Chapter 6 Radical Functions and Rational Exponents

Properties of Exponents

For any nonzero number a and any integers m and n,

$a^0 = 1$

$(ab)^n = a^n b^n$

$\dfrac{a^m}{a^n} = a^{m-n}$

$a^m \cdot a^n = a^{m+n}$

$a^{-n} = \dfrac{1}{a^n}$

$(a^m)^n = a^{mn}$

$\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$

nth Roots of nth Powers

For any real number a,

$$\sqrt[n]{a^n} = \begin{cases} a \text{ if } n \text{ is odd} \\ |a| \text{ if } n \text{ is even} \end{cases}$$

Properties of Rational Exponents

If the nth root of a is a real number and m is an integer, then $a^{\frac{1}{n}} = \sqrt[n]{a}$ and $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$. If m is negative, $a \neq 0$.

Like radicals have the same index and the same radicand. Use the distributive property to add and subtract them. Use the FOIL method to multiply binomial radical expressions.

Multiplication Property of Square Roots

For any numbers $a \geq 0$ and $b \geq 0$, $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$.

Division Property of Square Roots

For any numbers $a \geq 0$ and $b > 0$, $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$.

Combining Radical Expressions: Products

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers, then $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$.

Combining Radical Expressions: Quotients

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers and $b \neq 0$,

then $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$.

To **rationalize the denominator** of an expression, rewrite it so that the denominator contains no radical expressions.

To rationalize a denominator that is a square root binomial, multiply the numerator and denominator by the conjugate of the denominator.

Function Operations & Composite Functions

When performing function operations, you can use the same rules you used for real numbers, but you must take into consideration the domain and range of each function.

The composition of function g with function f is defined as $(g \circ f)(x) = g(f(x))$.

Inverse Functions

If a relation or a function is described by an equation in x and y, you can interchange x and y to get the inverse. The domain of a function becomes the range of its inverse, and the range of a function becomes the domain of its inverse.

Composition of Inverse Functions

If f and f^{-1} are inverse functions, then $(f^{-1} \circ f)(x) = x$ and $(f \circ f^{-1})(x) = x$ for x in the domains of f and f^{-1}, respectively.

Solving Radical Equations

To solve a **radical equation**, you must isolate a radical expression on one side of the equation. You can then rewrite the radical expression using a rational exponent and use the reciprocal of the exponent to solve the equation.

For example, to solve a square root equation, you square each side of the equation. Check all possible solutions in the original equation to eliminate extraneous solutions.

Radical Functions

	Square Root	nth Root
Parent	$y = \sqrt{x}$	$y = \sqrt[n]{x}$
Reflection across x-axis	$y = -\sqrt{x}$	$y = -\sqrt[n]{x}$
Stretch ($a > 1$) Shrink ($0 < a < 1$)	$y = a\sqrt{x}$	$y = a\sqrt[n]{x}$
Translation horizontal by h vertical by k	$y = \sqrt{x - h} + k$	$y = \sqrt[n]{x - h} + k$