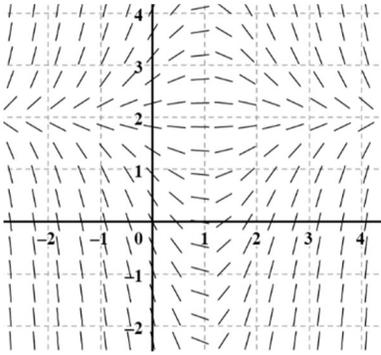
15. A population's rate of growth is modeled by the logistic differential equation  $\frac{dP}{dt} = \frac{1}{1000}P(500 - P)$ , where  $t$  is in weeks and  $P(0) = 10$ . What is the greatest rate of change for this population?

16. Using the logistic differential equation  $\frac{dP}{dt} = 0.2P - 0.001P^2$ , identify the carrying capacity.

17. Explain why the following slope field cannot represent the differential equation  $\frac{dy}{dt} = 0.4y$



18.



(A)  $\frac{dy}{dx} = y - 2x$       (D)  $\frac{dy}{dx} = xy^2$

(B)  $\frac{dy}{dx} = 1 + x + y$       (E)  $\frac{dy}{dx} = (x - 1)y^2$

(C)  $\frac{dy}{dx} = (1 - x)(y - 2)$

### Answers to Unit 7 Corrective Assignment (BC)

1. $\frac{dp}{dx} = 1.3416\sqrt{x}$	2. $y = Ce^{\frac{1}{2}x^2 + 7x}$	3. $y = \pm \sqrt{\frac{1}{x^2 + c}}$	4. $k \approx 0.0495$	5. $A(t) = 500e^{-0.8t}$
6. D	7. A	8. E	9. $y = 3e^{3x} - \sin x - 1$	10. $y = 8e^{4x}$
11. $y = -\frac{1}{4}\cos(2x) + \frac{1}{2}x^2 - \pi x + \frac{5}{4}$	12. $k = \frac{1}{4}$	13. $f(2.2) = 2$	14. $f(2.0) \approx 10.25$	
15. 62.5/week	16. 200	17. $\frac{dy}{dx} > 0$ when $y > 0$ , but the slope field shows line segments with negative slope.		18. C