

ALGEBRA 2 B – UNIT 6 – PROBABILITY & STATISTICS

Fundamental Counting Principle

If event M can occur in m ways and is followed by event N that can occur in n ways, then event M followed by event N can occur in $m \cdot n$ ways.

Definition: A permutation is an arrangement of items in a particular order.

Number of Permutations

The number of permutations of n items of a set arranged r items at a time is

$${}_n P_r = \frac{n!}{(n-r)!} \text{ for } 0 \leq r \leq n.$$

Using *factorial* notation, you can write $3 \cdot 2 \cdot 1$ as $3!$, read “three factorial.” For any positive integer n , *n factorial* is $n! = n(n-1) \cdot \dots \cdot 3 \cdot 2 \cdot 1$. The zero factorial is $0! = 1$.

Number of Combinations

The number of combinations of n items of a set chosen r items at a time is

$${}_n C_r = \frac{n!}{r!(n-r)!} \text{ for } 0 \leq r \leq n.$$

Definition: The theoretical probability of an event is the ratio of the number of ways that the event can occur to the total number of equally likely outcomes in the sample space.

probability

$$P(\text{event}) = \frac{\text{number of times the event occurs}}{\text{number of trials}}$$

Probability of A and B

If A and B are independent events, then

$$P(A \text{ and } B) = P(A) \cdot P(B).$$

Probability of A or B

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

If A and B are mutually exclusive events, then

$$P(A \text{ or } B) = P(A) + P(B).$$

Definition: A conditional statement is a sentence stating that the probability of one event depends on the occurrence of another event.

Conditional Probability

For any two events A and B with $P(A) \neq 0$, the probability of event B , given event A , is:

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

Measure	Definition
Mean	$\frac{\text{sum of the data values}}{\text{number of data values}}$
Median	For a data set listed in order: The middle values for an odd number of data values; the mean of the two middle values for an even number of data values
Mode	the most frequently occurring value(s)

A percentile is a number from 0 to 100 that you can associate with a value x from a data set. If x is at the 63rd percentile, then 63% of the data are less than or equal to x .

Mean, Variance, and Standard Deviation

$$\text{Mean: } \bar{x} = \frac{x_1 + x_2 + x_3 + \cdots + x_n}{n}$$

$$\text{Variance: } \sigma^2 = \frac{\sum(x - \bar{x})^2}{n}$$

$$\text{Standard deviation: } \sigma = \sqrt{\frac{\sum(x - \bar{x})^2}{n}}$$

- Find the mean, \bar{x} , of the n values in the data set.
- Find the difference, $x_i - \bar{x}$, between each value x_i and the mean.
- Square each difference, $(x_i - \bar{x})^2$.
- Find the average (mean) of these squares. This is the variance.

$$\sigma^2 = \frac{\sum(x - \bar{x})^2}{n}$$

- Take the square root of the variance. This is the standard deviation.

$$\sigma = \sqrt{\frac{\sum(x - \bar{x})^2}{n}}$$

Definition: Bias is an influence on someone or something in an unfair way.

Math Usage: A bias is a systematic error introduced by the sampling method of a survey.

Convenience sample Choose any members of the population who are conveniently and readily available.

Random sample All members of the population are equally likely to be chosen.

Self-selected sample Choose any members of the population who volunteer for the sample.

Systematic sample Order the population in some way, then select from it at regular intervals.

Binomial Probability

For repeated independent trials, each with a probability of success p and a probability of failure q (with $p + q = 1$), the probability of x successes in n trials is $P(x) = {}_n C_x p^x q^{n-x}$.

Binomial Theorem Using Combinations

For every positive integer n , use the combinations formula ${}_n C_r$ to expand $(a + b)^n$:

$$(a + b)^n = {}_n C_0 a^n + {}_n C_1 a^{n-1} b + {}_n C_2 a^{n-2} b^2 + \cdots + {}_n C_{n-1} a b^{n-1} + {}_n C_n b^n$$

Take note

Key Concept Normal Distribution

In a normal distribution,

- 68% of data fall within one standard deviation.
- 95% of data fall within two standard deviations.
- 99.7% of data fall within three standard deviations.

A normal distribution has a symmetric bell shape centered on the mean.

