Chapter 7 Exponential and Logarithmic Functions

Exponential Functions

Parent, $b > 0, b \neq 1$	$y = b^x$
Reflection across x-axis	$y = -b^x$
Stretch $(a > 1)$ Shrink $(0 < a < 1)$	$y = ab^x$
Translation horizontal by h vertical by k	$y = b^{x-h} + k$

The function $y = ab^x, a > 0, b > 1,$ models exponential growth.

$y = ab^x$ models exponential decay if $0 < b < 1$.

$$A(t) = a(1 + r)^t$$

- $A(t)$: Amount after t time periods
- r: Rate of growth $(r > 0)$ or decay $(r < 0)$
- a: Initial amount
- t: Number of time periods

Continuously Compounded Interest

$A(t) = P \cdot e^{rt}$, where $A(t)$ represents the total, P represents the principal, r represents the interest rate, and t represents time in years.

Logarithms are exponents. In fact, $\log_b a = c$ if and only if $b^c = a$.

The exponential function $y = b^x$ and the logarithmic function $y = \log_b x$ are inverse functions.

Logarithmic Functions

	Base b	Base e
Parents, $b > 0, b \neq 1$	$y = \log_b x$	$y = \ln x$
Reflection across x-axis	$y = -\log_b x$	$y = -\ln x$
Stretch ($a > 1$) Shrink ($0 < a < 1$)	$y = a \log_b x$	$y = a \ln x$
Translation horizontal by h vertical by k	$y = \log_b (x - h) + k$	$y = \ln (x - h) + k$

Properties of Logarithms

For any positive numbers m, n, and b where $b \neq 1$

Product Property: $\log_b mn = \log_b m + \log_b n$

Quotient Property: $\log_b \frac{m}{n} = \log_b m - \log_b n$

Power Property: $\log_b m^n = n \log_b m$

Change of Base Formula

For any positive numbers, m, b, and c, with $b \neq 1$ and $c \neq 1$, $\log_b m = \frac{\log_c m}{\log_c b}$.

When $b = 10$, the logarithm is called a **common logarithm**, which you can write as $\log x$.

The inverse of $y = e^x$ is the **natural logarithmic function** $y = \log_e x = \ln x$. You solve natural logarithmic equations in the same way as common logarithmic equations.

An equation in the form $b^{cx} = a$, where the exponent includes a variable, is called an **exponential equation**. You can solve exponential equations by taking the logarithm of each side of the equation. An equation that includes one or more logarithms involving a variable is called a **logarithmic equation**.

All content from Algebra 2, 2011, Prentice Hall, Pearson.